SELF-INSURANCE AND UNEMPLOYMENT BENEFIT IN A LIFE-CYCLE MODEL OF LABOUR SUPPLY AND SAVINGS

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Self-Insurance and Unemployment Benefit in a Life-Cycle Model of Labour Supply and Savings

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Abstract

This paper shows that flexibility in labour supply changes the incentives to save and the welfare cost of uncertainty in the presence of incomplete markets. Flexibility can lead to increased precautionary saving because the utility cost of saving is lower when labour supply is flexible. However, consumption is more smooth and welfare higher. Unemployment benefit provides alternative insurance against negative wage shocks. This reduces the need for asset holdings, but a more generous benefit programme can lead to increased asset accumulation because decreased participation means individuals need more assets in order to smooth consumption. Insurance through secondary earner labour supply becomes increasingly effective, and hence reduces precautionary saving, as the correlation between shocks becomes increasingly negative.

Keywords: precautionary saving, unemployment benefit, life-cycle labour supply, participation. *JEL Classification*: D91, H31

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Executive Summary

- This paper uses numerical methods to model individual choice of labour supply, consumption and savings across the life-cycle when wages are uncertain.
- Uncertainty leads to increased labour supply and reduced consumption when young and causes saving against the possibility of low wages in the future. In old age, individuals increase consumption and leisure.
- When labour supply is flexible, consumption is more smooth than when labour supply is fixed. Individuals can save through increased labour supply rather than just through reduced consumption and this reduces the need to give up consumption. The welfare cost of uncertainty is lower when labour supply is flexible.
- Saving is greater when labour supply is flexible than when it is fixed. A flexible labour supply means individuals can vary labour supply according to the realised wage, taking more leisure when the wage is low, and this reduces the need for saving. However, the cost of saving is less because saving can be provided by increased labour supply as well as reduced consumption.
- When households have two sources of income, secondary earners use their own labour supply to insure against shocks to primary earner income. This insurance becomes increasingly effective, reducing the need for precautionary saving, as the correlation of secondary earner wages with partner income becomes increasingly negative. A negative correlation means secondary earners will work a large number of hours for a high wage when primary earner income is low but not work at all when primary earner income is high.
- Unemployment benefit provides insurance and this reduces the need for precautionary saving. However, individuals choose to have periods when they are not working and therefore increase saving to smooth consumption across these periods. Thus, saving can increase with the generosity of benefits.

1 Introduction

When markets are incomplete, individuals save against shocks to income and the lack of an insurance market reduces their welfare. There is an extensive literature showing this in models where labour supply is fixed.[1] When labour supply is flexible, however, individuals can react to negative wage shocks by changing labour as well as by dissaving and this reduces the welfare cost of incomplete markets as well as changing the incentive to save. This paper addresses the question of whether individuals save greater or less amounts when labour supply is flexible and then calculates the welfare cost of the lack of insurance in a model with this flexibility. The lack of insurance against income uncertainty has led to unemployment benefit programs by governments and these also mitigate the cost of uncertainty. This raises the question of whether unemployment benefit programs reduce the incentives to save and to participate.

The basic model in this paper differs from the standard model used for precautionary savings analysis in three ways: first, there is the possibility of within-period substitution in addition to intertemporal substitution. As the degree of intertemporal substitution decreases, consumption within a period becomes a closer substitutes for leisure within that period relative to consumption in other periods and so the split between leisure and consumption in any given period becomes less important to lifetime utility than the level of total within period spending. The second change is that uncertainty is not only over income, but also over the relative price of the goods in the utility function. This means that the income loss associated with a low realisation of the wage is offset by leisure being cheaper. This reduces the cost of uncertainty and the extent of this reduction increases as the degree of within period substitutability increases. The final change relative to the model with a fixed labour supply is that the additional good, leisure, is constrained by the number of hours available and this introduces a participation choice and allows modelling of how uncertainty affects participation across the life-cycle. This basic model is then extended, first so that indi-

[1] For example, Zeldes (1989), Deaton (1991), Carroll (1992) and Attanasio *et al.* (1999).

viduals have an exogenous stream of unearned but stochastic income from their partner and second, so that individuals receive unemployment benefit if they are not-participating in the labour force.

The paper uses numerical methods to show five main results: first, in the basic model, uncertainty means individuals work more hours early in the life-cycle and fewer hours when old. Second, consumption is smoother when labour is flexible than it would be if the labour supply were fixed because precautionary accumulation occurs through giving up leisure as well as through giving up consumption. Third, asset accumulation is *greater* when labour supply is flexible. A flexible labour supply means that individuals can vary their labour supply according to the realised wage, taking more leisure when the wage is low, and this reduces the benefit of holding precautionary balances. However, the cost of accumulating precautionary balances is less because balances can be accumulated through either cutting consumption or increasing labour supply. For a wide range of parameter values considered, this second effect dominates and asset accumulation is greater. Fourth, when partner income is introduced, individuals use their own labour supply to insure against shocks to partner income. The effectiveness of this depends on the correlation of secondary earner wages with partner income: when these are negatively correlated secondary earners can effectively insure their income and will either work a significant number of hours or not participate at all. Finally, unemployment benefit has two effects on asset accumulation: the increased insurance reduces the need for precautionary balances; but this is offset because individuals choose to have periods when they are not working and use savings to smooth consumption across these periods. The balance of these two effects depends, in particular, on the degree of persistence in wage shocks and the generosity of benefits.

Section 2 reviews the literature; section 3 outlines the general formulation of the model; section 4 compares the flexible labour supply model, ignoring partner income and unemployment insurance, with a fixed labour supply model to show the role of labour supply in self-insurance. This comparison is made first, through showing life-cycle profiles of consumption and saving and second, through calculating lifetime expected cost functions. Section 5 introduces partner income as a form of insurance; section 6 introduces unemployment insurance and section 7 concludes.

2 Literature

There is wide literature on the life-cycle effects of uncertainty assuming labour supply is fixed. Deaton (1991) and others have shown how uncertainty leads to asset accumulation for precautionary reasons when marginal utility is convex. Modelling choice without the assumption that labour supply is fixed has two purposes: first, to show the implications for individual behaviour, and second, to be used in structural estimation of life-cycle labour supply models when λ-constant or two-stage budgeting techniques cannot be used to recover structural parameters.

Heckman (1974) examined life-cycle behaviour when labour supply and savings are chosen, but under certainty and ignoring non-participation. He showed that consumption can track income even if individuals are unconstrained by capital markets. The Euler equation for leisure shows that the marginal utility of leisure must be high when wages are high, but if consumption and leisure are additively separable then this must be satisfied by varying leisure alone and the consumption path will be flat. However, when consumption is not additively separable from leisure, the path of consumption depends on whether leisure and consumption are substitutes or complements: if they are complements then the high marginal utility of leisure when wages are high must reflect low leisure and low consumption. If leisure and consumption are substitutes, then the high marginal utility of leisure will reflect low leisure, but high consumption. In other words, consumption will be high when wages are high. As discussed in the introduction and returned to below, this issue of the substitutability of leisure and consumption within periods depends on the intertemporal elasticity of substitution. The simulations in section 4 are an attempt to extend this analysis to the case where wages are uncertain and participation is important.

There has been research estimating models of life-cycle labour supply either making the assumption that wages are certain and hence using Frisch elasticities (e.g. Heckman and MaCurdy, 1980; MaCurdy, 1981; Browning *et al.,* 1985) or taking savings to be a sufficient statistic for intertemporal behaviour and used two-stage budgeting (Blundell and Walker, 1986; Blundell *et al.*, 1993; MaCurdy, 1983). Further, Blundell, Magnac and Meghir (1997) model the joint labour supply-saving decision and show when solution of the dynamic program is not necessary.

The key point, however, is that if there is uncertainty, non-participation is significant and consumption and leisure are non-separable, then it is necessary to solve the stochastic dynamic program. The evidence from both the UK and the US shows that the participation issue is crucial, with significant changes over time in the participation rates, and in particular, large increases for women with high education levels (Blundell and MaCurdy, 1998). There has also been a decline over the past 30 years in the participation rates of older men. The evidence on the nonseparability of leisure and consumption is less investigated, but Browning and Meghir (1991) find that leisure in a given period is non-separable from consumption within the same period. Similarly, Blundell, Browning and Meghir (1994) show that it is necessary to condition on labour market status to obtain estimates from Euler equations which identify the true parameters.

Eckstein and Wolpin (1989) model the dynamic participation decisions of women, taking the income of partners to be exogenous. Participation in each period is the only control variable, and there is no saving or borrowing and no choice of hours conditional on participation. The key dynamic element in this model is that wages today depend on past labour market experience, and there is no role for uncertainty in changing participation. More generally, when the wage is dependent on experience, uncertainty may induce greater participation in the current period in order to increase utility tomorrow through a higher wage. The simulations in section 5 aim to look at the participation choices of secondary earners allowing for saving and a choice of hours.

However, these models abstract from any unemployment benefits and the effect that benefits can have on participation or asset accumulation. In particular, the fast (or indeed, immediate) withdrawal of benefits when individuals start work acts as a serious disincentive to begin work unless planning to work for a significant number of hours at a reasonable wage. This non-convexity in the budget set that individuals face is ignored in life-cycle analysis of labour supply and the simulations in section 6 attempt to address this issue. Social security also impacts on the participation decision through the timing of retirement (e.g. Rust and Phelan, 1997) but the model in the current paper abstracts from retirement to focus on the interaction of uncertainty with participation and savings. However, Hubbard *et al.* (1995) suggest that social security is a reason for reduced asset accumulation, because of asset-testing for retirement benefits, but they take participation and labour supply to be exogenous.

3 Model

The problem faced by the individual is to choose consumption and leisure for each period of her life in order to maximise expected utility. Her future wage rate is uncertain and uninsurable, but the interest rate is assumed to be known and constant. It is assumed that utility is additively separable across time periods, but that utility from consumption is not separable from utility from leisure within a period. The difference between spending on consumption and spending on leisure is that leisure in each period is constrained to be below the maximum number of hours available. Without this constraint, the individual would, in effect, be solving for an aggregate consumption good for each period, and the inter-period decision would be separate from the intra-period decision.

The general formulation of the model is given by the maximisation problem:

$$\max_{c_s, l_s} E_t \left[\sum_{s=t}^{T} \beta^{s-t} u\left(c_s, l_s\right) \right] \tag{1}$$

subject to,

$$A_{s+1} = \begin{cases} (1+r)\left[A_s + x_s + (1-l_s)(1-\tau)\,w_s - c_s\right] & \text{if working} \\ (1+r)\left[A_s + x_s + B - c_s\right] & \text{if not working} \end{cases} \tag{2}$$

where the discontinuity in moving from not-working to working arises only if $B > 0$. There is also a constraint on leisure,

$$1 \geq l_s \geq 0 \tag{3}$$

and a terminal condition,

$$A_{T+1} = 0 \tag{4}$$

where total time available in each period is normalised to 1. Time in period t is spent on leisure, l_t, or work. A_t is the amount of assets held at the start of period t, w_t is the real wage, x_t is partner income[2] and B the level of unemployment benefit, independent of age.[3] A tax on wages, τ, is levied to finance unemployment benefit. The terminal condition (4) is not at all innocuous in the presence of uncertainty. The individual knows the length of her life, but she cannot die in debt. This imposes a severe constraint on the borrowing that she may incur through her life. In particular, she will not borrow more than the discounted sum of the minimum income she will receive in each remaining period of her life (Hakansson, 1970). This restriction on non-collateral borrowing suggests there may be over-investment in goods that can be used as collateral, especially housing. This is not considered here, and the assumption that individuals must have a probability of zero of dying in debt is not relaxed.

The real wage is assumed to be determined by the process:

$$\ln w_t = \varrho \ln w_{t-1} + \alpha_t + \theta_t \ , \quad \theta_t \sim i.i.d.N\left(-\frac{\sigma^2}{2}, \sigma^2\right) \tag{5}$$

where θ_t is the shock to wages in age t, and α_t captures the deterministic drift of the wage equation. It is assumed that past labour market participation

[2] By partner income, I mean the income of the primary earner in the household.

[3] Making the benefit level age-dependent in a deterministic way is straightforward; making the benefit conditional on the asset stock is somewhat harder because the value function conditional on labour market status will not be monotonic in the asset stock.

does not affect the current wage, thus the model abstracts from human capital accumulation and this implies that the wage rate grows with age whether or not individuals participate. Since the shocks are multiplicative in the level of wages, shocks have a bigger impact in absolute terms when the wage is high. The variance of the shocks is assumed to be independent of age. In sections 4 and 5, $\varrho = 1$ and so the log of wages follows a random walk and all shocks to wages are permanent. When T is large, ignoring transitory income shocks is not likely to be a serious simplification because *i.i.d.* transitory shocks will average out across the lifetime and have little impact on precautionary saving. However, transitory shocks may have an impact on participation because intertemporal substitution is greater when shocks are transitory. This is highlighted in section 6 where the value of ϱ is varied.

Partner income is assumed to be exogenous (as in Eckstein and Wolpin, 1989) and determined by the non-stationary process:

$$\ln x_t = \ln x_{t-1} + d_t + \zeta_t, \quad \zeta_t \sim i.i.d.N\left(-\frac{\sigma_\zeta^2}{2}, \sigma_\zeta^2\right) \tag{6}$$

where d_t gives the deterministic drift in income. The shock to partner income, ζ_t, is correlated with the shock to individual wages, θ_t, and the coefficient of correlation is given by ρ.

The remaining assumptions are as follows:

(A1) r constant and $(1 + r)\beta = 1$

(A2) No demographics affecting preferences or labour supply.

(A3) Infinite demand for labour at a wage equal to the marginal product of labour. In other words, shocks to the wage are shocks to productivity.

Assumption $(A1)$ is to simplify the problem, and in addition the effects of interest rate uncertainty have been extensively analysed elsewhere (e.g., Skinner, 1988). Similarly, assumption $(A2)$ has been discussed in Attanasio *et al.* (1999). Assumption $(A3)$ rules out "involuntary unemployment", i.e. a wage of zero. Thus all periods of non-participation are chosen in response to the offered wage rate.

The within period utility function is assumed to be isoelastic and Cobb-Douglas between consumption and leisure in t,[4]

$$u(c_t, l_t) = \frac{\left(c_t^{\eta} l_t^{1-\eta}\right)^{1-\nu}}{1-\nu} \tag{7}$$

This imposes the restriction that shares of expenditure on the two goods in t remain constant as total spending in t increases, and this is exploited in the solution below.

The value function of the model is given by

$$V_t(A_t, w_t) = \max_{c_t, l_t} \{u(c_t, l_t) + \beta E_t[V_{t+1}(A_{t+1}, w_{t+1})]\} \tag{8}$$

subject to constraints (2) and (3). The finite horizon makes the value function non-stationary. Further, it will be kinked in the presence of unemployment benefit, and so the solution method in sections 4 and 5, where $B = 0$, differs from that in section 6. Both methods are discussed in the appendix.

4 Self-Insurance through Labour Supply

This section examines the model excluding partner income and without any unemployment benefit. The aim is to address the question of whether flexibility in labour supply reduces the need for precautionary saving and increases consumption smoothing. The section first discusses the solution to the problem, then shows life-cycle profiles in subsection 4.1 and finally makes welfare comparisons in subsection 4.2.

The first order conditions of the value function (8) can be reduced to:

$$\frac{\partial u(c_t, l_t)}{\partial c_t} = \beta(1+r) E_t\left[\frac{\partial u(c_{t+1}, l_{t+1})}{\partial c_{t+1}}\right] \tag{9}$$

[4] When utility is isoelastic, the coefficient of relative risk aversion determines both the extent of intertemporal substitution and also prudence which affects the degree to which substitution is reduced by uncertainty. This implies that the amount individuals care about future uncertainty cannot be separated from the degree of intertemporal substitution desired under certainty.

$$\begin{aligned}\frac{\partial u(c_t, l_t)}{\partial l_t} &\geq w_t \frac{\partial u(c_t, l_t)}{\partial c_t} \\ l_t &\leq 1\end{aligned} \tag{10}$$

The value function is concave when $B = 0$ so the solution to these first-order conditions is a maximum.

Using the particular form of the utility function (7) gives the Euler equation (9) to be

$$c_t^{\varepsilon-1}\, l_t^{\delta} = \beta(1+r) E\left[c_{t+1}^{\varepsilon-1}\, l_{t+1}^{\delta}\right] \tag{11}$$

where, for notational simplicity, $\varepsilon = \eta(1-\nu)$ and $\delta = (1-\eta)(1-\nu)$. Further, using the utility function, it is possible to write the solution for leisure, using (10), as

$$l_t = \min\left[\frac{(1-\eta)}{\eta}\frac{c_t}{w_t}, 1\right] \tag{12}$$

and this can be substituted into the Euler equation (11) to give one functional equation in one unknown function, $c_t(A_t, w_t)$, for each age t.

Consumption is a function of both the asset stock and the wage rate rather than simply cash-in-hand (as in Deaton, 1991) because the wage rate has two effects on the optimal decision: a higher wage rate in period t means more income in period t, but it also means leisure is relatively more expensive; further, the size of the substitution effect of a given wage rate depends on the amount of assets held and so both the wage and the asset stock are needed to calculate the optimal action. The expectations operator means equation (11) has to be solved numerically when marginal utility is non-linear and the numerical solution method is discussed in the appendix.

Before turning to life-cycle profiles using these solutions, it is necessary to discuss the degree of substitutability between consumption and leisure implied by the utility function (7). In an intertemporal context, the relevant notion of substitutability when wages are deterministic is Frisch substitutability, $\left.\frac{\partial c_t}{\partial w_t}\right|_{\lambda_t}$ where λ_t is the marginal utility of income in t. This is interpreted as the effect on the path of consumption of wages increasing across time. For a general utility function, assuming an interior solution for leisure,[5]

$$\left.\frac{\partial c_t}{\partial w_t}\right|_{\lambda_t} = -\frac{1}{u_{cc}u_{ll} - u_{cl}^2} u_{cl}\lambda_t \qquad \begin{cases} < 0 & \text{if } u_{cl} > 0 \\ > 0 & \text{if } u_{cl} < 0 \end{cases} \tag{14}$$

The denominator is positive because utility is strictly concave and the denominator is therefore the determinant of a negative-definite matrix. Using equation (7),

$$u_{cl}(c,l) = (1-\nu)\,\eta\,(1-\eta)\,c^{\eta(1-\nu)-1} l^{(1-\eta)(1-\nu)-1}, \tag{15}$$

which is negative if $\nu > 1$, indicating within period consumption and leisure are Frisch substitutes. When $\nu < 1$, consumption and leisure are Frisch complements. This dependence on the value of ν arises because ν determines the extent of desired utility smoothing: a higher ν means increased utility smoothing, and less exploitation of possibilities of intertemporal substitution. The desire for intertemporal substitution arises because wage growth means utility tomorrow becomes more expensive. When $\nu > 1$, the income effect of a higher wage tomorrow dominates the intertemporal substitution effect: in other words, utility will remain smooth, but will be higher in all periods. This means either consumption or leisure will be high in each period, with consumption being high when the wage is high: this is the key result of Heckman (1974). By contrast, when $\nu < 1$, the intertemporal substitution of utility dominates the income effect and so utility will be increased in periods of low wages by simultaneously higher consumption and higher leisure.[6]

[5] This term for Frisch substitutability can be derived as follows: assuming an interior solution, the first order condition (10) can be decomposed into two equations: $u_l(c_t, l_t) = \lambda_t w_t (1+r)$ and $u_c(c_t, l_t) = \lambda_t (1+r)$, where λ_t is the Lagrange multiplier on the intertemporal budget constraint (2). Taking the total derivative of these two equations and putting into matrix form, gives

$$\begin{pmatrix} u_{cc} & u_{cl} \\ u_{cl} & u_{ll} \end{pmatrix} \begin{pmatrix} \frac{\partial c_t}{\partial w_t} \\ \frac{\partial l_t}{\partial w_t} \end{pmatrix} = \begin{pmatrix} 0 \\ \lambda_t \end{pmatrix} \tag{13}$$

Inverting the first term on the left hand side and rearranging gives the expression for Frisch substitutability in the text.

[6] This dependence is exactly analogous to the conflicting substitution and income ef-

However, when wages are uncertain, Frisch substitutability is no longer valid because the marginal utility of wealth depends on the realised wage rate. This is the reason why recourse to full stochastic dynamic programing is necessary to establish how consumption and labour supply respond to anticipated and unanticipated changes in wages. However, the intuition behind Frisch substitutes can be used to explain the stochastic results: a higher value of ν increases desired utility smoothing under uncertainty, and this again increases the effective substitutability of within period leisure and consumption. Similarly, when $\nu < 1$, within period leisure and consumption are effectively complements.

4.1 Life-Cycle Profiles

The first aim here is to show the life-cycle profiles of consumption and labour supply under uncertainty when labour supply is flexible. These profiles show how consumption, leisure and saving vary with the wage rate under uncertainty. The second aim is to show that a flexible labour supply leads to *greater* precautionary saving than a fixed labour supply, and also to greater consumption smoothing, for reasonable parameter values. The results are based on average profiles for 50,000 simulations using the relevant numerical solution to the Euler equation (11) and parameter values are varied to show behaviour at realistic and less-realistic values.

Effect of Uncertainty. The intertemporal behaviour of consumption and labour supply comprises three effects: first, planned intertemporal substitution due to the deterministic trend in the wage rate; second, planned intertemporal substitution due to prudence leading to the deferral of utility; and finally, the effect of particular wage realisations. These effects cannot be separated, and the aim of the simulations shown in figure 1 is to show

fects on consumption following an interest rate decrease. When $\nu > 1$, consumption in the current period decreases, because the income effect dominates; but when $\nu < 1$, consumption in the current period increases because the substitution effect dominates. The increase in the wage rate in the next period is analogous to an interest rate fall because the price of utility in the next period increases relative to today.

the combined effect on intertemporal consumption and labour supply.

Result 1 *Uncertainty leads individuals to defer utility by giving up both consumption and leisure when young and increasing both when old, relative to their choice under certainty.*

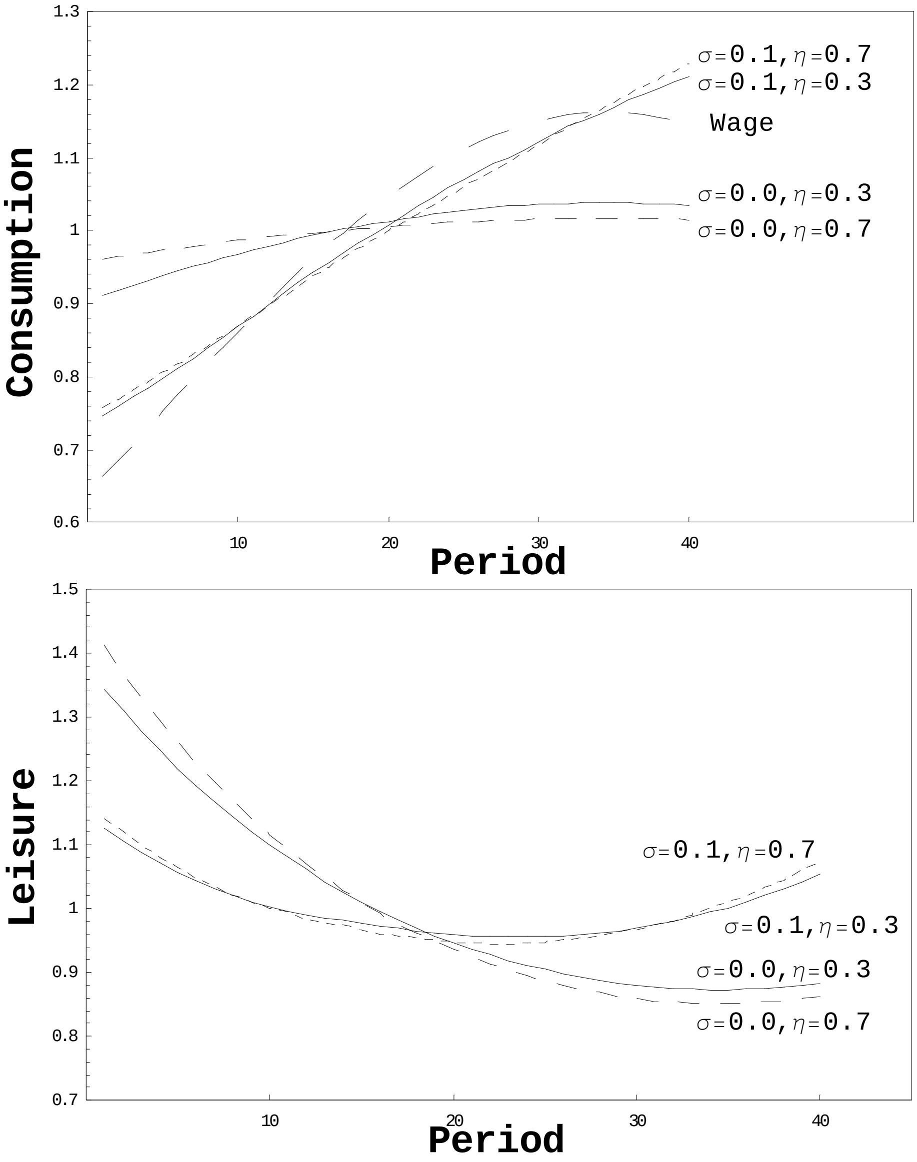
Uncertainty has less of an effect on both consumption and leisure when η is equal to 0.3, relative to the case when it is 0.7. The intuition for this is as follows: the variance in the wage means that income varies but it also means that the price of leisure relative to consumption varies. If individuals care mainly about leisure, then a high wage means more income, but this is partly negated by the higher price of leisure. Similarly, the negative effect of a low wage reducing income is partly negated by the fall in the price of leisure. However, if individuals care mainly about consumption, then the high wage means more income and a lower price of consumption relative to leisure. Thus, the more individuals care about leisure relative to consumption, the less important is uncertainty in the wage.

Figure 1 also shows that the consumption path is more concave when $\eta = 0.3$ than when $\eta = 0.7$ both under certainty and uncertainty. In other words, consumption tracks the wage more closely with a lower η. Given utility is being smoothed across time and leisure is more important to utility when $\eta = 0.3$, changes in the wage rate will be translated primarily into changes in consumption rather than changes in leisure.

It is possible to vary ν to show the effect on consumption smoothing of increased risk aversion, and also increased prudence. However, changes in ν when there is wage growth have two distinct effects: first, a change in ν changes the degree of Frisch substitutability and hence the planned extent of intertemporal substitution of utility: a higher ν means utility must be smoother and so consumption is higher in periods of high wages. Second, changes in ν change the amount individuals care about uncertainty, independent of wage growth. This latter effect is discussed below where the absence of wage growth makes the analysis clearer.

In these simulations it is very rare for individuals to choose not to participate. In the simulations reported above, all individuals participate in

Figure 1: Life-Cycle Paths for Consumption and Leisure

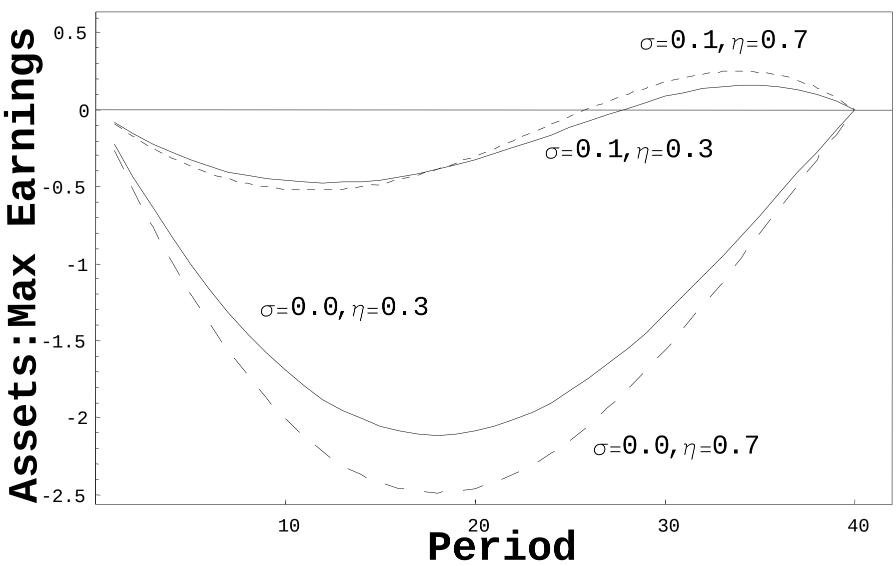


The lines in both graphs are normalised around a value of 1 by dividing actual values by their means. Mean consumption and leisure depend on η. The graphs show paths for two values of η, 0.3 and 0.7, holding $\nu = 1.5$. The graphs show paths under certainty (labelled $\sigma = 0.0$) and under uncertainty where $\sigma = 0.1$.

all periods. There are two effects here which negate each other: on the one hand, individuals will not-participate when the wage is low, which in the simulations above means when individuals are young; on the other hand, uncertainty makes individuals defer leisure and so makes them more likely to participate when young. However, even with no deterministic wage growth, non-participation still only reaches about 1.5% when $\eta = 0.3$ and is still not observed when $\eta = 0.7$. The underlying reason is that wages are the only primary source of income, and non-participation would require a large difference in wages between different periods.

Figure 1 shows that the effect of uncertainty on both consumption and leisure is greater the larger the size of η. However, it is not possible to see the differences in asset accumulation because both graphs are normalised to give lines centered on a value of one to allow comparability. Figure 2 therefore shows the extent of asset accumulation for the different scenarios.

Figure 2: Asset Accumulation across the Life-Cycle



Individuals begin and end their lives with no asset holdings. The figure shows the ratio of the end-of-period asset stock to maximum earnings, under certainty (labelled $\sigma = 0.0$) and under uncertainty (labelled $\sigma = 0.1$) and for two different values of η. Maximum earnings is the average across periods and across individuals of maximum earnings in each period (i.e. when $l_t = 0$). This denominator is chosen rather than the mean of actual earnings because maximum earnings is independent of preferences. $\nu = 1.5$.

When the wage rate is certain, the growth in wages means that individuals want to borrow against their future earnings. This is shown in figure 2 by the two lines labelled $\sigma = 0.0$. A larger value of η implies greater borrowing because the average value of labour supply is greater and so the individual will have greater earnings to borrow against. Uncertainty reduces the extent of borrowing because of the possibility of a series of negative shocks to wages. Such a very bad outcome could leave an individual with zero consumption if she has borrowed too much, and so she chooses to constrain borrowing. Indeed, in periods when the wage is high, the individual pays off all borrowing and saves against remaining shocks. As implied by figure 1, the effect on borrowing is greater when η is larger.

Comparison of a Flexible with a Fixed Labour Supply. In this section the effect of a fixed versus a flexible labour supply is analysed for given values of η. It is assumed that there is no wage growth and that the initial asset stock is zero. For any η, therefore, the fixed value of leisure is set equal to $(1 - \eta)$ which would be optimal under the given assumptions if wages were certain.[7] This means that any difference between the models in the degree of consumption smoothing and precautionary saving is due to the use of labour supply to reduce the effect of uncertainty.

It is well known (e.g. Deaton, 1992) that when labour supply is fixed, increases in the coefficient of relative risk aversion, ν, lead to faster consumption growth: increases in ν increase prudence and so individuals care more about uncertainty, and will therefore defer more consumption to the future in order to accumulate precautionary balances, and this decreases consumption smoothing. When labour supply is flexible, however, increases in the coefficient of relative risk aversion ν lead to *slower* consumption growth if $\nu < \bar{\nu}$. If $\nu > \bar{\nu}$, increases in ν lead to faster consumption growth. The value $\bar{\nu}$ varies with η : when $\eta = 0.3$, $\bar{\nu} \approx 5$; when $\eta = 0.7$, $\bar{\nu} \approx 1.5$ [8] This is shown in table 1, which shows average per period growth rates of consumption.

A higher ν increases prudence, but this is offset by the increase in substitutability between leisure and consumption with a higher ν. When leisure and consumption are closer substitutes, variation in the wage has less of an effect on utility: the effect of a low wage can be offset by increasing leisure and lowering consumption, and this reduces the variation in utility. This leads to increased consumption smoothing as the degree of substitutability increases. As ν becomes very large, however, the effect of increased prudence decreasing smoothing comes to dominate the increased substitutability of leisure and consumption.

[7] One alternative to this is to assume that labour supply is fixed equal to the average path of labour supply under uncertainty. There would still be a difference in the two models because the fixed labour supply leaves no scope to vary labour in response to a given wage realisation.

[8] These are approximations to $\bar{\nu}$ based on the evidence of simulations around $\bar{\nu}$. Simulations suggest that $\bar{\nu}$ is unique, but this cannot be shown analytically.

Table 1: Average Consumption Growth, Varying ν

	Coefficient of Relative Risk Aversion, ν							
Model	**0.5**	**0.75**	**1.5**	**3.0**	**5.0**	**10**	**15**	**25**
Flexible, $\eta = 0.3$	1.24	0.97	0.66	0.53	0.52	0.57	0.63	0.74
Fixed, $\eta = 0.3$	0.77	0.80	0.87	1.0	1.20	1.60	1.93	2.48
Flexible, $\eta = 0.7$	0.91	0.84	0.82	1.0	1.21	1.64	1.96	2.44
Fixed, $\eta = 0.7$	0.70	0.76	0.92	1.26	1.62	2.33	2.87	3.66

The table gives the average per period consumption growth using the average simulated profiles.

Table 1 also shows that the effect of increases in η on consumption smoothing depends on whether ν is greater than or less than 1. When $\nu > 1$, increases in η lead to faster consumption growth both when labour is fixed and when it is flexible. When $\nu < 1$, increases in η lead to slower consumption growth both when labour is fixed and when it is flexible. There are two effects here: first, a larger value of η means the utility value of leisure is less relative to consumption and hence intertemporal substitution will be undertaken more by leisure and less by consumption, leading to smoother consumption when η is high; second, a larger η means the cost of the fall in consumption due to a low wage is not offset as much by higher leisure as when leisure is highly valued. The first, intertemporal substitution effect, dominates when $\nu < 1$, but the second, uncertainty effect, dominates when $\nu > 1$. This discussion leads to the following result:

Result 2 *When labour supply is flexible, consumption is more smooth than when labour is fixed if $\nu > 1$; but is less smooth if $\nu < 1$.*

If $\nu > 1$, the non-separability of leisure and consumption in the utility function means that when the realised wage rate is low, utility can be increased by substituting leisure for consumption. This ability to use leisure to equalise marginal utility across time means that the consumption path is smoother when labour supply is flexible. This occurs even for very large ν. By contrast, when $\nu < 1$, consumption is less smooth when labour supply is flexible, because leisure and consumption are complements, and this means that when leisure is low consumption will also be low.

These effects arise only when within period utility is non-separable, because additive separability implies that the marginal utility of consumption is independent of leisure and so the Euler equation (11) must be satisfied by sacrificing consumption today. Intuitively, the increased smoothing when $\nu > 1$ arises because precautionary balances can be provided partly through increased labour and so reducing the amount of consumption that has to be sacrificed. Further, in old age, accumulated wealth is used not just to increase consumption, but also to increase the amount of leisure taken.

This result on consumption smoothing raises the question of whether precautionary saving is greater when labour is flexible or when it is fixed. When the labour supply is flexible, individuals can *ex-post* adjust their labour supply depending on the realised wage. This reduces the welfare loss of uncertainty in the wage and means that holding precautionary balances is less beneficial. However, *ex-ante* precautionary balances can be accumulated by sacrificing leisure as well as by sacrificing consumption. This means that the utility cost of accumulating a given amount of precautionary balances is less. In other words, when labour is flexible an increased asset stock tomorrow does not decrease the marginal utility of consumption tomorrow by as much as when labour is fixed; but increasing savings today does not increase the marginal utility of consumption today by as much. To equalise a given difference between marginal utility today and expected marginal utility tomorrow, this means that savings have to be greater when labour is flexible.

The corollary of the discussion on consumption smoothing is that when labour is fixed, increases in ν lead to increases in asset accumulation. Further, when labour is flexible and $\nu < \bar{\nu}$, increases in ν lead to decreases in asset accumulation. As before, when $\eta = 0.3$, $\bar{\nu} \approx 5$; when $\eta = 0.7$, $\bar{\nu} \approx 1.5$.[9]

[9]When consumption smoothing is at a maximum, precautionary saving appears to be

The reversal of the effect of higher ν between the two cases arises because, when labour is flexible, the higher ν implies leisure and consumption are closer substitutes and this lessens the negative impact of uncertainty. When $\nu > \bar{\nu}$, the benefit of increased substitutability is dominated by the increased prudence caused by an increase in ν. These points are illustrated in figure 3. The scales on the two graphs are different: the scale of the first graph is twice that of the second because there is a level difference between the pictures: asset accumulation is greater for all ν shown when labour is flexible. When $\eta = 0.3$ and labour is fixed, the maximum asset stock never exceeds half the mean of maximum earnings (i.e. when $w = \bar{w}$ and $l_t = 0$). When $\eta = 0.7$ and labour is flexible, the maximum asset stock rises to twice the mean of maximum earnings, for $\nu = 1.5$.

Asset accumulation also depends on the value of η analogous to the dependence of consumption on η: when $\nu < 1$, a higher η means less asset accumulation, and when $\nu > 1$, a higher η means increased asset accumulation. This holds for both the fixed and flexible labour supply cases. This leads to the following result:

Result 3 *Precautionary saving is greater when labour supply is flexible if $\nu < \tilde{\nu}$. When $\eta = 0.3$, $\tilde{\nu} \approx 25$; when $\eta = 0.7$, $\tilde{\nu} \approx 10$.*[10]

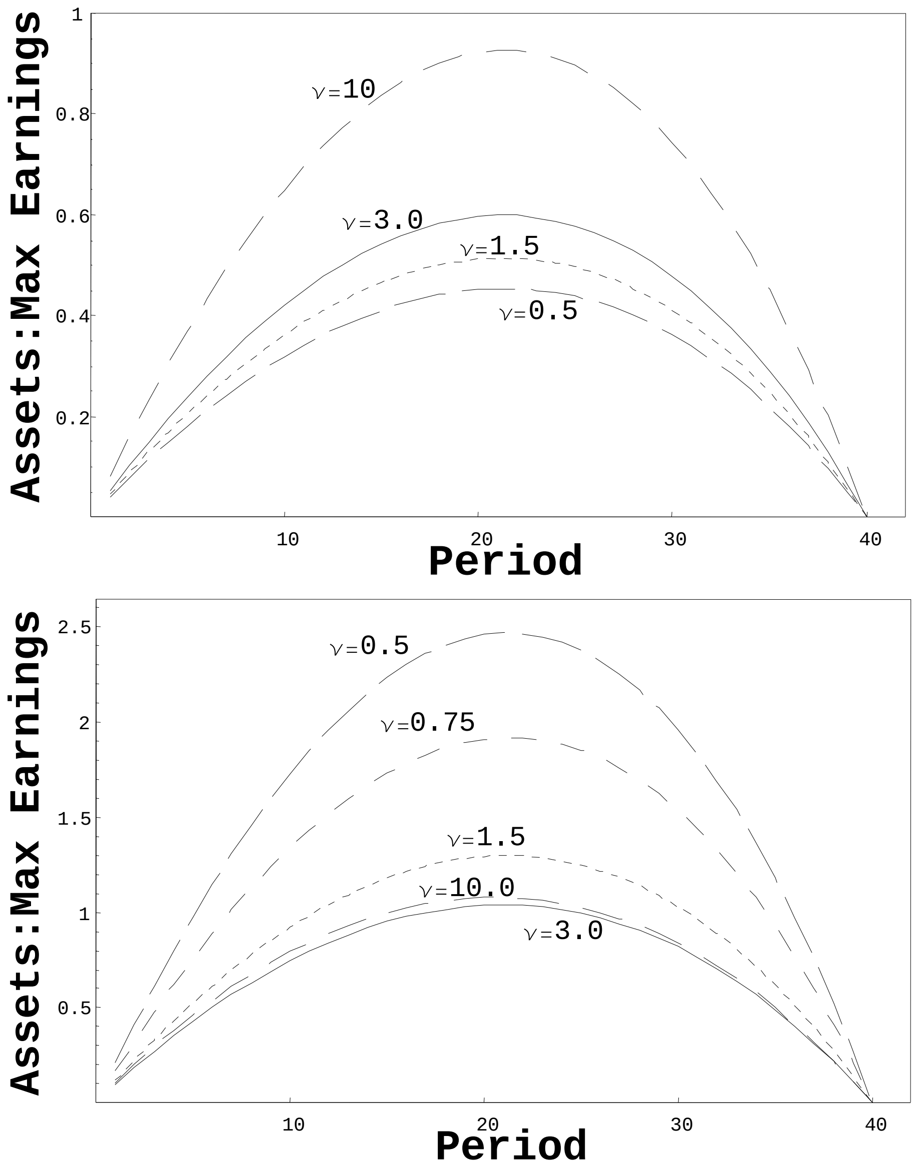
This result is true even when consumption is more smooth with a flexible labour supply. The reason is that labour supply of the young will be greater when flexible and this offsets the greater consumption smoothing and the net effect on saving can be positive. As ν becomes very large, however, precautionary accumulation increases faster with ν when labour is fixed than the increase when labour is flexible. Thus precautionary saving will be greater with a fixed labour supply only if ν is sufficiently large.

The point of analysing the extent of precautionary saving is to show the extent to which individuals defer utility because of uncertainty. The

at its minimum.

[10]The values $\tilde{\nu}$ are different from the $\bar{\nu}$ earlier in the text. The values of $\tilde{\nu}$ are approximations: with an exogenous labour supply, savings are accumulated until later in the life-cycle, so the extent of precautionary saving is hard to compare.

Figure 3: Precautionary Saving: Fixed and Flexible Labour Supply



Individuals begin and end their lives with no asset holdings. The graphs show the ratio of the end-of-period asset stock to maximum earnings. The first graph keeps labour supply fixed; the second graph allows labour supply to vary. Five values of ν (0.5,0.75,1.5,3 and 10) are used; $\eta = 0.3$. and $\sigma = 0.1$.

accumulation of physical capital is sufficient to show this only under the following assumptions: first, the wage today is independent of past employment status, ruling out returns to tenure and on-the-job learning; second, there is no explicit human capital accumulation; and third, there are no intertemporal nonseparabilities in utility. Models analysing these three issues, but ignoring savings, are discussed in Blundell and MaCurdy (1998).

4.2 Welfare Effects

The aim of this section is to calculate the expected monetary cost of achieving a given amount of expected utility both when labour is fixed and flexible and for different utility valuations of leisure. The expected consumption profile is not sufficient for determining welfare because of the non-separability of consumption and leisure. The welfare cost of uncertainty is calculated against the benchmark of a deterministic wage profile. It is assumed that there is no wage growth so the fixed and flexible labour supply models coincide when wages are certain.[11] The difference in the expected cost of achieving a given amount of expected utility must therefore be due to the use of leisure for precautionary reasons alone.

In the calculations below, for a given variance and mean of the shock to wages, the size of the *initial asset stock* is computed so that expected utility is equal to $\bar{U}$. This gives the expected cost needed to compensate for the uncertainty of σ :

$$E\left[c\left(w,\bar{U}\left|\sigma\right.\right)\right]=A_1+E\left[\sum_{t=1}^{T}\frac{w_t}{\left(1+r\right)^{t-1}}\right] \qquad (16)$$

This expected cost can be compared to the cost when $\sigma^2=0$. Further, this expected cost can be compared to the cost when labour is fixed.[12]

[11] The value of the fixed amount of labour that is supplied is clearly very important in determining the size of the welfare benefit of a flexible labour supply. Any choice will be arbitrary, and the choice in the text suggests labour supply is set optimally for certainty, despite the uncertain state of nature. The important point about this though, is that labour supply is assumed constant throughout life.

[12] The advantage of solving for expected cost by varying A_1 is that the model only has to be solved once for each set of parameter values. Once the model has been solved, the value

Table 2: Expected Cost of achieving Expected Utility, $\bar{U}$

		$\sigma = 0.1$, Flexible N^s	$\sigma = 0.1$, Fixed N^s
$\eta = 0.7$	$\bar{U} = -42.75$ (2.43)	*6.13* (11.12)	*6.61* (11.81)
	$\bar{U} = -43.5$ (-2.42)	*6.75* (6.75)	*7.11* (7.33)
	$\bar{U} = -44.25$ (-7.03)	*7.41* (2.78)	*7.71* (3.18)
$\eta = 0.3$	$\bar{U} = -58$ (7.28)	*1.35* (9.26)	*2.33* (10.70)
	$\bar{U} = -58.75$ (3.56)	*1.73* (6.03)	*2.38* (6.97)
	$\bar{U} = -59.5$ (-0.026)	*2.13* (2.95)	*2.52* (3.48)

The first column gives the value of $\bar{U}$ for each row. Underneath the value of $\bar{U}$ is the value of the initial asset stock, A_1, needed to achieve $\bar{U}$ when the wage is certain. The main numbers in the second and third columns are the percentage mark-ups of the expected cost of achieving $\bar{U}$ when wages are uncertain over the cost when wages are certain. This mark-up is found by varying the initial asset stock, and the values of A_1 necessary to achieve $\bar{U}$ are given in brackets. The second column shows the mark-up when leisure is flexible, the thrid column when leisure is fixed and set to the optimal value for leisure under certainty. The value of ν is held constant at $\nu = 1.5$

It is clear that the welfare cost of uncertainty will be lower when labour is flexible because individuals have an additional choice variable, but table 2 shows the extent of the difference for different values of η. When $\eta = 0.7$, the cost of uncertainty dominates the issue of whether or not labour is flexible.[13] However, when $\eta = 0.3$, the cost of uncertainty is greatly reduced by making labour flexible. It is, however, difficult to compare the model with $\eta = 0.3$ with the model $\eta = 0.7$ because the utility functions are different. The table also shows that the effect of uncertainty on the expected cost of achieving $\bar{U}$ depends on the level of $\bar{U}$: uncertainty has a greater impact on expected cost when lifetime welfare is lower both for $\eta = 0.3$ and $\eta = 0.7$.

The extent of the expected cost mark-up, and the difference between the two models depends on the degree of persistence in the wage process. If wages followed an *i.i.d.* process instead of the random walk in logs, then the uncertainty over the wage rate would have a very small cost as the shock in any one period would have little impact on lifetime wealth.

5 Insurance from Partner Income

This section introduces the exogenous, stochastic income stream, x_t, into the intertemporal budget constraint (2). This can be interpreted as introducing partner income, where the partner has no flexibility over hours of work. Households therefore choose household consumption and secondary earner labour supply.[14] The aim of this is to examine the effect of the in-

of A_1 that gives $\bar{U}$ is found by choosing the A_1 that solves the equation $E\left[U\left|\sigma^2\right.\right] = \bar{U}$, where $E\left[U\left|\sigma^2\right.\right]$ is computed by calculating the average utility of a large number of simulated life-cycles, where the simulations use the earlier solution.

[13] To judge the importance of the reduced cost associated with a flexible labour supply, it would be necessary to calculate the reduction in σ for the fixed labour supply case that would give a similar mark-up.

[14] The purpose of this analysis is neither to model household sharing nor to enter the debate about household equivalence scales. These issues are clearly important, but the assumptions that I adopt here are selected primarily to enable clear comparisons between the household ($x_t > 0$) and individual models ($x_t = 0$). The value of household consumption to household utility is the same as the value of individual consumption to individual utility; and similarly, the value of secondary earner leisure to household utility is identical to

surance introduced by having two sources of income and to show the extent to which individuals use their own labour supply to insure against shocks to the income of their partner.

The Euler equation and solution for leisure generated by the optimisation problem when $x_t > 0$ are the same as equations (11) and (12), but the solution for the consumption function changes because of the change in the budget constraint. Simulations are shown varying ρ, the correlation coefficient between the shock to wages and the shock to partner income and varying ν, the coefficient of relative risk aversion.

The key assumption is that comparisons are made holding expected lifetime wealth constant. Increases in lifetime wealth would reduce the cost of uncertainty when utility is isoelastic simply because prudence depends on the level of wealth. If lifetime wealth were different between the household and individual models, it would not be possible to separate out the effect on precautionary saving of this difference in wealth from the effect on precautionary saving of having two sources of income. It is also assumed that there is no deterministic growth in wages or in partner income and that the unconditional expected value of the wage equals the unconditional expectation of partner income. The superscript ^{p} on w_t indicates this is the wage of the secondary earner in the household model. Average income in the individual model, $\bar{w}_t^I$, is equal to average income in the household model, $\bar{w}_t^I = \bar{w}_t^p + \bar{x}_t$. Under certainty, these assumptions mean

$$c_t = \eta \left(A_t + x_t + w_t^p\right) \tag{17}$$

$$l_t = (1-\eta)\left(\frac{A_t + x_t + w_t^p}{w_t^p}\right) \tag{18}$$

When $A_0 = 0$, leisure in the individual model is half the value of secondary earner leisure in the household model (respectively, $(1-\eta)$ and $2\,(1-\eta)$). This is simply a price effect due to the lower wage necessary to keep lifetime wealth constant; alternatively, this can be explained by the increased

the value of individual leisure to individual utility. This implicitly assumes that there are no economies of scale in household utility, but these assumptions make the interpretation of results much clearer.

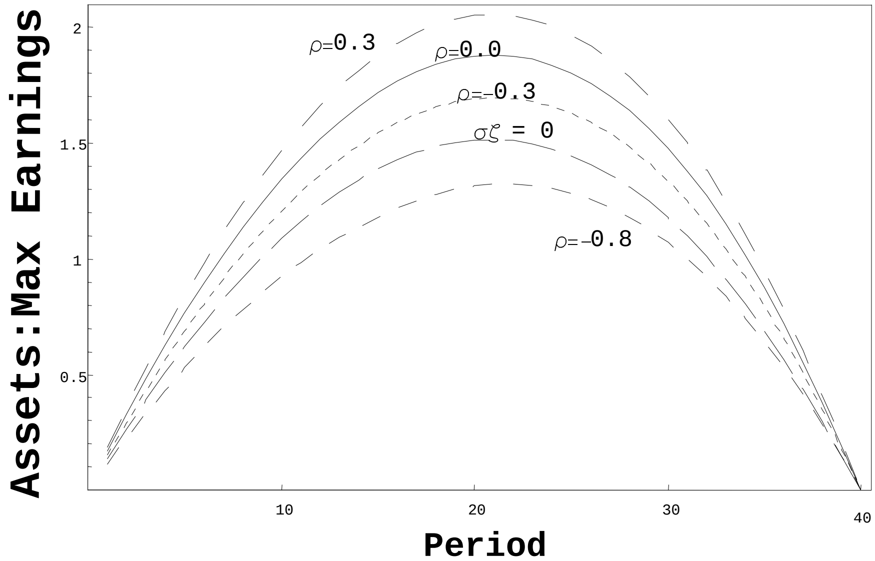
proportion of unearned income leading to increased leisure. Consumption depends only on the sum of x_t and w_t^p and so is identical in the two models.[15]

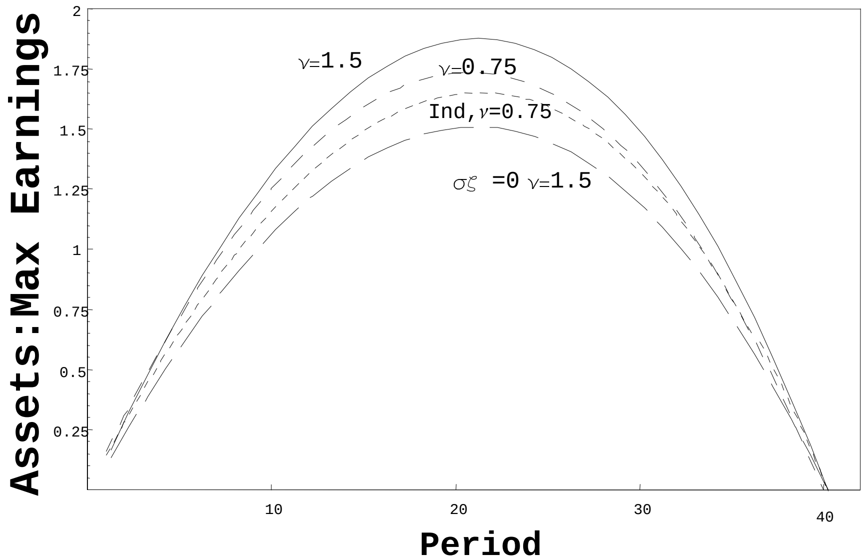
Asset Accumulation. Uncertainty over primary and secondary earner wages means that individuals defer utility through reducing consumption and leisure today and accumulating precautionary balances. The graphs in figure 4 show the extent of this asset accumulation for various scenarios. Precautionary balances rise to at least one and a half times mean income. This level of accumulation partly reflects the size of the variance, but it also reflects the value of η (set to 0.7). As shown in section 4, a smaller value of η means the level of precautionary balances is lower because the cost of variation in the wage is less, given $\nu = 1.5$. The first graph shows that asset accumulation will be greater the larger ρ, the coefficient of correlation between partner income and wages. Further, if ρ is sufficiently negative, precautionary saving is lower with uncertainty over partner income than with certainty over partner income.

The second graph shows the effect of increasing ν. In contrast to the discussion in section 4, precautionary saving increases with ν. An increase in ν has two effects in both the individual and household models: first, it increases the desired degree of smoothing in utility and this has the effect of making consumption and leisure within each period closer substitutes; and second, it increases prudence. When only half of income is due to wages, the benefit of this increased substitution is less, and the effect is dominated by the increased prudence associated with the increase in ν. The second graph also shows that asset accumulation in the household model is greater than asset accumulation in the individual model, unless ρ is sufficiently negative. This is shown for $\nu = 0.75$. It also holds for $\nu = 1.5$ and the difference in asset accumulation is even greater than when $\nu = 0.75$. This result holds

[15] In the simulations that follow, the variance of the wage, σ^2 is assumed to equal the variance of partner income, σ_ζ^2, and both are constant across time. Both the shock to wages and the shock to partner income are multiplicative, and the standard deviation is set equal to 0.1. The variance of secondary earner wages is the same whether partner income is included or excluded, this means that the variance of total income $(w_t + x_t)$ will differ according to the value of ρ and according to whether partner income is included.

Figure 4: Household Asset Accumulation





The y-axis of both graphs shows the ratio of the end-of-period asset stock to the mean across households of maximum earnings ($\bar{w}^p_t + \bar{x}_t$). The first graph shows the asset stock varying ρ, holding $\nu = 1.5$. Four values of ρ (-0.8,-0.3, 0.0 and 0.3) are used. The line labelled "$\sigma_\zeta = 0.0$" shows the case when primary earner income is certain, but the secondary earner's wage is uncertain. The second graph shows household asset accumulation for two values of ν (0.75 and 1.5) and individual asset accumulation, labelled *"Ind,ν = 0.75"*, when $\bar{w}^I_t = \bar{w}^p_t + \bar{x}_t$. The line for the individual model when $\nu = 1.5$ is marginally below the line for $\nu = 0.75$. In both graphs, $\sigma_\zeta = \sigma = 0.1$ and $\eta = 0.7$. If both primary and secondary earner wages are certain, $A_t = 0, \forall t$.

eventhough the variance of total income when partner income is included is less than the variance of total income without partner income. Increased balances tomorrow have less of a benefit when partner income is included because having two random variables offers some insurance. However, this reduced benefit of balances in the future is offset by the reduced scope to react to shocks in the future because only the labour supply of the secondary earner can be varied. When the latter effect dominates, precautionary saving will be greater when partner income is included. When $\rho = -0.8$, the insurance from having the two random variables outweighs the reduced scope to react to shocks, and so precautionary saving is greater in the individual model without partner income. Consumption smoothing reacts analogously: higher values of ρ mean less smoothing and, further, there is less consumption smoothing than in the individual model unless ρ is sufficiently negative.

Participation and Labour Supply. Figure 5 shows life-cycle profiles of leisure for the household and individual models. In both cases, leisure is lower when young than when old because uncertainty induces individuals to defer utility. The path of leisure is flatter, and more concave, the lower ρ. The flatter path arises because a lower ρ means uncertainty causes less variation in income and so, as shown above, there is less need for precautionary balances and this means less leisure has to be sacrificed when young. The concavity arises because the participation constraint is being hit. This means leisure cannot rise by as much in old age as would be the case if unconstrained. When ρ is negative, there is greater variance in leisure, a higher chance of not participating and hence a more concave path for leisure. A positive shock to partner income means income is greater and this increases leisure. Similarly, a negative shock to wages tends to increase leisure by reducing the price of leisure. Thus, when wages and partner income are negatively correlated, and so high partner income will coincide with a low secondary earner wage, the variance in secondary earner leisure will be greater. This can be seen in the x_t/w_t term in equation (17). When $\rho = -0.8$, for example, individuals will either work a large number of hours or very few hours, whereas when $\rho = 0.3$, the variance of leisure is less. This is reflected in figure 6 which shows that non-participation increases with age and the proportion of non-participants is greater when ρ is smaller. This discussion leads to the conclusion:

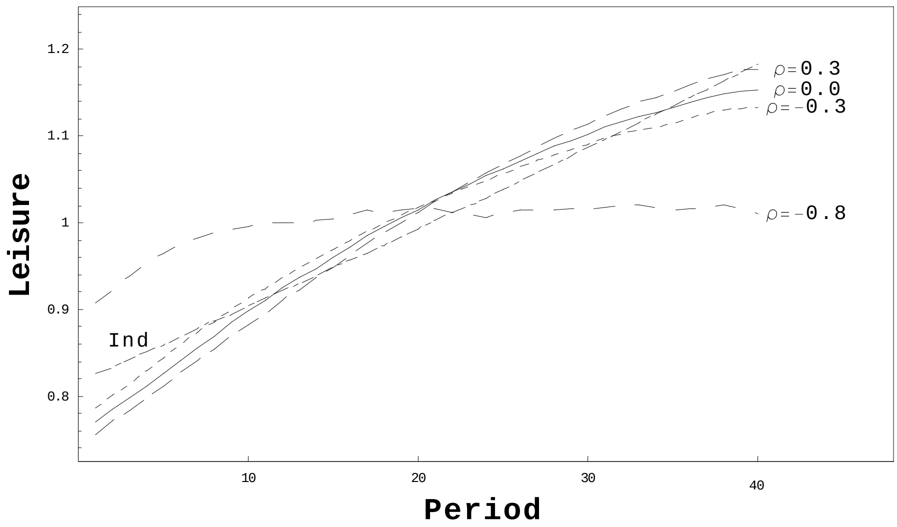
Result 4 *Insurance through secondary earner labour supply becomes increasingly effective as ρ becomes increasingly negative. This implies lower precautionary saving, a greater variance in hours of leisure and an increased proportion of secondary earners not participating.*

6 Unemployment Insurance

This section introduces unemployment benefit, B, into the model, but leaves out the exogenous income stream, x_t. The aim of this is to address the questions firstly, of how unemployment benefit affects participation across the life-cycle and secondly, of whether unemployment benefit decreases asset accumulation. The section begins by explaining the Bellman equation for the individual problem and showing how the reservation wage changes with age. Life-cycle profiles of participation and asset accumulation are then shown.

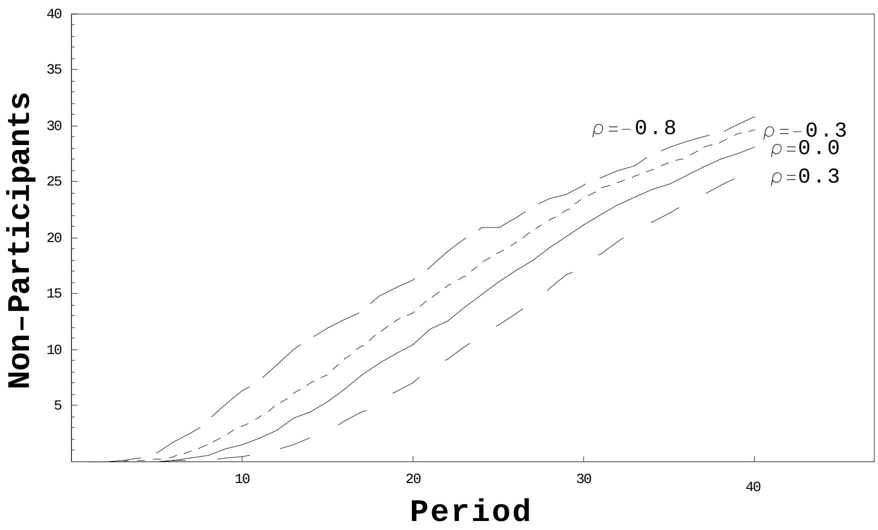
One problem with carrying out simulations for different benefit levels in this partial equilibrium framework is that total spending on the benefit may differ from total revenue. Without changing the tax rate to reflect the change in benefits, it is possible to show spurious effects on precautionary saving and on participation: an uncompensated increase in benefits will reduce both precautionary saving and participation because income is higher and the degree of prudence decreases with income. If the change in benefits is uncompensated it is not possible to separate this effect from the effect on the variance of income. Therefore, the model assumes that the benefit is paid for by a wage tax and that the net present value of revenue raised by the tax across 50000 individuals balances total payments to those same individuals across their lifetimes. The model is still only a partial equilibrium story, however, because the pre-tax wage rate and interest rate are exogenous, and

Figure 5: Individual and Household Leisure Smoothing



The line labelled *"Ind"* shows leisure in the individual model. The remaining lines show secondary earner leisure in the household model for different values of the correlation coefficient, ρ. The lack of smoothness in the $\rho = -0.8$ line arises partly because of the high variance in leisure in the simulations. In all cases, $\sigma_\zeta = \sigma = 0.1$, $\eta = 0.7$ and $\nu = 1.5$. When σ_ζ =0.0, the path for leisure lies between $\rho = -0.8$ and $\rho = -0.3$.

Figure 6: Non-Participation across the Life-Cycle



The y-axis shows the percentage of individuals who choose not to participate in each period. In the individual model, there is full participation in all periods. The lines show the extent of non-participation for different values of the correlation coefficient, ρ. In all cases, $\sigma_\zeta = \sigma = 0.1$, $\eta = 0.7$ and $\nu = 1.5$. Since $\eta = 0.7$, leisure when both wages and partner income are certain equals 0.6 and so everyone participates in all periods.

in particular the wage is independent of labour supply and the interest rate is independent of the asset stock. The "revenue neutrality" of any change in benefits is therefore only an approximation, but it means that changes in behaviour following changes in the level of benefits are not due to an arbitrary change in income. Table 3 shows the values of τ that are needed to balance the budget for different values of B. Unsurprisingly, more generous benefits require higher taxes. Further, a greater value of ϱ, meaning shocks to wages are more persistent, means a higher tax rate is required for any given benefit level.

Table 3: Tax Rates to Pay for Social Security

	Autoregression Coefficient ϱ			
$\frac{B}{\bar{w}}$	**0.0**	**0.7**	**0.8**	**0.95**
0.2	0.032	0.23	0.42	0.81
0.22	0.22	0.77	1.01	1.59
0.24	0.49	1.44	2.07	3.23
0.26	2.01	4.08	5.21	6.72
0.27	2.30	5.48	7.27	11.70

The table gives the percentage tax rate on wages necessary to equate revenue with the cost of the given benefit. The value of the benefit is given as the ratio of the benefit to the mean value of maximum earnings in one period (i.e. earnings if $l_t = 0$ and $w_t = \bar{w}$). When $\frac{B}{\bar{w}} > 0.29$ for $\rho = 0.0$ and $\frac{B}{\bar{w}} \geq 0.28$ for $\rho = 0.7, 0.8$ and 0.95, it is not possible to balance the budget.

The positive value of B makes the budget constraint (2) discontinuous at 0 hours of work. This means that the value function will be kinked and the policy function discontinuous at the point where the individual chooses to participate in the labour force. The value of working can be written as

$$V_t^E (A_t, w_t) = \max_{c_t, l_t} \{u(c_t, l_t) + \beta E_t [V_{t+1} (A_{t+1}, w_{t+1})]\} \tag{19}$$

where $A_{t+1} = (1 + r) [A_t + (1 - l_t)(1 - \tau) w_t - c_t]$. Similarly, the value of not working can be written as

$$V_t^N (A_t, w_t) = \max_{c_t} \{u(c_t, 1) + \beta E_t [V_{t+1} (A_{t+1}, w_{t+1})]\} \tag{20}$$

where $A_{t+1} = (1+r)[A_t + B_t - c_t]$. Where $V_t^E(A_t, w_t) = V_t^N(A_t, w_t)$, the individual is indifferent between working and not working.

Proposition 1 *For any given wage rate, w_t, there is a unique level of the asset stock, $A_t^{R,w}$, such that*

$$V_t^E\left(A_t^{R,w}, w_t\right) = V_t^N\left(A_t^{R,w}, w_t\right) \tag{21}$$

In other words, the reservation asset stock, $A_t^{R,w}$, is the level of the asset stock where the individual is indifferent between working and not working given the wage, w. If $A_t > A_t^{R,w}$, then the individual will not participate. If $A_t < A_t^{R,w}$, then the individual will participate.

Proof: See Appendix.

It is therefore possible to write the Bellman equation conditional on labour market status for each t as

$$\begin{aligned} & V_t^N(A_t, w_t) && (22)\\ = & \max_{c_t}\left\{u(c_t,1) + \beta E_t\left[\left\{\begin{array}{ll} V_{t+1}^N(A_{t+1}, w_{t+1}) & \text{if } A_{t+1} > A_{t+1}^{R,w} \\ V_{t+1}^E(A_{t+1}, w_{t+1}) & \text{if } A_{t+1} \leq A_{t+1}^{R,w}\end{array}\right.\right]\right\} \end{aligned}$$

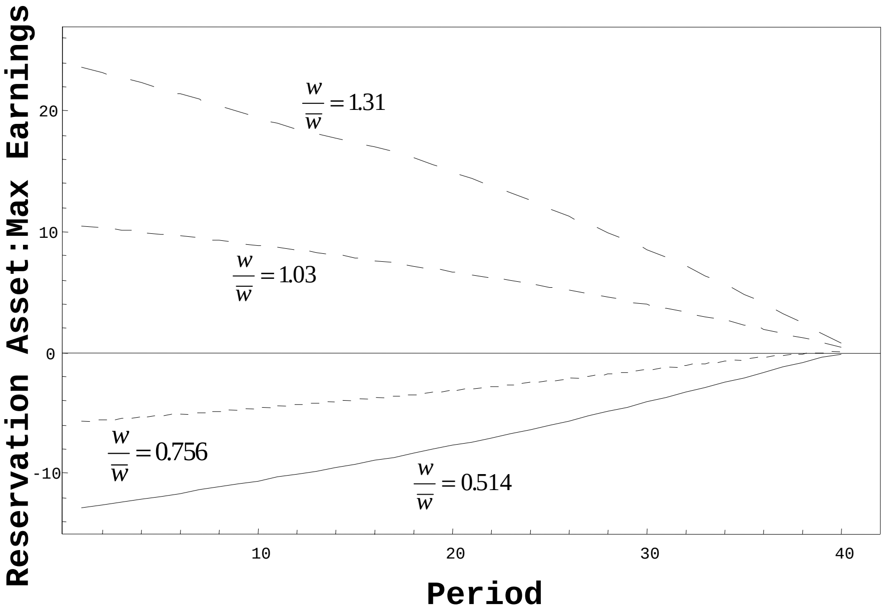
$$\begin{aligned} & V_t^E(A_t, w_t) && (23)\\ = & \max_{c_t, l_t}\left\{u(c_t,l_t) + \beta E_t\left[\left\{\begin{array}{ll} V_{t+1}^N(A_{t+1}, w_{t+1}) & \text{if } A_{t+1} > A_{t+1}^{R,w} \\ V_{t+1}^E(A_{t+1}, w_{t+1}) & \text{if } A_{t+1} \leq A_{t+1}^{R,w}\end{array}\right.\right]\right\} \end{aligned}$$

These value functions can be solved recursively as described in the appendix. Part of the solution involves solving for the reservation asset stock, $A_t^{R,w}$, for each t.

Figure 7 shows how $A_t^{R,w}$ varies with t for different values of w for a particular set of parameters.[16] A lower reservation asset stock means participation is less likely. Figure 7 makes two points: first, when the wage

[16] The results also hold for different benefit levels, and for values of the auotregression coefficient $\varrho \in \{0.0, 0.7, 0.8, 0.95\}$.

Figure 7: Reservation Asset Levels across the Life-cycle



The y-axis shows the ratio of the reservation asset stock to the mean of maximum earnings in any one period (i.e. $\bar{w}$). Similarly, the values of w given are divided by $\bar{w}$. When the asset stock is greater than the reservation asset stock for any given wage rate in any particular period, the individual will choose not to participate. $\frac{B}{\bar{w}} = 0.27$, $\varrho = 0.7$, $\sigma = 0.1$, $\eta = 0.7$ and $\nu = 1.5$.

rate is above the mean wage, $\bar{w}$, the reservation asset stock falls with age making participation less likely and second, when the wage rate is below the mean wage, $\bar{w}$, the reservation asset stock rises with age making participation more likely. In other words, if $A_t > 0$, the reservation wage is increasing with age, whereas if $A_t < 0$, the reservation wage is decreasing with age.

These points arise because a given wage realisation in a particular period has two effects: first, it leads to intertemporal substitution of labour, with a high wage leading to increased labour supply now in the expectation of a low wage in the future when labour supply will be less. The extent of intertemporal substitution depends on the number of periods remaining and the degree of persistence in shocks: the fewer the number of periods remaining, the lower the probability of a reversal in the wage rate, and this reduces the intended intertemporal substitution; further, the greater the persistence in shocks to the wage, the lower the probability of a reversal in the wage rate and this again reduces the intended intertemporal substitution. The second effect is the effect of a given wage realisation on lifetime wealth: a high wage increases lifetime wealth and this reduces labour supply in all periods. The extent of the effect of a given wage realisation on lifetime wealth also depends on the number of remaining periods and the degree of persistence: the greater the number of periods remaining, the less the effect on lifetime wealth of a given wage realisation; whereas the greater the degree of persistence in wages, the greater the effect on lifetime wealth.

These effects explain figure 7: when individuals are young, the intertemporal substitution effect dominates because the effect of the wage on lifetime wealth is small. Thus, when the wage is above the mean, individuals will exploit this intertemporal substitution and participate, unless their asset stock is very high, with the intention of not participating at some later age. Similarly, unless they are heavily in debt, individuals will not participate when the wage is below the mean, but with the intention of participating in the future when the wage is higher. As the individuals become older, however, the possibilities for intertemporal substitution of labour are less and the wealth effect comes to dominate. This means that if the wage is above the mean, individuals are less likely to participate in any of the remaining periods, whereas if the wage is below the mean, individuals are more likely to participate in all the remaining periods.

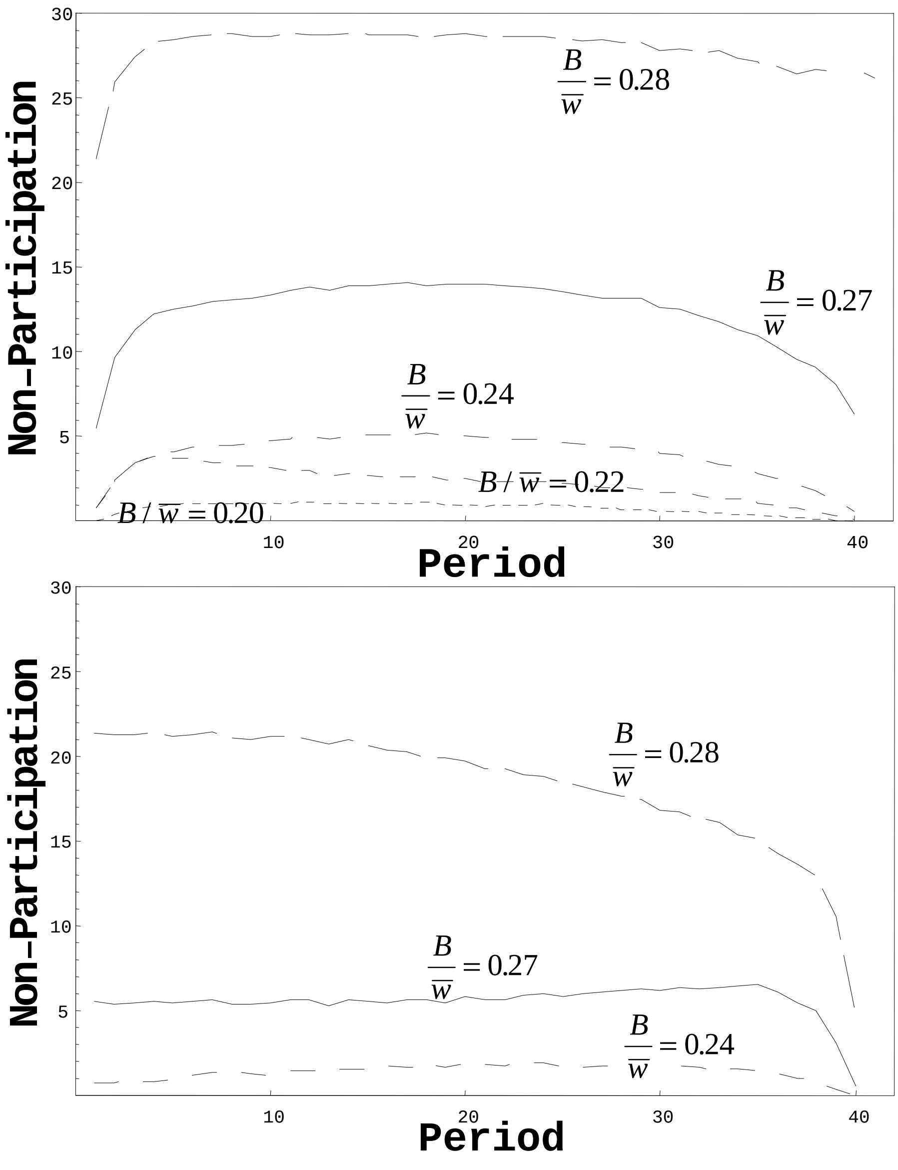
This raises two further questions: first, what happens to the reservation asset stock as the generosity of benefits changes? Increasing the generosity of benefits has a straight income effect: in any period and for any wage, the reservation asset level is lower and the reservation wage higher. Second, what happens as the degree of persistence changes? As discussed, a reduction in persistence increases the possibilities for intertemporal substitution and reduces the wealth effect of a given wage realisation. This makes the dispersion of the lines in figure 7 even greater. The dispersion of reservation asset levels still reduces with age, but the reduction occurs later in the life-cycle as the effect of deviations from the mean wage have little impact on remaining life-time wealth until close to the final period. By contrast, when wages are more persistent, a deviation of the wage today from the mean would lead to less intertemporal substitution than shown in figure 7 because the expectation of the wage rate tomorrow changes correspondingly.

Life-cycle Profiles. Results in this section are the implications for participation and asset accumulation of simulations varying the generosity of unemployment benefit and for different values of ϱ, the autoregression coefficient in the wage equation. Figure 8 shows the effect of unemployment benefit on participation across the life-cycle for $\varrho = 0.7$ and $\varrho = 0.0$. Unsurprisingly, this shows that increased generosity of benefits increases the average level of non-participation at all ages, but the path of the average level of non-participation hides the behaviour of particular individuals. The key point about the behaviour of particular individuals is that the probability of participation in the next period is higher if the individual has not participated in the current period. This is because non-participation involves the intertemporal substitution of labour supply and the use of savings to maintain consumption in periods of non-participation. Thus, following a period of non-participation, savings will be lower and the individual is more likely to participate. An increase in the generosity of benefits increases the number of periods of non-participation, but as discussed below, it also increases hours of work in periods of participation.

Further, for a given generosity of benefits, the average level of non-participation is greater when shocks are more persistent. Finally, when $\varrho = 0.7, 0.8$ *or* 0.95 the average level of non-participation increases with t when young, and then falls with t when old. When $\varrho = 0.0$, the average level of non-participation falls with t when old, and there is mild evidence that it increases with t when young. These points arise because the variance between individuals in realised wage profiles and hence in lifetime wealth is greater if the persistence of wage shocks is greater. At the start of the life-cycle, everyone has the same asset stock, but individuals who receive a positive wage shock increase their assets, and so persistence in wage shocks means asset stocks will diverge quickly. The accumulation of assets increases the probability of non-participation in subsequent periods, and thus non-participation increases with age early in the life-cycle because individuals who have had positive wage shocks have been able to accumulate assets before receiving negative shocks. When there is little persistence, there is not the same opportunity for asset accumulation. Towards the end of the life-cycle, non-participation falls with t because there is not the same opportunity for intertemporal substitution of labour. This effect occurs later in the life-cycle when there is no persistence because there is a higher probability that the wage shock will be reversed in subsequent periods if there is no positive correlation in shocks across time.

This increase in participation when old is in contrast to the results of section 5 and to the results of Eckstein and Wolpin (1989). These latter two sets of results are driven by a similar rationale: towards the end of the life-cycle the benefit of participation falls. In section 5, this occurs because uncertainty has been resolved and so there is less need to accumulate savings; in the model of Eckstein and Wolpin, the benefit of participation falls as the number of remaining periods declines because the return to accumulating experience and hence increasing the wage in subsequent periods is less. The key difference in the current model with unemployment benefit is

Figure 8: Non-Participation with Unemployment Benefit



The first graph shows the extent of non-participation when $\varrho = 0.7$ and the second graph when $\varrho = 0.0$. The graphs show the percentage of individuals who choose not to participate in each period for different values of $\frac{B}{\overline{w}}$. $\sigma = 0.1$, $\eta = 0.7$ and $\nu = 1.5$.

that periods of non-participation are the result of intertemporal substitution interacting with the benefit system.

When shocks are idiosyncratic and there is unemployment benefit, there is negligible asset accumulation. This is because the uncertainty over the wage averages out across the life-cycle and so there is little need for precautionary balances. Even with $\varrho = 0.7$, the level of asset accumulation is still low, but when $\varrho = 0.95$, the average level of asset balances reaches the average of maximum earnings in one period (i.e. $\bar{w}$).

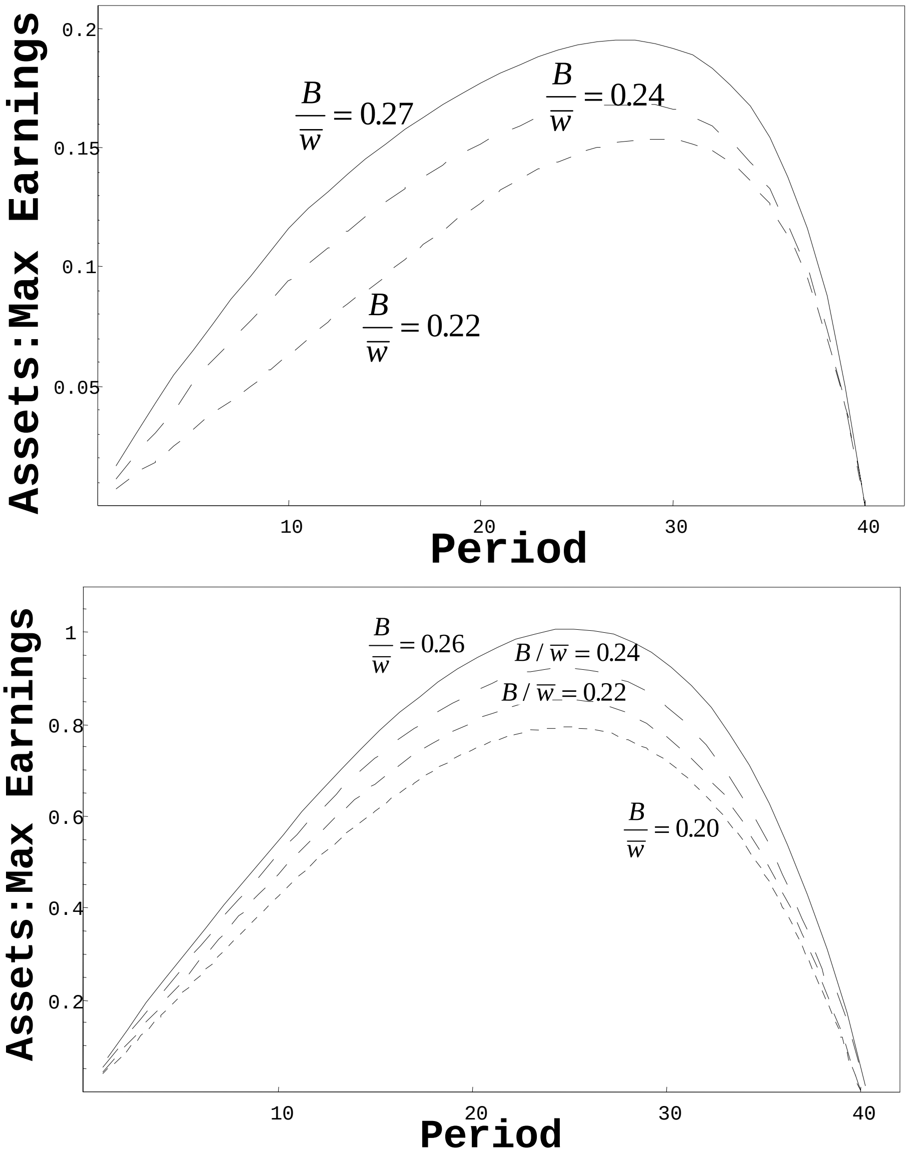
Result 5 *Asset accumulation increases with the generosity of benefits when* $\varrho = 0.7, 0.8$ *or* 0.95*.*

This result is shown in figure 9 for $\varrho = 0.7$ and $\varrho = 0.95$. An increase in the generosity of benefits has two effects: first, it increases insurance against shocks to the wage rate, thus reducing the cost of uncertainty and the need for precautionary balances; second, it increases the number of periods of non-participation, and this means increased saving in order to maintain consumption levels in periods of non-participation. In the particular examples shown in figure 9, the insurance aspect is dominated by the desire to smooth consumption.

As the generosity of benefits increases, the greater saving shown arises because labour supply when individuals actually work is greater. Lifetime labour supply is lower, however, and so consumption conditional on participation is lower when the benefit system is more generous because lifetime wealth will be lower due to the lower lifetime labour supply. Both consumption and leisure conditional on participation increase with t but the paths are fairly flat.

These results raise the question of which benefit level is optimal. It is possible to carry out explicit welfare calculations of the expected cost of achieving a given expected utility, as in section 4.2; however, between the revenue neutral benefit changes considered above, lifetime utility is increasing in the benefit level.

Figure 9: Asset Accumulation with Unemployment Benefit



The figure shows the ratio of the end-of-period asset stock to the mean of maximum earnings (i.e. $\bar{w}$) for different values of $\frac{B}{\bar{w}}$. The first graph shows $\varrho = 0.7$, and the second shows $\varrho = 0.95$. In both cases, $\sigma = 0.1$, $\eta = 0.7$ and $\nu = 1.5$.

7 Conclusion

The simulations in this paper aim to show the effect of non-market insurance on life-cycle behaviour. This is in a model where individuals choose labour supply and savings in each period. Three forms of insurance are considered. The first form is self-insurance where individuals undertake precautionary savings or precautionary labour supply: the key result is that increased flexibility in the labour supply can lead to increased precautionary saving. This arises because the utility cost of accumulating assets is lower when increased labour supply when young can be used to increase savings. A further point, however, is that consumption is more smooth and welfare higher when labour supply is flexible. This suggests that individuals who are constrained to work fixed hours through their lives will have accumulated fewer assets and have lower welfare than those who are unconstrained.

The second form of insurance is through household income: it is assumed that households have two income streams, primary earner income, where there is no flexibility over hours, and secondary earner income, with flexibility over hours and participation. The two sources of income provide increasingly effective insurance, and hence reduced precautionary saving, as the correlation between shocks to the two sources becomes increasingly negative. When shocks are negatively correlated, secondary earners participate and work long hours when the primary earner receives low income shocks, although participation falls across the life-cycle.

The third form of insurance considered is unemployment benefit paid for by a wage tax. Unemployment benefit gives the individual insurance against negative wage shocks and this reduces the need for precautionary balances (as discussed by Varian, 1980). However, in periods of non-participation, individuals use up their savings in addition to receiving unemployment benefit (which is not asset-tested), and this increases desired asset holdings. In this way, a more generous benefit program can lead to increased asset accumulation because the decreased participation means individuals need greater saving in order to smooth consumption.

These results raise a number of further research avenues: the introduction of an explicit period of retirement would introduce a further motive for saving which will interact with the precautionary motives discussed. Retirement could be introduced through a sufficiently large productivity downturn when old, and this would leave the actual date of retirement endogenous. In this context, it would make sense to introduce the asset testing of benefits. The results of Hubbard *et al.* (1995) show that asset-testing of retirement benefits leads to reduced saving by sections of the population. By making labour supply flexible, it is possible to see whether this reduced saving occurs through reduced labour supply, early retirement or increased consumption.

8 Appendix: Solution Method

Without Unemployment Benefit. Sections 4 and 5 use the same, standard technique to solve for the optimal policy functions which give leisure and consumption as functions of the wage and the initial asset level for that age, conditional on the assumed wage process:

$$c_t = c_t\left(w_t, A_t \left| \varrho, \sigma^2 \right.\right)$$

$$l_t = l_t\left(w_t, A_t \left| \varrho, \sigma^2 \right.\right)$$

Since l_t can be written as a function of c_t as in equation (12), the problem reduces to solving for c_t. The solution is found, as in Deaton (1991), through backward induction, starting with the terminal constraint (4). Once the policy functions for every age have been found, the model is then simulated starting from the initial period to give a profile of how consumption and leisure evolve across the lifetime. The expectations operator in equation (9) and the non-linear form of marginal utility mean that it is not possible to find analytic solutions to the policy functions and so numerical solution methods are necessary to obtain the policy functions. There are two state variables, the asset level, A_t, and the wage rate.

In sections 4 and 5, ϱ is assumed to be equal to 1 and so the wage rate is non-stationary and hence the solution for consumption is non-stationary. However, as in Deaton (1991), it is possible to transform the variables in equations (11) and (12) to make them stationary. This is done by dividing both equations by w_t, and substituting c_t/w_t in place of actual consumption. Solving for c_t/w_t gives the same solution as solving for c_t as utility is homothetic. The solution to each age-specific functional equation is found by using the finite element method (McGrattan, 1996; Reddy, 1993). This is a local approximation method which approximates well any kinks in the policy function.[17]

There remain some implementation issues: to impose a minimum value on the wage rate, it was assumed that the shock to log wages, θ, is within five standard deviations of the mean. Shocks are multiplicative, so this is a restriction on the proportion by which wages can change between t and $t+1$.[18] Since the wage is non-stationary, the state space is defined across A_t/w_{t-1} and w_t/w_{t-1}. The grid values for w_t/w_{t-1} are determined primarily by the abscissae for the Gaussian quadrature and the deterministic wage growth term, given by α_t. The grid spacing for A_t/w_{t-1} needs to be more dense in areas where the gradient of the policy function changes quickly (close to $\underline{A_t}$, the minimum feasible asset stock in t). The lifetime budget constraint imposes a restriction on the extent of spending in any one period. As discussed in Attanasio *et al.* (1999), this is information available to the individual and so should be exploited in the solution method. To be specific, the individual cannot spend more than minimum income for remaining periods, where this minimum income is determined by the minimum wage in each period. This is because the marginal utility of having zero within period income to spend is infinite and the individual is constrained not to die in debt. Hence, the solution for within period spending lies between 0 and minimum income.

The following two propositions show that the solution will be well-behaved and also act as a check on the solution.

Proposition 2 *The age of an individual and the asset level at the start of the period define a unique wage at which the constraint on leisure binds exactly. That is, there is a unique reservation wage.*

Proof: The reservation wage is unique if, for any wage greater than the reservation wage, desired hours of leisure, l^d, are less than the maximum, and if, for any wage less than the reservation wage, desired hours of leisure are greater than the

[17] Fortran code is available from the author on request. The code uses the NAG library to solve the underlying nonlinear equations.

[18] In Carrol (1992) and others, the worst outcome in a given period is an income of zero, and this occurs with probability p, which both papers set equal to 0.005. This means individuals never borrow because the probability of the worst outcome occurring is set at a significant level. Borrowing in the current paper will be very limited because minimum income is very low, although not zero.

maximum (for a given initial level of assets). In other words,

$$\begin{aligned}(i)\text{ if }w &> w_R,\ l^d<1\\ (ii)\text{ if }w &< w_{R,}\ l^d>1\end{aligned}$$

where w_R is the reservation wage.

Given the marginal utility of leisure is positive, the necessary and sufficient condition for these two conditions to be met is that desired hours of leisure decrease with the wage rate at $l^d = 1$. This relationship can be seen to hold from the Slutsky equation for hours of work, $h = 1 - l$.

$$\frac{dh}{dw}=\left.\frac{\partial h}{\partial w}\right|_{\bar{y}}-\frac{\partial h}{\partial y}h$$

where y is within period spending. At $l^d = 1$, $h = 0$ and so the last term in this equation drops out. Thus, the sign of dh/dw at $l^d = 1$ is determined only by the substitution effect. When the wage rate increases, the substitution effect always leads to a fall in leisure and so $\left.\frac{\partial l}{\partial w}\right|_{l^d=1} < 0.$■

Proposition 3 *The reservation wage is nondecreasing in the level of asset holdings, assuming that leisure is a normal good.*

Proof: The reservation wage is implicitly defined by

$$l\left(w_R, A\right)=1$$

Taking the total derivative of this gives,

$$\frac{\partial l\left(\cdot,\cdot\right)}{\partial w}dw_R+\frac{\partial l\left(\cdot,\cdot\right)}{\partial A}dA=0$$

Rearranging gives,

$$\frac{dw_R}{dA}=-\frac{\partial l\left(\cdot,\cdot\right)/\partial A}{\partial l\left(\cdot,\cdot\right)/\partial w}$$

The numerator is positive if leisure is normal. The denominator is the Hicksian substitution effect which is negative. Hence, $\frac{dw_R}{dA} > 0.$■

With Unemployment Benefit. The problem with introducing unemployment benefit, or indeed any benefit conditional on labour market status, is that it makes the budget set facing the individual non-convex. This in turn implies that the value function is non-concave and the policy functions for leisure and consumption will be discontinuous. The extra wage income from working is less than the benefit lost if hours of work are very low and this means that the individual will choose either to work a significant number of hours or not to work at all.

In section 6, the wage process is simplified by assuming that w_t can only take a discrete range of values. This range of values is independent of w_{t-1}, but the transition probability of observing w_t depends on w_{t-1} and the value ϱ. In other words, the wage process follows a first order Markov process, which mimics the continuous process described by equation (5). As ϱ tends to 1, however, the discrete approximation differs increasingly from the continuous process. The advantage of this discretisation is that the solution is for a number of one dimensional policy functions, with the number of policy functions determined by the number of discrete points used in the quadrature process, rather than for one two dimensional policy function.

The main problem in the solution is dealing with the discontinuity in the policy functions. Interpolation across a discontinuous policy function will be very imprecise. To avoid this, it is necessary to solve first for the point at which the policy function becomes discontinuous and then to approximate separately the solution at either side of the discontinuity. This means solving for the states of the world in which the individual is indifferent between working and not-working, i.e. where

$$V_t^E\left(A_t^{R,w}, w_t\right) = V_t^N\left(A_t^{R,w}, w_t\right) \tag{24}$$

Proposition 1 claims that the value of $A_t^{R,w}$ such that this indifference holds is unique. This can be proved as follows:

Proof: It is necessary to prove the following:

$$\left.\frac{\partial\left(V^N\left(A_t, w_t\right) - V^E\left(A_t, w_t\right)\right)}{\partial A_t}\right|_{A_t = A_t^{R,w}} > 0 \tag{25}$$

If this holds for any candidate $A_t^{R,w}$, then at $A_t^{R,w}$ the value of not participating is increasing faster than the value of participating, and so, since the value functions are equal at $A_t^{R,w}$ this means that $V^N\left(A_t, w_t\right) < V^E\left(A_t, w_t\right)$ if $A_t < A_t^{R,w}$ and $V^N\left(A_t, w_t\right) > V^E\left(A_t, w_t\right)$ if $A_t > A_t^{R,w}$. If equation (25) holds, the value functions can only cross once, because the value functions conditional on labour market status are continuous, and so $A_t^{R,w}$ is unique. The proof of (25) requires showing that

$$\frac{\partial u\left(c_t^N, 1\right)}{\partial c_t^N} > \frac{\partial u\left(c_t^E, l_t\right)}{\partial c_t^E} \tag{26}$$

where c_t^N is the solution to (20) and c_t^E, l_t is the solution to (19).[19] At $A_t^{R,w}$,

$$A_t^{R,w} + (1 - l_t)(1 - \tau) w_t > A_t^{R,w} + B \tag{27}$$

If this did not hold, equation (24) could not hold because the value of not participating would have to be greater than the value of participating since leisure confers utility.[20] This implies that consumption in t is higher when employed than when not-participating because consumption in every period is a normal good. If within period utility were additively separable between leisure and consumption, this would be sufficient to prove (26), but the lower leisure when participating raises the marginal utility of consumption when participating.

Therefore, the final step uses the fact that indirect utility is concave in total within period spending, so increases in income have a diminishing effect on indirect utility. Since equation (27) shows that income is greater when the individual is participating, the marginal effect of income on indirect utility, $\partial v(w, y)/\partial y$, will be greater when individuals are not participating. Further,

$$\frac{\partial v^E(w,y)}{\partial y} = \frac{\partial u\left(c^E, l\right)}{\partial c^E}\frac{dc^E}{dy} + \frac{\partial u\left(c^E, l\right)}{\partial l}\frac{dl}{dy} \tag{28}$$

$$= \frac{\partial u\left(c^E, l\right)}{\partial c^E}\left(\frac{dc^E}{dy} + w\frac{dl}{dy}\right) \tag{29}$$

$$= \frac{\partial u\left(c^E, l\right)}{\partial c^E} \tag{30}$$

where the first step uses the fact that there is an interior solution for leisure, and the second step comes from the definition of $y = c + wl$. Further, $\partial v^N(w, y)/\partial y$ is similarly related to $\partial u\left(c^N, 1\right)/\partial c^N$ and this proves the inequality (26).■

Given this uniqueness of $A_t^{R,w}$, it is possible to write the conditional value functions as in equations (22) and (23) in the text. The model can then be solved recursively from period T with the solution in each t found in two steps. First, for

[19] Using the envelope theorem,

$$\frac{\partial V^N}{\partial A_t} = (1+r)\beta E_t\left[\frac{\partial V_{t+1}}{\partial A_{t+1}}\right]$$

and so, using the first order condition,

$$\frac{\partial u\left(c_t^N, 1\right)}{\partial c_t} = (1+r)\beta E_t\left[\frac{\partial V_{t+1}}{\partial A_{t+1}}\right],$$

it is possible to generate the left-hand side of the inequality (26) in the text. An analogous derivation can be made for $\frac{\partial V^E}{\partial A_t}$.

[20] However, this would not necessarily be true if the utility from leisure were not intertemporally additvely separable.

each wage rate on the grid, the value $A_t^{R,w}$ is found by solving equation (24) using equations (22) and (23). Second, the value functions conditional on labour market status are solved using the Bellman equations. This is done for values above $A_t^{R,w}$ for the value function when not working (equation (22)) and for values below $A_t^{R,w}$ for the value function when working (equation (23)). The value function is not discontinuous at $A_t^{R,w}$ but it is non-concave, and a more precise solution is given by solving first for $A_t^{R,w}$.[21] In addition to storing the value functions in t, it is also possible to store the conditional policy functions, and this speeds up the subsequent simulations.

Tax Requirement. The solution method just described works for a given value of B and a given value of τ. However, it is necessary that, for a given value of B, the tax rate must be set to balance the budget, otherwise results may be due to changes in expected income rather than the increased B per se. This wage tax will have distortions on intertemporal and labour market behaviour in addition to the distortion of the benefit payment. Therefore, for any given level of benefit payment, the rate of the wage tax necessary to make benefit changes revenue neutral is found by simple iteration: an initial guess is made at τ and the dynamic program is solved and simulated to give the realised cost of the benefit program and the realised revenue raised by τ, both discounted to period 1. Depending on whether there is a deficit or surplus, a new value of τ is tried, and this process continues until there is budget balance.[22] At the solution, τ^*, it is necessary to check (numerically) that $\partial R/\partial\tau > 0$ to ensure that τ^* is the minimum tax rate that balances the budget. It is also necessary to check that $\tau = 0$ is not a solution: increases in τ increase non-participation, hence increasing required revenue and leading to spurious solutions. Table 3 in the text shows the necessary tax rates for different values of the generosity of benefits found using this method.

[21] One additional problem is that it is not possible to use the Euler equation to solve the maximisation sub-problem in (22) and (23) because of the non-differentiability of the value function at $A_{t+1}^{R,w}$. Instead, a simple optimisation method is used.

[22] It is not necessary for the budget to balance, but it is necessary for the deficit (or surplus) to be the same for all scenarios to keep revenue neutrality. A balanced budget is simplest as it allows comparison with the model with no benefits.

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