EMPIRICAL PATTERNS OF FIRM GROWTH AND R&D INVESTMENT:

A QUALITY LADDER MODEL INTERPRETATION

Tor Jakob Klette
Zvi Griliches

THE INSTITUTE FOR FISCAL STUDIES
WP 25/99

Empirical patterns of firm growth and R&D investment: A quality ladder model interpretation

by

Tor Jakob Klette[*]
(University of Oslo and Statistics Norway)

and

Zvi Griliches
(Harvard University and NBER)

October 19, 1999

Abstract

We present a model of endogenous firm growth with R&D investment and stochastic innovation as the engines of growth. The model for firm growth is a partial equilibrium model drawing on the quality ladder models in the macro growth literature, but also on the literature on patent races and the discrete choice models of product differentiation. We examine to what extent the assumptions and the empirical content of our model are consistent with many of the the findings that have emerged from empirical studies of growth, productivity, R&D and patenting at the firm level. The analysis shows that the model fits well with a number of empirical patterns such as (i) a skewed size distribution of firms with persistent differences in firm sizes, (ii) firm growth independent of firm size, as stated in the so-called Gibrat's law, and (iii) R&D investment proportional to sales.

---

[*] Previous versions of this paper have been presented at workshops at Statistics Norway, Institute for Fiscal Studies (London), the "Economics and econometrics of innovation" conference in Strasbourg (June, 1996), the NBER Summer Institute 1996 and the "Innovation, competition and employment" conference at Crete (August, 1997). We have received a number of useful comments at those presentations and from Philippe Aghion, Petter Frenger, Per B. Maurseth, Tore Nilssen, Trond E. Olsen, Atle Seierstad and, in particular, Jarle Møen. The paper has also benefited from many suggestions from an anonymous referee. The research of Klette and Griliches was supported by the Norwegian Research Council (Nærings-LOS) and the Mellon Foundation, respectively.

1 Few theories on firm growth and firm heterogeneity

Empirical research on firm and plant level data has revealed a large amount of heterogeneity within narrowly defined industries. This heterogeneity is striking in a number of dimensions such as size (sales, employment), firm growth rates, rates of job creation and job destruction, and also in variables such as capital intensity and R&D intensity[1]. Much of the heterogeneity, e.g. in terms of sales and R&D investments, is quite persistent over a number of years. Recently, researchers have addressed the question of how we can reconcile this persistent heterogeneity with theories based on optimizing agents (see the survey by Sutton, 1997).

This paper presents a theoretical model with heterogeneous firms within an industry, where R&D investments and stochastic innovation are the engines of firm growth. The model presented is, for specific parameter values, consistent with at least three widely studied empirical regularities of R&D investments, firm sales and firm growth: (i) R&D intensities are independent of sales[2]. This result is derived from our model which treats R&D investments as a non-rival input in production. (ii) Firm growth is, to a first approximation, independent of size. This relationship is often referred to as Gibrat's law[3]. (iii) The size distribution of firms is highly skewed with persistent differences in firm sizes. This is true both for sales and other variables such as R&D investments. This third regularity is closely related to the second regularity, as has been emphasized by Simon and his co-workers (see Ijiri and Simon, 1977). Our contribution is to show that these three regularities can all be related and derived from a fully specified model of endogenous firm growth, based on optimizing agents. We examine whether the empirical content of the assumptions required to derive the model make sense in view of what has been learned from microeconometric research on innovation, patents and R&D, and we also consider a number of additional empirical implications that we derive from the model.

Our objective is to develop a framework for empirical studies incorporating the observed heterogeneity across firms in R&D investments and various measures of performance, e.g. productivity, patenting, profitability and sales growth. Currently, econometric studies of R&D investments and firm performance are typically based on econometric models without much theoretical content, while theorists build their models of R&D and innovation focusing on macro issues and a few stylized facts. Our model integrates the stochastic process for R&D investments into the model of firm performance, and the model allows us to consider how the behavior of a

---

[1] See Geroski (1998) for a survey of the large, empirical literature on firm heterogeneity, emphasizing that "the heterogeneities in performance between firms persist into the long run more or less regardless of how performance is measured."

[2] See Cohen and Klepper (1996) for a recent review of this issue.

[3] See Sutton (1997) for a comprehensive survey.

firm evolves over time, and also how firm behavior interact with the decisions and performance of its competitors. Through such a broad view, we hope that this framework will help us to obtain some deeper insights into the rich set of information contained in many panel data sets on firm performance in R&D intensive industries which, over the last decade, have become widely available.

Much of the recent work in macro growth theory is inspired by the observation that poor countries do not catch up with the richer countries. This is somewhat parallel to the micro observation that firm growth is largely independent of firm size, and the specification of the model we present below is inspired by the macro-models of endogenous growth, in particular the version of the quality ladder model developed by Barro and Sala-i-Martin (1995, ch.7). Previous versions of the quality ladder model by Aghion and Howitt (1992) and Grossman and Helpman (1991a,b) imply that each new innovation is introduced by a new firm. Barro and Sala-i-Martin (1995, ch.7) and Thompson (1996) notice that it is hard to reconcile this property with the observed pattern with persistent dominance of established firms, at least on an annual time scale[4]. We have therefore developed an alternative model where incumbent firms persistently innovate and grow or decline over a number of years. The relationship between our model and the previous literature on the quality ladder model is discussed in appendix A, where we also list a number of other studies related to our analysis[5].

Compared to the early work by Simon and others on firm growth and Gibrat's law[6], our framework gives some value added in that the random walk process for the firms' demand is not imposed on the model as an *a priori* assumption, but emerges as an endogenous result from quality changes resulting from R&D activities which are treated in the model as forward looking investment behavior.

The next section spells out the model and discusses the validity of the assumptions of the model in view of existing empirical studies. Additional empirical implications of the model are derived and examined in section 3. Section 4 elaborates on the specification of the demand side of our model, the nature of competition across firms within the whole industry and the optimal price setting. Section 5 provides conclusions and discussion of future research.

---

[4] See e.g. Gruber (1992). Geroski (1998) surveys a large number of empirical studies of the performance of large firms.

[5] Ericson and Pakes' (1995) study has a focus similar to ours in that they develop a theoretical model of R&D investment and firm growth through innovation consistent with a number of empirical observations. Another related study is Cohen and Klepper (1996) which provides an interpretation of the empirical patterns of R&D investments similar to our model, but they do not address the issue of firm growth. See appendix A for further remarks.

[6] See e.g. Ijiri and Simon (1977) and the survey by Sutton (1997).

2 Our version of the quality ladder model

This section presents a theoretical model of R&D investments and firm growth. As pointed out above, our analysis is a variation of the quality ladder model, with some new elements inspired by the patent race literature concerned with the persistence of monopoly. The analysis derives explicit expressions for the firm's R&D investments and the rate of firm growth. The focus in this section is on an individual firm doing R&D to avoid direct competition from potential entrants, while section 4 shows how the analysis easily can be extended to a simultaneous study of a many firms competing within an industry.

2.1 Dynamic optimization and firm growth

The essential process in the model is the upgrading of the quality of a firm's product, where the quality of the product is increased step-wise through cumulative innovations. For concreteness, we assume that the first firm to innovate receives a patent lasting at least until the improved version of the product arrives. Consider first the incumbent firm's participation in the race for innovation $k+1$, given that it holds the patent for the product at stage k. Denote the incumbent's hazard rate of innovation by λ, corresponding to the level of R&D measured in efficiency units, per unit of time. Considering a small length of time, Δt, we have a Bellman-type equation for the value function for the incumbent firm

$$V(k) = \max_{\lambda} \left\{ [\pi(k) - w\lambda]\Delta t + \frac{1}{1 + r\Delta t} E_{k'} \left[V(k' | \lambda, \Delta t) \right] \right\}, \tag{1}$$

where $\pi(k)$ and w are the profit flow from product k and the flow of R&D costs per unit of λ, respectively. k' is the state of the firm after the Δt units of time have elapsed.

If the firm makes an innovation, it can choose between stopping its R&D effort and earn a profit flow $\pi(k+1)$ until the innovation $k+2$ arrives, or carrying on its R&D effort one more step. If it chooses the latter, the firm has to pay a sunk cost, $F(k+1)$, for new R&D equipment needed to participate in stage $k+1$ of the innovation race. In the first case, with $e^{-\lambda(k+1)\tau}$ as the probability that the innovation $k+2$ has not arrived at time τ, the firm's expected value is

$$\int_0^{\infty} e^{-[r+\lambda(k+1)]\tau} \pi(k+1) d\tau = \frac{\pi(k+1)}{r + \lambda(k+1)}. \tag{2}$$

Our model also incorporates the threat of innovation by an outside competitor, which innovates with a hazard rate $\tilde{\lambda}$. Hence, the firm can be in four different situations or states after Δt: (i) Nothing happens and the incumbent continuous as before, (ii) the incumbent innovates and continuous its R&D, (iii) the incumbent innovates and stops its R&D, and (iv) the competitor makes a breakthrough and replaces the incumbent. All these four possibilities are accounted for by the expression

$$E_{k'}\left[V(k')|\Delta t\right] = [1-\lambda\Delta t]V(k) + \lambda\Delta t\, W(k+1) + \tilde{\lambda}\Delta t\left[\bar{V}(k+1) - F(k+1) - V(k)\right] \qquad (3)$$

where

$$W(k+1) \equiv \max\left\{\frac{\pi(k+1)}{r+\lambda(k+1)}; V(k+1) - F(k+1)\right\}.$$

$\bar{V}(k+1)$ in the last term in (3) is the value of being an outside competitor in stage $k+1$ of the innovation race. Assuming free entry such that the value of the potential entrant is equal to the R&D sunk costs, i.e. $\bar{V}(k+1) = F(k+1)$, the last term in (3) is reduced to $-\tilde{\lambda}\Delta t V(k)$. Inserting (3) into (1), multiplying both sides with $(1+r\Delta t)$, and neglecting terms of second or higher order in Δt, it follows that

$$rV(k) = \max_{\lambda}\left[\pi(k) - w\lambda + \lambda\{W(k+1) - V(k)\} - \tilde{\lambda}V(k)\right]. \qquad (4)$$

Similarly, the Bellman equation for the potential entrant in the race for innovation $k+1$, leads to

$$r\bar{V}(k) = -w\tilde{\lambda} + \tilde{\lambda}\left[W(k+1) - \bar{V}(k)\right] - \lambda\bar{V}(k).$$

This expression can be rewritten as

$$\begin{aligned}\bar{V}(k) &= \frac{\tilde{\lambda}[W(k+1)-w]}{r+\tilde{\lambda}+\lambda} \\ &= F(k) = \tilde{\lambda}f\end{aligned} \qquad (5)$$

where the last line reflects the free entry condition. The R&D sunk costs are assumed to increase linearly with the scale of the innovative activity (cf. lab. costs)[7]. From (5), and accounting for $\tilde{\lambda}$ not being negative, we have that

$$\begin{aligned}\tilde{\lambda} &= \max[0; \lambda^*(k) - \lambda] \\ \text{where } \lambda^*(k) &\equiv \frac{1}{f}[W(k+1)-w] - r.\end{aligned} \qquad (6)$$

This equation shows that as the incumbent's R&D effort, λ, increases, the potential entrant's R&D effort, $\tilde{\lambda}$, will decrease with the same amount so that the sum is constant.

The incumbent firm has a first mover advantage, and, hence, the incumbent firm's optimal R&D level can be derived by inserting (6) into (4):

$$rV(k) = \max_{\lambda}\left[\pi(k) - \lambda\{W(k+1) - V(k) - w\} - \max\{0; \lambda^* - \lambda\}V(k)\right]. \qquad (7)$$

---

[7] Allowing the R&D sunk costs also to include a fixed (size-independent) component may be more realistic, but complicates the mathematical analysis considerably.

The λ that maximizes the right hand side of this expression is given by $\lambda^*(k)$, assuming that the condition $0 < W(k+1) - w < V(k)$ is satisfied.

In this case $\tilde{\lambda} = 0$, i.e. the incumbent preempts the innovation race. Notice that since the probability rate at which the incumbent's current profits is displaced, is fixed independent of the incumbent's own R&D, due to the free entry condition, (5), the so-called Arrow-effect will not arise here. The Arrow effect is the incumbent's tendency to hold back its current R&D effort to avoid loosing its current profit; cf. Arrow (1962). Gilbert and Newbery (1984) made the same observation about the disappearance of the Arrow effect in their argument with Reinganum (1983), and they emphasized that first mover advantage for the incumbent is essential for the argument.

Equation (7) can now be restated

$$\begin{aligned} V(k) &= \frac{1}{r+\lambda^*(k)}\{\pi(k) + \lambda^*(k)\,[W(k+1) - w]\} \\ &= \frac{\pi(k)}{r+\lambda^*(k)} + F(k), \end{aligned} \tag{8}$$

where the last equality follows from the definition of $\lambda^*(k)$ in (6). Equation (8) says that the value of the incumbent firm is equal to the expected profit from holding the current innovation plus the sunk R&D costs associated with the race for innovation $k+1$. It follows that when making innovation $k+1$, the incumbent is indifferent between stopping its R&D effort thereby earning a profit flow $\pi(k+1)$ until the innovation $k+2$ arrives, or carrying on its R&D effort one more step. If, however, the incumbent has slightly lower R&D sunk costs than the potential entrant (a little epsilon will do), it will choose to continue, and we assume that this is the case. Then $W(k+1) = V(k+1) - F(k+1)$. Using this and (8) for $V(k+1)$, the definition of $\lambda^*(k)$ in (6), the incumbent firm's optimal level of R&D, can now be restated as

$$\lambda^*(k) = \frac{1}{f}\left[\frac{\pi(k+1)}{r+\lambda^*(k+1)} - w\right] - r. \tag{9}$$

2.2 Some additional functional form assumptions

Some additional functional form assumptions are required in order to derive predictions consistent with the empirical observations discussed in the introduction, and which we will consider in more detail in section 3. We start by assuming that the value of holding a patent, the flow and set up costs of R&D all increase as the product becomes more advanced, such that

$$\pi(k) = \pi_0 e^{\eta_\pi k}; \quad w(k) = w_0 e^{\eta_w k}; \quad f(k) = f_0 e^{\eta_f k}.$$

The first assumption is straight forward, while the empirical support for the last two assumptions will be discussed in section 2.3.

The incumbent's R&D effort, given by (9), can now be rewritten as

$$\lambda^*(k) = \frac{\pi_0/f_0}{r + \lambda^*(k+1)} e^{\gamma_\pi(k+1) - \gamma_f k} - \frac{w_0}{f_0} e^{(\gamma_w - \gamma_f)k} - r. \tag{10}$$

A stationary state In the general case, the rate of innovation by the incumbent firm, $\lambda^*(k)$ given by (10), depends on the firm's product quality, k. However, if $\gamma_\pi = \gamma_w = \gamma_f$, the rate of innovation will be independent of k, i.e. $\lambda^* = \lambda^*(k) = \lambda^*(k+1)$, and from (10) it follows, after a little algebra, that

$$\lambda^* = \sqrt{\frac{\pi_0 e^\gamma}{f_0} + \left(\frac{w_0}{2f_0}\right)^2} - \frac{w_0}{2f_0} - r. \tag{11}$$

There are two offsetting forces that make λ^* independent of k in this case. On the one hand, innovation is assumed to get harder and harder when k increases, as reflected by $w'(k)$ and $f'(k)$ being positive, while on the other hand, the expected returns to an innovation also increases with k (cf. $\pi'(k) > 0$).

Notice that the steady state level of R&D effort, λ^*, is an increasing function of the (basic) rate of profit, π_0; and the size of each innovative step as captured by the parameter γ. The costs per efficiency unit of R&D are captured by the two parameters w_0 and f_0, and if these two parameters increase, the steady state rate of innovation will decrease. Finally, the steady state rate of innovation declines with the discount rate.

The knife edge case, $\gamma_\pi = \gamma_w = \gamma_f$; is analytically very attractive, and it also provides predictions consistent with a number of empirical observations that we will discuss in section 3. What will, however, happen if this strict relationship doesn't hold? If γ_π is less than γ_w or γ_f, the innovation rate, $\lambda^*(k)$; will decline as k increases since the effect that innovation becomes more costly as k increases dominates the increasing profitability of the innovations as k increases. This imply that the changes in k will slow down as k increases. The other case, where γ_π is larger than γ_w and γ_f, has the opposite implication, i.e. that the changes in k will speed up as k increases. Below we assume that firm size is determined by product quality. The first case will then correspond to smaller firms growing faster than larger firms, while the second case will correspond to larger firms growing faster than smaller firms. The first case has some empirical support, but the rest of the paper focuses on the knife edge, steady state case as it simplify the analysis considerably.

The R&D intensity in steady state As in the quality ladder literature, we assume that a firm's sales (or market share) increase with its product quality, such that

$$S(k) = S_0 e^{\gamma_s k}. \tag{12}$$

The R&D expenditure per unit of sales is now

$$\begin{aligned}\frac{w(k)\lambda(k)}{S(k)} &= \frac{w_0\lambda^*}{S_0}e^{(\eta_w-\eta_s)k} \\ &= \frac{w_0\lambda^*}{S_0}, \qquad (13)\end{aligned}$$

where the last equality holds if an additional steady state assumption is imposed, i.e. $\eta_s = \eta_w$. In this steady state case, the R&D intensity is independent of the firm's product quality, k, and, consequently, firm size.

The expression (13) covers only the R&D flow costs, which typically accounts for 80-90 percent of R&D expenditures. The point that the R&D intensity is independent of the firm's product quality, k, will, however, hold more generally, since the R&D sunk costs also increase at the same rate as the firm's product quality, k.

2.3 Short and long run diminishing returns in R&D

The model introduces diminishing returns in R&D in two different ways; one concerning the relationship between the firm's level of R&D and the expected waiting time to the next innovation, and the other relates to the increasing costs of doing R&D as the product becomes increasingly advanced.

The assumption that the R&D costs, $w(k)\lambda(k)$, increase linearly with level of R&D activity, $\lambda(k)$, is analytically very convenient as it permits an explicit solution for λ^*. It implicitly imposes diminishing returns in innovation, as the expected waiting time to the next innovation, given by $1/\lambda$, is reduced at a declining rate with level of R&D activity. Declining returns to scale in R&D is also introduced through the assumption that costs per efficiency unit of R&D increase when the products become more and more advanced.

The issue of "returns to scale in R&D" has been the subject of much research on the basis of patent statistics; see Griliches (1990) for a survey. In a study of the relationship between patents and R&D in a panel of U.S. firms, Hall, Griliches and Hausman (1986) report estimates of the elasticity between 0.3 and 0.6 in the longitudinal dimension, suggesting rather sharply diminishing returns. However, Blundell, Griffith and Windmeijer (1999) have recently revisited this issue and they find a higher long run elasticity (0.6-0.9) based on their new, preferred specification of the dynamic process. Finally, Griliches (1990) argues that the appearance of diminishing returns in the longitudinal dimension could be an artifact due to the incompleteness of the underlying data rather than a reflection of the characteristics of the underlying innovation process.

Other studies have found some support for diminishing returns to R&D using cross-sectional data and macro data. Bound et al. (1984) report that the number of patents per dollar of R&D is significantly lower for firms with larger R&D budgets[8]. Similarly, Acs and Audretsch (1991) find a negative relationship between innovations per R&D dollar and the level of R&D investments[9]. Using time series data at the macro level, several researchers point out a similar pattern, i.e. the increasing ratio of R&D per patent; see Caballero and Jaffe (1993), Kortum (1993) and Griliches (1994). One interpretation of these findings is that they reflect diminishing returns to R&D. Cohen and Klepper (1996) discuss at greater length the diminishing returns to R&D issue and the relevant empirical literature.

Finally, we should keep in mind that even if the estimated elasticity of patents with respect to R&D expenditures had been precisely estimated in the patent studies, it is not clear how that should be translated to the form of the innovation function in our model. That is, what is the relationship between a patent and making a step on the quality ladder; does a patent increase demand with a certain percentage, or is the percentage increase in demand from a patent dependent on the stock of patents and the firm's size? This functional form question remains open[10].

3 Main propositions derived from the model and the empirical evidence

Our model of firm growth has a number of steady-state predictions that fits well with the empirical patterns of firm growth and R&D investments, as we will show in this section. In our discussion, we draw on the empirical findings from several studies, in addition to our new empirical results derived from a sample of Norwegian R&D-performing firms in high-tech industries, 1985-95. An attractive aspect of the Norwegian data set is that variables are reported at the line-of-business level within each firm. This fits our framework better than the more widely used sets of data where the firm is the unit of observation, since many firms are conglomerates producing a large number of different products. Appendix B provides details on sample and variable construction.

---

[8] Griliches (1990) discusses alternative interpretations of this finding by Bound et al., and he argues that the pattern might be an artifact due to a sample selectivity problem for small firms.

[9] Acs and Audretsch's (1991) result is based on a cross sectional analysis of a comprehensive data set on innovations collected by the Small Business Administration.

[10] The relationship between sales and patents might be so noisy that it is hard to discriminate between alternative functional forms on the basis of the available data. The standard assumption in the literature is a log-log relationship between patent counts and sales, rather than a linear-log relationship; see e.g. Griliches, Hall and Pakes (1991).

3.1 Patterns across firms within the same industry

3.1.1 R&D intensity is independent of firm size

The R&D intensity in steady-state is given in (13), and since the right hand side of (13) does not vary with k, we have established that

Proposition 1 R&D increases proportionally with firm size so that R&D intensity is independent of size.

The empirical literature that relates to this issues is vast as is clear, c.f. the survey in Cohen and Klepper (1996)[11]. A simple illustration of the relationship between R&D and firm size can be derived from our sample of Norwegian R&D-performing line-of-businesses. Regressing the log of R&D expenditures, $\ln R_{it}$, against log of sales, $\ln S_{it}$, where i and t refer to line-of-business and year, respectively, we obtained the relationship

$$\begin{aligned} \ln R_{it} &= \underset{(0.07)}{0.89} \cdot \ln S_{it} + \text{dummies} + \text{error term}, \end{aligned}$$

where the number in parenthesis is (heteroskedastic adjusted) standard error[12]. The R-square is 0.49, and the relationship include year and industry dummies. Even though the elasticity of R&D with respect to size is less than one, the difference is not statistically significant. Figure 1 graphs the relationship between log R&D and log sales together with a smoothed average and a straight line corresponding to an elasticity equal to one. The graph shows that R&D is close to proportional to firm size, in particular above a minimum threshold.

A more elaborate study is provided by Bound et al. (1984), based on the Compustat file covering firms in US manufacturing, and they conclude, after checking a number of econometric issues, that R&D increases proportionally to size. They find deviations from this pattern among very large and very small firms, which tend to be more R&D intensive than the rest. However, as they point out, very small firms on the Compustat files are likely to be more innovative and do more R&D than the average small firm in US manufacturing[13]. Cohen, Levin and Mowery (1987) confirm the main conclusion in Bound et al. at the firm level, and also at the line of business level for the sample of R&D performing business units. They report, however, some positive relationship between the R&D intensity and the size of the business unit for the sample of all business units. In their survey, Cohen and Levin (1989) emphasize that the size

---

[11] See also the surveys by Cohen and Levin (1989) and Cohen (1995).

[12] Estimates where $\ln S_{it}$ was instrumented with lagged values or the number of employees gave very similar results.

[13] Griliches (1990) elaborates on this argument.

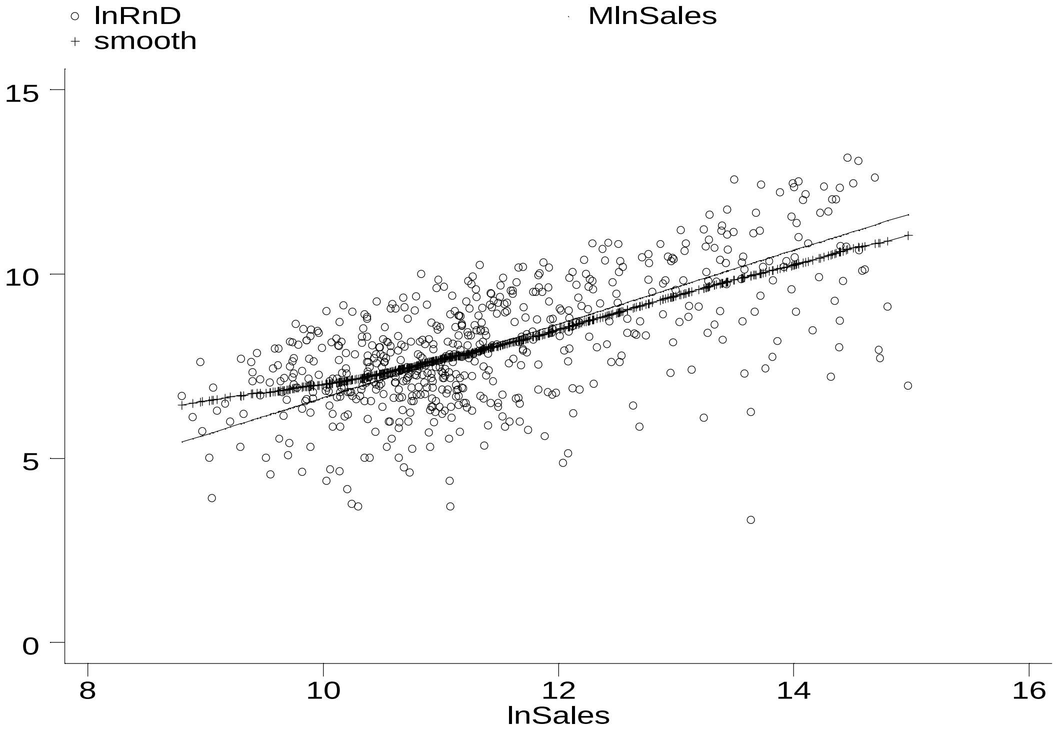

Figure 1: The relationship between log R&D and log sales toghether with a line reflecting the smoothed average (cf. crossed line) and a line corresponding to an elasticity equal to one (cf. continuous line).

effects in the R&D intensities, even in cases where they are found to be statistically significant, are "minute both in terms of the variance explained and the magnitude of the coefficients". Similarly, Stylized fact 3 in Cohen and Klepper (1996) states that "in most industries it has not been possible to reject the null hypothesis that R&D varies proportionally with size across the entire firm size distribution". We conclude that Proposition 1, and hence the model presented in this paper, is a reasonable first approximation to at least one widely studied pattern of R&D investments.

3.1.2 Firm growth is independent of firm size

In (11), we showed that λ is independent of k in steady state. Since k determines the size of the firm according to (12) and λ is the Poisson parameter in the stochastic process for k, we have established the following proposition:

Proposition 2 The model implies firm growth independent of firm size, in accordance with Gibrat's law.

There is a long line of research on the empirical relationship between firm size and firm growth, see the survey by Sutton (1997). In our Norwegian sample, we found that log of firm size is very close to a random walk, i.e.

$$\begin{aligned} \ln S_{it} &= \underset{(0.013)}{0.97} \cdot \ln S_{i;t-2} + \text{error term} \end{aligned}$$

essentially consistent with no relationship between firm growth and firm size. The R-square for this relationship is 0.92. Similarly, regressing $\ln(S_{it}/S_{i;t-2})$ on $[\ln(S_{it}) + \ln(S_{i;t-2})]/2$ gave a statistically insignificant estimate equal to 0.01 and an R-square less than one percent.

Two more elaborate studies for US manufacturing are Evans (1987) and Hall (1987). Evans (1987) concludes that departures from Gibrat's law might be significant for small firms and long time periods, but one "might not go too wrong by maintaining Gibrat's law" for "short run changes in the growth and size distribution of the largest firms". Hall (1987) has carried out a careful analysis considering sampling bias and measurement errors, and she concludes that "Gibrat's law is weakly rejected for the smaller firms ... and accepted for the larger firms". The evidence suggests that Gibrat's law is not universally true, and it would not make sense to take that pattern for granted, but we consider it desirable that our model can be made consistent with this benchmark case.

3.1.3 R&D follows a random walk

From equation (13), showing that the R&D intensity is independent of product quality, and Proposition 2 we have that:

Proposition 3 R&D follows a random walk (Gibrat's law in R&D).

Our sample showed that log of R&D investments is not quite a random walk, but quite close:

$$\begin{aligned} \ln R_{it} &= \underset{(0.027)}{0.89} \cdot \ln R_{i,t-2} + \text{error term} \end{aligned}$$

with an R-square equal to 0.81. Figure 2 confirms that R&D is close to a random walk, and almost exactly so when R&D is above a minimum threshold. Notice that measurement errors in $\ln R_{i,t-2}$ is likely to bias the estimated coefficient downwards. An alternative approach, less affected by random measurement errors, is to regress $\ln(R_{it}/R_{i,t-2})$ on $[\ln(R_{it}) + \ln(R_{i,t-2})]/2$, and this gave an insignificant parameter estimate equal to -0.02 and an R-square less than one percent.

In a study of US manufacturing firms (covering 8 years), Hall, Griliches and Hausman (1986) conclude that "R&D investment [is] essentially a random walk with an error variance which is small relative to the total variance of R&D expenditures between firms".

3.1.4 The size distribution of firms is skewed

It is well known that a process of firm growth according to Gibrat's law, will generate a skewed size distribution of firm size:

Proposition 4 The model will generate a highly skewed distribution of firm size.

Using observations of log of sales for the business units in our Norwegian sample we have performed a skewness test for log-normality separately for each of the years 1985 and 1995. The results show that log-normality can be rejected at the 0.5 and 0.2 percent significant levels, respectively, for the two years. Examining the distribution of log-sales in a standardized normal probability plot, it is clear that the distributions of firm sizes have thicker upper tails than the log-normal[14]. To our knowledge, there has been little systematic analysis of the skewness of the firm size distribution in high-tech industries (or other industries for that matter) since the book by Ijiri and Simon (1977). See, however, the discussions in the surveys by Geroski (1998) and Sutton (1997).

---

[14] In the working paper version of this paper we documented similar skewness in the size distribution within separate 4-digit (ISIC) industries.

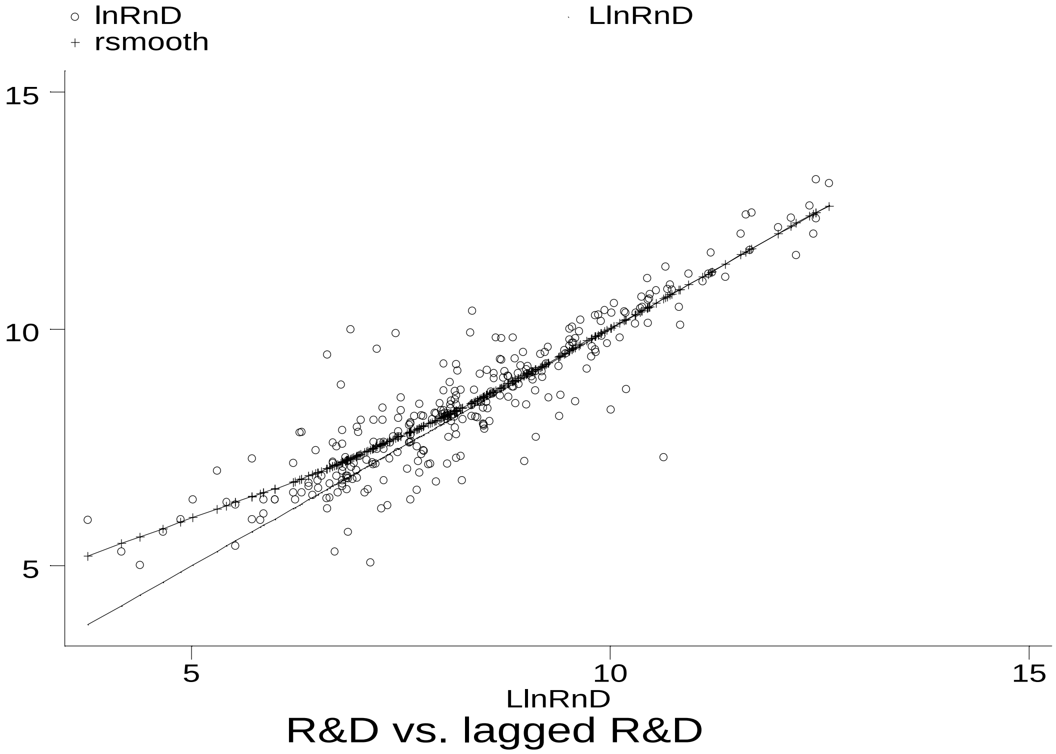

Figure 2: The relationship between log R&D in year t (vertical axis) and t ¡ 2 together with smoothed average (cf. crossed line) and a line corresponding to an elasticity equal to one (cf. continuous line).

3.1.5 The variance of the firm size distribution increases over time

Since the intensity parameter (λ) for the arrival of new innovations is independent of the level of k, the process will generate a Poisson distribution for k with a parameter which increases in proportion with the "age" of the process[15]. We consider the firms (within an industry) to be different realizations of such processes, and hence:

Proposition 5 The model will generate a more and more widely spread distribution of firm sizes as the average firm age increases.

The increasing spread generated by Gibrat's law has been the focus of much research and it has largely been considered an unrealistic implication; see Steindl (1968) for a survey of the early attempts to create models for the firm size distribution, based on random walk growth but with a stable (non-increasing) spread of the distribution, and McCloughan (1995) for a recent study and further references. Entry and exit of firms can be one stabilizing force, as was early demonstrated by Simon and his coworkers; see Ijiri and Simon (1977).

However, the increasing spread of the firm size distribution, as suggested by Proposition 5, may not be unrealistic in many cases. The growth of IBM in the mainframe industry and Microsoft in the software industry are two, well-known examples of increasing dominance by the leading firms as industries mature. Klepper (1996) cites several empirical studies emphasizing the pattern of increasing concentration and dominance by leading firms over the industry life cycle. Finally, notice that our model predicts an increasing spread of the firm size distribution only if the average firm age increases. The issue of average firm age is, of course, largely determined by the pattern of entry and exit, which is left open in the model presented above[16].

3.2 Differences in R&D and innovation across industries

The propositions above referred to regularities between firms within the same industry, presumably facing similar innovative opportunities, profitability of R&D and degree of competition. However, the innovative opportunities, profitability of R&D and degree of competition clearly differ across industries and the model in section 2 has implications for how such differences between industries will affect R&D expenditures and the rate of innovation.

---

[15] The parameter in the Poisson distribution is proportional to (λt) in the case with a stationary intensity parameter (λ), where t is the age of the process. More generally, with a non-stationary intensity parameter, we have that the spread (and the mean) in the Poisson distribution is $\int_0^t \lambda(s)\,ds$.

[16] Pakes (1994) considers structural analysis of firm growth incorporating entry and exit decisions.

3.2.1 Competition, R&D and innovation

There is an old and still ongoing debate about the impact of competition on R&D expenditures and the rate of innovation, and our model captures two aspects of this debate[17]. The first relates to competition in the race to innovate. Our analysis in section 2.1 of the incumbent's tendency to preempt the race, shows that it is the threat of entry by an outsider that pushes forward the innovative effort by the incumbent, and the incumbent's innovative effort would have been reduced in the absence of such a threat. Hence, competition in the innovative race stimulates innovation, as has also been emphasized in the patent race literature. The second aspect of competition relates to competition in the product market once the innovation has taken place. This is reflected in the flow of profit to the innovative firm. A higher flow of profit will give higher R&D expenditures and more innovation, as can be seen from (11). That high profits from innovation stimulates the innovative effort is not a new observation; it was recognized already when the patent law was introduced in the fifteenth century and forcefully pointed out by Schumpeter (1942). To summarize:

Proposition 6 Competition in the race to innovate (ex ante) stimulates innovation and R&D expenditures, while competition in the product market (ex post) reduces the innovative effort, due to reduced profitability of innovations.

The literature on R&D and competition has also stressed a third issue, – the role of the costs of internal versus external finance in R&D investments, emphasizing that external funds impose higher costs of finance, especially for R&D investments due to problems of asymmetric information. This line of argument, first attributed to Schumpeter (1942), suggests that high profits stimulate R&D spending and innovation, as the profits provide scope for internal finance[18]. A higher discount rate reduces the rate of innovation in our model as shown in (11).

It is empirically hard to unravel the relationship between competition and innovative effort, as the various aspects of competition often are closely linked and pull in opposite directions, as stated in Proposition 6. In their survey of the relationship between innovation and market structure, Cohen and Levin (1989) conclude that "the empirical results bearing on the Schumpeterian hypotheses [about innovation, firm size and market concentration] are inconclusive, in large part because investigators have failed to take systematic account of more fundamental sources of variation ... [across] firms and industries".

---

[17] See Nickell (1996) for a wider discussion and further references.

[18] However, Aghion, Dewatripont and Rey (1999) have argued that when managers goal is survival rather than profit maximization, large funds of internal finance may reduce the incentives to innovate.

The problem of interpretation is illustrated by some recent, empirical studies of the relationship between profit margins (i.e. cash flow relative to sales), innovation and R&D-intensity across firms, including the studies by Geroski, Machin and van Reenen (1993), Brouwer and Kleinknecht (1994) and Himmelberg and Petersen (1994). Brouwer and Kleinknecht's study is a pure cross sectional analysis of the relationship between R&D and the profit margin, while the studies by Geroski et al. and Himmelberg and Petersen use longitudinal data sets. Brouwer and Kleinknecht find a positive relationship between R&D intensity and the profit margin in their cross section, which can reflect both the positive effect of internal funding and the (ex post) product market competition effect. Similarly, several other studies, including Himmelberg and Petersen (1994), report a positive relationship between R&D intensity and the cash flow relative to sales, i.e. the profit margin, which also can reflect either of the two positive effects.

Geroski et al. (1993) identify a positive relationship between innovation counts and the profit margin, and they emphasize a longitudinal pattern suggesting a causal relationship that runs from changes in innovative activity to changes in the profit margin. They also point out, however, that the longitudinal effects are small compared to persistent cross sectional differences in the profit margins. In the model presented above, the causal relationship runs the other way, i.e. from expected profits per unit of output to R&D investments, because product markets with higher expected profits will cet.par. attract more R&D investments. In section 4, we will show how our model can be extended to accommodate a causal pattern where a new innovation increases the profit margin, which then gradually decreases as the innovation gets older. This extension allows also for a causal relationship running both ways, between R&D and innovation on the one hand, and higher profit margins on the other.

3.2.2 Innovative opportunities stimulate innovation

In discussing (11), we noticed that the rate of innovation is higher when the innovative opportunities are larger, i.e. with larger innovative steps (cf. ´) or when the costs per efficiency unit of R&D are lower (cf. w_0 and f_0). Furthermore, we can show that the R&D intensity also will be higher with larger innovative steps and lower costs per efficiency unit of R&D. Proving the second part of this last claim is not entirely straightforward, but it can be done easily when the ratio, μ, between f_0 and w_0 is low. This is quite realistic as investment costs in R&D (laboratories and equipment) typically amount to less than 10-20 percent of total R&D. In this case, from (11), we find that

$$\lambda^* = \frac{1}{2\theta}\sqrt{1 + 4\theta\frac{\pi_0 e^{\eta}}{w_0}} - \frac{1}{2\theta} - r$$
$$\approx \frac{\pi_0 e^{\eta}}{w_0} - r,$$

which, inserted in (13), implies that the R&D intensity, $w_0\lambda^*/S_0$, declines when the costs per efficiency unit of innovation, w_0, increase. Hence, we have established that our model is consistent with the following proposition:

Proposition 7 Industries with higher innovative opportunities will have higher rate of innovation and higher R&D intensity.

This proposition is intuitive and there is a large body of research suggesting that higher innovative opportunities spur innovative activities and R&D investments; see e.g. Pakes and Schankerman (1984) and the surveys by Cohen and Levin (1989) and Cohen (1995, section 4).

3.2.3 Firm growth rates are more dispersed in innovative industries

The variance of firm growth rates is proportional to $\eta^2\lambda^*$, if we neglect firm entry and exit. Hence, ignoring the issue of firm entry and exit for argument's sake, we have that

Proposition 8 The variance of firm growth rates is higher in industries with a higher rate of innovation.

This proposition is consistent with Klette and Førre's (1998) finding that more R&D intensive industries have higher rates of job reallocation.

4 Product market competition

In the analysis of firm behavior in section 2, we did not consider other firms producing similar products in the same industry, but this section shows how the analysis easily can be extended to a simultaneous study of many, heterogenous firms competing within an industry with vertically and horizontally differentiated products. That is, the model of firm behavior presented in section 2 can be embedded in a model of product market competition based on discrete choice theory for the demand of differentiated products. This model of market competition allows for simultaneous growth and decline of different firms in the same industry. Indeed, a breakthrough in one firm, leading to growth in demand, will reduce the demand and market shares for the products of its competitors.

Consider a firm producing a product i of quality k_i and with price P_i. The firm's sales are determined by the demand function

$$S_i = M \frac{e^{v(k_i;P_i)}}{\sum_{j \in J} e^{v(k_j;P_j)}} \tag{14}$$

with $v(k_i;P_i) = \theta_s k_i - \alpha \ln P_i$. The parameter θ_s was introduced in (12), α reflects the price elasticity of demand and M is the size of the market, while J refers to all firms in the market. This demand system has been extensively discussed by others; see e.g. Anderson, dePalma and Thisse (1992), Berry (1994) and their references. Ignoring the price effects for the moment, firm growth or decline is driven by cumulative improvements in product quality of the firm's own product (k_i) relative to improvements in the quality of competing products. Notice that it is a firm's relative product quality as well as its relative price that determines its market share. This property suggests that a firm has to upgrade the quality of its product at the pace of its competitors in order to preserve its market share.

The model presented in section 2 focuses on the quality, k_i, of a single firm and how the changes in k_i is generated by the firm's R&D investments. Formally, the specification in (12) corresponds to (14) with

$$S_0 = M \frac{e^{-\alpha \ln P_i}}{\sum_{j \in J} e^{v(k_j;P_j)}}: \tag{15}$$

The analysis in section 2 assumes that the firm considers the development of S_0 to be independent of changes in its product quality. Section 5 will elaborate on the relationship between product quality and price setting, cf. the numerator (15). Concerning the denominator in (15), we assume that the firms are small and consider its development as exogenously given, independent of e.g. their own R&D decisions, in the spirit of monopolistic competition. In contrast, notice that the models in Pakes and McGuire (1994) and Ericson and Pakes (1995) emphasize the strategic interactions in R&D investment decisions across producers within the industry. R&D investments are also driven by strategic considerations in our model, but each firm confines its strategic considerations to deterring outside entry into its product line.

5 Optimal price setting and product quality

So far, we have ignored the important issue of price setting and, in particular, the relationship between optimal price setting and product quality. Our analysis of steady state assumes that the flow of profits, $\pi(k)$; and sales, $S(k)$, increase at the same rate as k increases, implying that the price is independent of product quality, k. This section discusses the theoretical justification and empirical support for this price setting assumption. In the pricing stage, a firm is facing price competition from two different margins: (i) Competition from other differentiated products as captured by the denominator in (14), and (ii) competition from the lower quality versions of the same differentiated product. When the price setting of a new product is constrained by competition from the lower quality versions of the same differentiated product, the innovation is labelled as a non-drastic innovation, following Arrow's (1962) terminology, and the innovation is termed drastic if this is not the case.

5.1 Non-drastic innovations

Let us denote the constant, marginal production costs for firm i by c_i, which are independent of k_i. Below we discuss cases where the production costs vary with product quality. The lower quality variety of product i (denoted $k_i - 1$) can be produced by competing firms, and we assume for simplicity that they have the same marginal costs as firm i. If the monopoly price for firm i exceeds $c_i e^{\eta_s/\alpha}$, the new (high-quality) product will be outcompeted by the old variety[19]. This is the case of a non-drastic innovation. In this case, as shown by Arrow (1962), the optimal price is the so-called "limit price", determined such that the lower quality product is just competed out of the market, i.e.

$$P_i = c_i e^{\eta_s/\alpha}$$

minus a little "epsilon". Notice that this limit price prevents the lower quality variety of the product to enter, and it is consequently only the highest quality variety of each product which is present in the market. Clearly, this price is determined independently of k_i, and the basic flow of profit is

$$\begin{aligned}
\pi_0 &= (P_i - c_i)\, S_0 \\
&= M \frac{e^{-\alpha \ln c_i - \eta/\alpha}}{\sum_{j \in J} e^{\eta k_j - \ln c_j - \eta/\alpha}} c_i \left(e^{\eta/\alpha} - 1 \right) \\
&= M \frac{c_i^{1-\alpha}}{\sum_{j \in J} e^{\eta k_j} c_j^{-\alpha}} \left(e^{\eta/\alpha} - 1 \right);
\end{aligned}$$

which is higher for low-cost firms, since $\alpha > 1$ (i.e. elastic demand). Consequently, from (11), the rate of innovation is higher for low-cost firms.

5.2 Drastic innovations

In the case of drastic innovations, a firm with a new innovation can charge the monopoly price as it is not constrained in its price setting by the presence of lower quality varieties of the same

---

[19] This follows from the demand system (cf. eq. 14), as consumers will choose the low quality variety at price c_i if $v(k_i; P_i) < v(k_i - 1; c_i)$:

product, i.e. its monopoly price is below the limit price, $c_i e^{\eta s/\alpha}$. The optimal price in the case with a drastic innovation and Bertrand competition, given demand as specified in (14), is

$$\begin{aligned} P_i &= c_i \left[1 - \frac{1}{\alpha(1-s_i)}\right]^{-1} \\ &\approx c_i \left(1 - \frac{1}{\alpha}\right)^{-1} \end{aligned}$$

where s_i is the firm's market share[20], and the last approximation is valid when the firm has a small market share, i.e. the monopolistic competition case. A firm's market share clearly depends on its product quality, and, hence, with drastic innovation it is only under monopolistic competition that the price is independent of product quality. Notice that this assumption of small market shares is consistent with our previous assumption that the firm considers the development of the denominator in (14) to be exogenous. In this case, the basic flow of profit is

$$\pi_0 = M \frac{c_i^{1-\alpha}}{\sum_{j\in J} e^{\eta k_j} c_j^{-\alpha}} \left(\frac{1}{1-1/\alpha} - 1\right)$$

which is higher for low-cost firms, as in the non-drastic case.

5.3 Some empirical evidence

The profit margin in our analysis above, is argued to be independent of product quality, and since product quality determines market shares, there should be no relationship between profit margins and market shares. This seems to be consistent with the survey of the empirical evidence on the relationship between profit margins and market shares by Schmalensee (1989), who conclude that "profitability is not generally strongly related to market shares" within industries, with a few exceptions. Similarly, Griliches and Cockburn (1994) show that increased competition and drastic changes in market shares due to entry of new generic drugs in the pharmaceutical industry they consider, did not seriously affect the price charged by the incumbent firm. Metrick and Zeckhauser (1996) also provide empirical evidence suggesting that the profit margins are independent of product quality in the markets for mutual funds and in the automobile industry.

Our sample of Norwegian firms, however, did reveal a statistically significant, positive relationship between profit margins[21] and market shares

$$\left(\frac{P-c}{P}\right)_{it} = \underset{(.009)}{0.06} \ln S_{it} + \text{dummies} + \text{error term},$$

---

[20] Notice that $M = \sum_{j\in J} S_j$, i.e. total industry sales as can be found by summing both sides of (14) over all firms.

[21] The profit margin is constructed as operating surplus divided by sales. See Appendix B for further details.

and the positive relationship remained statistically significant also when we instrumented $\ln S_{it}$ by other measures of size, such as employment.

5.4 Flows of profit over the product life cycle

While the quality of an existing product variety may not affect a firm's profit margin, the introduction of a new, innovative product variety seems to affect the profit margin. That is to say, empirical studies, e.g. Mansfield, Schwartz and Wagner (1981), suggest that the profit margin is high for a new high-quality product variety, but that imitation eliminates innovative rents within a few years. Similarly, Geroski, Machin and van Reenen (1994) find that innovations increase the profit margin for the innovating firms, while Berndt, Griliches and Rosett (1993) show that the rate of price change for a product is negatively related to the age of the product[22].

The pattern that the profit margin declines gradually after a new product variety is introduced can be incorporated in our model. In the case of non-drastic innovations, the marginal costs may gradually decline for potential producers of the lower quality variety, e.g. as the knowledge of how to produce the lower quality variety diffuses to low cost producers. Similarly, for a product that initially was a drastic innovation, the profit margin may turn out to be reduced as the competition from the lower quality variety intensifies when the production costs for this old, lower quality variety have been sufficiently reduced.

Formally, this idea corresponds to the profits per unit of time declining with the age of the innovation according to $\pi(k)\,e^{-\rho t}$, where t is the time elapsed since the product of quality k was introduced and ρ is the rate of cost reduction for the lower quality producer. The expected value of holding the patent for innovation k is now (cf. equation 2)

$$\int_{t=0}^{\infty} e^{-[r+\lambda(k)+\rho]t}\pi(k)\,dt = \frac{\pi(k)}{r+\lambda(k)+\rho},$$

and (8) should in this case be rewritten as

$$V(k) = \frac{\pi(k)}{r+\lambda(k)+\rho} + F(k).$$

Extending the model in this way does not require any substantial change in the analysis in section 2, and it allows for a causal relationship running from R&D to innovation to profit margins, with the ratio between price and marginal costs declining with the age of the most recent product variety. However, profits per unit of output remain independent of the level of the product quality also in this extended model, which is essential in order to preserve the simple mathematical structure of the model spelled out in section 2.

---

[22] See also Kuznets (1930).

5.5 R&D as a non-rival input and product quality

So far we have assumed that production costs are independent of the product quality. That is, our model assumes that once a new product improvement is developed and introduced, no additional resources are needed to produce this improved product as compared to the older version. Romer (1990) has emphasized this aspect of R&D, by labeling R&D, or, more precisely, knowledge as a non-rival input in production. Improvements in computer software packages can serve as examples of non-rival improvements in product quality.

Our framework hides a second dimension of quality, which can be altered by spending more resources per unit of production. This quality aspect is a rival output dimension; it is related to the notion of a rival good in Romer's (1990) terminology. More horsepower and leather seats in cars are examples of quality differences in "rival" dimensions. The rival dimension of quality has implicitly been buried in the measurement of output quantities and prices, and in the marginal costs. It is not surprising that the costs related to the rival part of quality increase with sales, and this is why the rival dimension of quality has largely been ignored in our analysis.

The more interesting aspect of quality improvements from the view of firm growth and performance, is the non-rival dimension related to product development and research. It is changes in the non-rival dimension that introduces a cumulative growth process in our model of firm performance.

6 Conclusions and remaining issues

We have presented a fully specified, theoretical model of endogenous firm growth, where R&D and innovations are the engines of growth. The model is tightly specified and the many specific assumptions were introduced in order to rationalize a number of empirical regularities that have been established from empirical research on firm growth and innovation. We have also examined the empirical content of the assumptions and argued that they are good or, at least, acceptable approximations to the findings in much available empirical research.

On the basis of our analysis, we have argued that our model is promising as a benchmark model to understand:

1. Why the size distribution of firms is highly skewed, with persistent heterogeneity.
2. How Gibrat's law can be reconciled with optimizing behavior.
3. Why R&D intensities across firms are largely independent of size, even in cases where R&D is a non-rival input.

Our model is able to address these three issues within a single, integrated framework.

We must admit that the correspondence between the empirical literature and the empirical content of our model is not perfect. For instance, our model ignores important aspects of imitation[23]. Empirically, it is widely observed that even in high tech industries there is a large fraction of firms reporting no R&D activity, and these firms presumably survive by imitation[24].

While our model accounts for much heterogeneity within industries, it still neglects considerable heterogeneity that is widely observed in empirical studies. For instance, we observe empirically large and persistent differences in R&D intensities between firms in most industries also at a disaggregated industry level. The sources of differences in R&D intensities have been the subject of much empirical research[25]. Our model can be extended to accommodate such differences, by introducing differences in productions costs, product market competition and innovative opportunities across firms and over time. Identifying and modeling these different sources of heterogeneity is a crucial step towards structural estimation of the parameters in the model. The stochastic structure of such an extended model is probably quite complex and may be econometrically tractable only through simulations. However, our model provides explicit mathematical expressions for the key variables, at least in the steady state case, which suggests that simulation of the model is quite tractable. A natural next step is to explore inference and testing of the model through simulations.

7 References

Acs, Z.J. and D.B. Audretsch (1991): R&D, firm size, and innovative activity. In Acs, Z.J. and D.B. Audretsch (eds.): Innovation and Technological Change: An international Comparison. New York: Harvester Wheatsheaf.

Aghion, P., M. Dewatripont and P. Rey (1999): Competition, financial discipline and growth, Review of Economic Studies (forthcoming).

Aghion, P. and P. Howitt (1992): A model of growth through creative destruction, Econometrica 60, 323-51.

Andersen, S.P., A. de Palma, J.-F. Thisse (1992): Discrete Choice Theory of Product Differentiation, Cambridge (U.S.): MIT Press.

Arrow, K.J. (1962): Economic welfare and the allocation of resources to inventions, in R.R. Nelson (ed.): The rate and direction of inventive activity, Princeton: Princeton University Press.

Barro, R. and X. Sala-i-Martin (1995): Economic Growth, New York: McGraw-Hill.

---

[23] Notice, however, that the threat of entry which plays an important role in the model, implicitly assumes that imitation of existing products and know-how is easy since a new competitor potentially can make a step on the quality ladder from the same level as the incumbent firm.

[24] See e.g. Nelson (1988) for a discussion of this point.

[25] See Pakes and Schankerman (1984), Cohen (1995) and Klette and Johansen (1998).

Berndt, E.R., Z. Griliches and J. Rosett (1993): Auditing the producer price index: Micro evidence from prescription pharmaceutical preparations. Journal of Business & Economic Statistics, 11, 251-64.

Berry, S.T. (1994): Estimating discrete choice models of product differentiation, RAND Journal of Economics 25, 242-62.

Blundell, R, R. Griffith and F. Windmeijer (1999): Individual effects and dynamics in count data models. IFS Working Paper W99/3.

Bound, J., C. Cummins, Z. Griliches, B. Hall, A. Jaffe (1984): Who does R&D and who patents? in Z. Griliches (ed.): R&D, Patents and Productivity, Chicago: University of Chicago Press

Brouwer, N.M. and A. Kleinknecht (1994): Monopoly profits from innovation and wage differentials. A micro-econometric investigation. Mimeo, Free University of Amsterdam.

Caballero, R.J. and A. Jaffe (1993): How high are the giants shoulders: An empirical assessment of knowledge spillovers and creative destruction in a model of economic growth. In O.J. Blanchard and S. Fischer (eds.): NBER Macroeconomic Annual 1993. Cambridge (U.S.): MIT Press.

Cohen, W.M. (1995): Empirical studies of innovative activity. In P. Stoneman (ed.): Handbook of Innovation and Technological Change. Oxford: Blackwell.

Cohen, W.M. and S. Klepper (1996): A reprise of size and R&D, Economic Journal 106, 925-51.

Cohen, W.M. and R. Levin (1989): Empirical studies of innovation and market structure. Ch. 18 in R. Schmalensee and R. Willig (eds.): Handbook of Industrial Organization, Vol. 2. Amsterdam: North Holland.

Cohen, W.M., R. Levin and D. Mowery (1987): Firm size and R&D intensity: A re-examination. Journal of Industrial Economics, 35, 543-65.

Dasgupta, P. (1986): The theory of technological competition, in J.E. Stiglitz and G.F. Mathewson (eds.): New developments in the analysis of market structure. Cambridge (U.S.): MIT Press.

Ericson, R. and A. Pakes (1995): Markov perfect industry dynamics: A framework for empirical analysis, Review of Economic Studies 62, 53-82.

Geroski, P.A. (1998): An applied econometrician's view of large company performance. Review of Industrial Organization 13, 271-93.

Geroski, P.A., S. Machin, J. Van Reenen (1993): Innovation and firm profitability, RAND Journal of Economics 24, 198-211.

Gilbert, R.J. and D.M.G. Newbery (1982): Preemptive patenting and the persistence of monopoly, American Economic Review 72, 514-26.

Gilbert, R.J. and D.M.G. Newbery (1984): Uncertain innovation and the persistence of monopoly: Comment, American Economic Review 74, 238-42.

Griliches, Z. (1990): Patent statistics as economic indicators: A survey. Journal of Economic Literature 28, 1661-707.

Griliches, Z. (1994): Productivity, R&D, and the data constraint, American Economic Review 84, 1-23.

Griliches, Z. and I. Cockburn (1994): Generics and new goods in pharmaceutical price indexes, American Economic Review 84, 1213-32.

Griliches, Z., B.H. Hall and A. Pakes (1991): R&D, patents, and market value revisited: Is there a second (technological opportunity) factor?, Economics of Innovation and New Technology 1, 183-201.

Grossman, G.M. and E. Helpman (1991a): Innovation and Growth in the Global Economy, Cambridge (U.S.): MIT Press

Grossman, G.M. and E. Helpman (1991b): Quality ladders in the theory of economic growth, Review of Economic Studies, 58, 43-61.

Gruber, H. (1992): Persistence of leadership in product innovation. Journal of Industrial Economics, 40, 359-75.

Hall, B.H. (1987): The relationship between firm size and firm growth in the US manufacturing sector, Journal of Industrial Economics, Vol.. 35, 583-606

Hall, B.H., Z. Griliches and J.A. Hausman (1986): Patents and R&D: Is there a lag?, International Economic Review, 35, 583-606.

Himmelberg, C. and B.C. Petersen (1994): R&D and Internal Finance: A Panel Study of Small Firms in High-Tech Industries, Review of Economics and Statistics, 76, 38-51.

Ijiri, Y. and H. Simon (1977): Skewed size distributions and the sizes of business firms. Amsterdam: North Holland

Jovanovic, B. (1982): Selection and the evolution of industry, Econometrica 50, 649-70.

Kihlstrom, R.E. and J.J. Laffont (1979): A general equilibrium theory of firm formation based on risk aversion, Journal of Political Economy 87, 19-48.

Klepper, S. (1996): Entry, exit, growth, and innovation over the product cycle. American Economic Review, 86, 562-83.

Klette, T.J. and S.E. Førre (1998): Innovation and job creation in a small open economy – evidence from Norwegian manufacturing plants 1982-92. The Economics of Innovation and New Technology 5, 247-72.

Klette, T.J. and Z. Griliches (1998): Empirical patterns of firm growth and R&D investment: A quality ladder model interpretation. NBER Working Paper no. 6753.

Klette, T.J. and F. Johansen (1998): Accumulation of R&D capital and dynamic firm performance: a not-so-fixed effect model. Annales d'Economie et de Statistique. No. 49/50, 389-419.

Kortum, S. (1993): Equilibrium R&D and the decline in the patent-R&D ratio: U.S. evidence. American Economic Review (Papers and Proceedings) 83, 450-57.

Kuznets, S. (1930): Secular movements in production and prices, Boston: Houghton Mifflin Co.

Lippman, S.A. and R.P. Rummelt (1982) Uncertain imitability: An analysis of interfirm differences under competition, Bell Journal of Economics 13, 418-38.

Lucas, R.E. (1978): On the size distribution of firms, Bell Journal of Economics 9, 508-23.

Mansfield, E. M. Schwartz and S. Wagner (1981): Imitation costs and patents: An empirical study. Economic Journal, 91, 907-18.

McCloughan, P. (1995): Simulation of concentration development from modified Gibrat growth-entry-exit processes. Journal of Industrial Economics, 43, 405-33.

Metrick, A. and R. Zeckhauser (1996): Price versus quantity: Market clearing mechanisms when sellers differ in quality. NBER Working Paper 5728.

Nelson, R.R. (1988): Modeling the connections in the cross section between technical progress and R&D intensity. RAND Journal of Economics 19, 478-85.

Nickell, S.J. (1996): Competition and corporate performance. Journal of Political Economy, 104, 724-46.

Pakes, A. (1994): Estimation of dynamic structural models. Problems and prospects II: Mixed continuous-discrete models of market interactions. In C. Sims and J.J. Laffont (eds.): Proceedings of the 6th World Congress of the Econometric Society. Cambridge (U.K.): Cambridge University Press.

Pakes, A. and R. Ericson (1998): Empirical implications of alternative models of firm dynamics. Journal of Economic Theory, 79, 1-45.

Pakes, A. and P. McGuire (1994): Computing Markov-perfect Nash equilibria: numerical implications of a dynamic differentiated product model, RAND Journal of Economics 25, 555-89.

Pakes, A. and M. Schankerman (1984): An exploration into the determinants of research intensity, in Z. Griliches (ed.): R&D, Patents and Productivity, Chicago: Chicago University Press .

Reinganum, J.F. (1983): Uncertain innovation and the persistence of monopoly, American Economic Review 73, 741-8.

Reinganum, J.F. (1985): Innovation and industry evolution, Quarterly Journal of Economics 100, 81-98.

Romer, P.M. (1990): Endogenous technological change, Journal of Political Economy 98, S71-S102.

Schmalensee, R. (1989): Inter-industry studies of structure and performance. Ch. 16 in R. Schmalensee and R. Willig (eds.): Handbook of Industrial Organization, Vol. 2. Amsterdam: North Holland.

Schumpeter, J.A. (1942): Capitalism, Socialism and Democracy, London: George Allen & Unwin Ltd.

Steindl, J. (1968): Size distributions in economics. In D.L. Sills (ed.): International Encyclopedia of Social Sciences. London: MacMillan.

Sutton, J. (1997): Gibrat's Legacy. Journal of Economic Literature 35. 40-59.

Thompson, P. (1996): Technological opportunity and the growth of knowledge: A Schumpeterian approach to measurement. Journal of Evolutionary Economics, 6, 77-97.

Appendix A: Previous literature

The specification of our model is inspired by the macro-models of endogenous growth developed by Aghion and Howitt (1992), Grossman and Helpman (1991a, 1991b) and Barro and Sala-i-Martin (1995, ch.7). The quality ladder models introduced by Aghion and Howitt (1992) and Grossman and Helpman (1991a, 1991b) have the property that each innovation is introduced by a new firm, while the producers of the old product varieties are driven out of the market. It is difficult to reconcile this property with the persistence of large firms that we tend to see in empirical data, at least at an annual frequency; see surveys of the empirical literature by Sutton (1997) and Geroski (1998). Thompson (1996) makes the same criticism of the quality ladder growth model. He develops a complete model of R&D investment and firm growth, somewhat different from the model we present. On the basis of his model, he presents an empirical analysis of the relationship between R&D and the stock market value of the firm.

The model presented in Barro and Sala-i-Martin (1995, ch.7), emphasizing the persistent dominance of the established firms rather than the continuous replacement of the leading firm, is similar to our model. There is, however, a serious error in the model by Barro and Sala-i-Martin (1995, ch. 7.4), and this is why we have developed a new model. Specifically, the second order condition associated with the first order condition in their equation (7.39) is not satisfied. That is to say, the level of R&D investment that Barro and Sala-i-Martin identify for the incumbent is not the optimal level of R&D investment.

Our (partial equilibrium) version of the quality ladder model is closely related to patent race models discussing the persistence of monopoly; see in particular Gilbert and Newbery (1982, 1984). Reinganum's (1985) model of industry evolution is also related to our analysis. Her model has the same property as the quality ladder models referred to above, where each innovation is introduced by a new firm. The model presented below extends the analysis by Reinganum and Gilbert-Newbery by allowing for competition between horizontally differentiated products. In our model each competing product (variety) is upgraded separately, i.e. each competing product is moving up its own quality ladder.

Our analysis has a focus similar to the work by Ericson and Pakes (1995), Pakes and Ericson (1998) and Pakes and McGuire (1994). They also present a fully specified model of firm growth with investment in innovative activities as the determinant of firm growth. Their model is more sophisticated than ours in that it allows for strategic interactions and considerations between the firms in their R&D investments decisions. They also have a more complete analysis with respect to firm entry and exit. However, the cost of this sophistication is that the Ericson-Pakes model is analytically difficult to handle, and the model must be examined through simulations, while our simpler model is analytically more tractable. The two models are to some extent complementary in the sense that our model can most easily be justified in situations where the firms considered are small relative to the market, while the case with many firms creates problems for the simulations of the Ericson-Pakes model (see Pakes and McGuire, 1994).

There are also some earlier studies of firm heterogeneity that are related to our study, such as Jovanovic (1982) and Lippman and Rummelt (1982). Jovanovic's analysis is a dynamic version of Lucas' (1978) model, which is similar to the model by Kihlstrom and Laffont (1979). Both Lucas and Kihlstrom-Laffont present theoretical models that are consistent with heterogeneous firms, but they are both static and the sources of the heterogeneity are given exogenously, as is also the case for Jovanovic's model. The relationship between Jovanovic's model and the Ericson-Pakes model is extensively discussed in Pakes and Ericson (1998), and their discussion is equally relevant for the relationship between Jovanovic's model and the model we present.

Cohen and Klepper (1996) present a model that can rationalize a number of empirical regularities regarding R&D investments and firm size. Our model is in several respects similar to their analysis of the relationship between R&D and size, but it is somewhat more complete as it rationalizes why each firm's profits from its next innovation are constrained by its current size.

More generally, our framework explicitly models firm growth, while firm growth is treated rather briefly in Cohen and Klepper's model. Klepper (1996) also examines a related model of firm growth and industry evolution driven by innovation. Klepper's model is able to rationalize a number of interesting empirical regularities of firm growth and industry evolution, but the model is highly stylized at the individual firm level and therefore difficult to reconcile with empirical studies based on firms as the unit of observation.

Dasgupta (1986) is to some extent also related to our paper, in that he presents a theoretical model consistent with a number of the empirical observations we also consider (and some others). However, Dasgupta's analysis is based on a static model and does not make any predictions about patterns of firm growth and firm heterogeneity. Indeed, his model assumes identical firms.

Appendix B: The sample of Norwegian R&D firms

Our sample of Norwegian R&D firms covers the high-tech industries ISIC 382, 383 and 385, with observation for the years 1985, 87, 89, 91, 93 and 95. The sample is constructed by merging data from the bi-annual R&D-survey with data from the annual manufacturing census, and the unit of observation is a line-of-business within a firm at the three digit level. Only line-of-businesses with at least 20 employees are included as the R&D survey covers a non-representative sample of smaller firms. Some observations with very high R&D-intensity have also been eliminated[26]. The sample contains 586 observations for 265 line-of-businesses, and only observations with non-zero R&D investments are included. The same data sources were used in Klette and Johansen (1998), where further details are provided.

The sales variable is the value of gross production corrected for taxes and subsidies, while operating surplus is this measure of sales minus materials, energy and labor costs, also corrected for taxes and subsidies. R&D includes all intramural and extramural R&D expenditures. As mentioned, all variables are broken down by line-of-business within each firm.

---

[26] The results are based on observations with R&D-intensity below 50 percent, but the results are very similar if the threshold is increased to 100 percent.