

Survival pessimism and the demand for annuities

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Abstract

The "annuity puzzle" refers to the fact that annuities are rarely purchased despite the longevity insurance they provide. Most explanations for this puzzle assume that individuals have accurate expectations about their future survival. We provide evidence that individuals mis-perceive their mortality risk, and study the demand for annuities in a setting where annuities are priced by insurers on the basis of objectively-measured survival probabilities but in which individuals make purchasing decisions based on their own subjective survival probabilities. Subjective expectations have the capacity to explain significant rates of non-annuitization, yielding a quantitatively important explanation for the annuities puzzle.

Keywords: Annuity Puzzle, Subjective Expectations, Survival Probabilities

JEL Codes: D14, D84, D91, J14

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1 Introduction

Annuities insure individuals against longevity risk by allowing them to exchange wealth for an income stream guaranteed until death. Theory predicts that under general conditions, risk-averse individuals will purchase a fairly-priced annuity (Yaari (1965); Davidoff et al. (2005)). Few households, however, ever purchase an annuity. This divergence between theory and experience has become known as the "annuity puzzle".

Most of the explanations that have been proposed for this puzzle² attempt to rationalize non-purchase by individuals who are assumed to have accurate perceptions of their survival probabilities. Individuals are not, however, well-informed about their survival-expectations (Hurd and McGarry (1995), Elder (2013), Wu et al. (2015)).

We study the demand for annuities in a setting where those annuities are priced by insurers on the basis of objectively-assessed survival probabilities but in which individuals make purchasing decisions on the basis of their own subjective survival probabilities. We estimate survival curves for a sample of older individuals using directly-measured expectations. We find a pattern of significant under-estimation of individuals' chances of survival through their 50s, 60s and 70s, and over-estimation of survival probabilities through their late 80s and beyond, on average. Overall, pessimism dominates, and most respondents would perceive an annuity that is priced fairly from an actuarial point of view as one which is unfairly-priced.

Individuals might still purchase an annuity that is unfairly-priced as the longevity insurance provided by the annuity might be worth the low apparent (to them) 'money's worth', depending on their preferences. To assess the quantitative importance of survival pessimism for annuity purchases, we embed these subjective survival curves in a lifecycle model of consumption, saving and annuitization choice. We assess which of our sample members,

¹Lockwood (2012) reports that less than 5% of a sample of single retirees in the US own an annuity; Inkmann et al. (2011) show that only 6% of older households in the UK voluntarily purchase an annuity.

²These explanations including adverse selection (Brugiavini (1993); Finkelstein and Poterba (2004); Finkelstein and Poterba (2014)), bequest motives (Lockwood (2012); Gan et al. (2015)), precautionary saving for medical and long-term care expenses (Ameriks et al. (2011)), existing annuity provision from social security income (Pashchenko (2013)), and costs of administration (Mitchell et al. (1999)). See Brown (2007) for a general review of this literature.

conditional on their idiosyncratic subjective survival probabilities, would choose to annuitize their wealth. With plausible levels of risk aversion and patience, we are able to explain high rates of non-annuitization – at typical preference parameters, we find that approximately half of individuals would not choose to annuitize. We compare the effect of our introducing subjective survival probabilities to the effect of actuarially unfair pricing, caused by adverse selection (or transaction costs or other market imperfections). We find that survival pessimism is quantitatively as important or more important than the higher prices caused by adverse selection.

This result does not depend on other explanations that have been given for non-annuitization – in our model, individuals have only modest social security income, do not have bequest motives, face no medical cost risk, do not have access to means-tested income floors and annuities are priced fairly given objectively-measured survival rates. The difference between 'objective' and individual-specific 'subjective' survival curves is large enough to outweigh the insurance value of an annuity for a large proportion of individuals.

Subjective expectations of survival have been shown to be empirically important in explaining a number of economic decisions. Hurd et al. (2004) find that those with particularly low expectations of survival are more likely to retire earlier and to claim Social Security benefits earlier. Bloom et al. (2006) find that a higher subjective probability of survival is associated with higher wealth levels. Gan et al. (2015) finds that a model of wealth decumulation and bequests including subjective survival expectations better fits decumulation and bequest behavior than does one with life table survival probabilities. Heimer et al. (forthcoming), build a life-cycle model with subjective mortality beliefs and show that 'pessimism' about survival to older age, combined with 'optimism' at the oldest ages can explain both under-saving for retirement and slow decumulation of wealth at the end of life. None of these papers considers an annuitization choice, as we do.

There is evidence of a correlation between individual life expectancies – realized as well as expected – and decisions around annuitizations. Examining the voluntary market for

annuities in the United Kingdom, Finkelstein and Poterba (2004) find a positive association between ex-post survival and features of annuities purchased (for example those who buy back-loaded annuities are longer-lived). Inkmann et al. (2011) find that those with higher stated expectations of survival are more likely to purchase an annuity than those who are more pessimistic.³ That paper also experiments with subjective survival probabilities in a life cycle model – and shows that if subjective survival probabilities are reduced by 10% at each age (a quantity that is not empirically grounded) no households would demand an annuity. However, to the best of our knowledge, no paper has quantified the importance of the observed divergence between average reported survival expectations and life table survival rates for the annuitization decision, which is the focus of this paper.

We proceed as follows. Section 2 outlines the data. Section 3 compares average reported survival expectations to official life tables, and sets out our method for constructing 'subjective' survival curves from stated beliefs. In Section 4 we outline the model of annuitization and the impact of introducing subjective survival curves on predicted rates of annuitization. Section 5 concludes.

2 Data

We draw on data from the English Longitudinal Study of Ageing (ELSA), a biennial panel representative of the English household population aged 50 and above. ELSA is part of a network of longitudinal aging studies around the world, modelled on the US Health and Retirement Study. Seven waves of the survey are currently available, covering the period 2002/03 to 2014/15. One module of the survey asks individuals about their expectations that certain events will happen in future, including whether or not they will leave an inheritance, whether they will still be in work at a certain age and whether at some point in the future they will not have enough resources to meet their financial needs. This battery of questions opens with the following statement:

³However, Brown (2001) finds no evidence of this phenomenon.

"Now I have some questions about how likely you think various events might be. When I ask a question I'd like you to give me a number from 0 to 100, where 0 means that you think there is absolutely no chance an event will happen, and 100 means that you think the event is absolutely certain to happen."

As part of this module, individuals are asked a question of the form "What are the chances that you will live to be age X or more?", where the age X depends on the current age of the respondee. All individuals aged 65 and under are asked about survival to age 75. Those aged 66 or older are asked about the age which is between 11 and 15 years ahead of their current age and is a multiple of 5. For example, those aged 75-79 are asked about survival to age 90. Additionally, from wave 3 onwards, all individuals aged under 70 were asked a second question about survival to age 85. We denote individual i's reported probability of survival to age α as $R_i(\alpha)$.

Over the seven waves of ELSA, 16,345 unique individuals are asked one or more survival questions in 67,201 separate interviews.

2.1 Evaluating the content of subjective reports

Before using individual responses to survival probability questions in analysis, we wish to assess whether individuals appear to understand the meaning of these questions and to be able to engage with the probabilistic concepts involved. Secondly, assuming that the nature of these questions is understood, we would like to establish as far as possible whether answers constitute considered, reflective judgements that might plausibly guide behavior, or are instead picked with little prior thought.

In just 1.5% of interviews, individuals answer "Don't know" to one or more survival probability questions, suggesting a willingness to answer in almost all cases. Figure 1 shows the distribution of reported survival probabilities for the full sample of first questions asked by 10 percentage point categories. We split the sample into those aged below 65 and those aged 65 and above, with the younger group much more likely to report high chances of

survival.

0%

0-9

10-19

20-29

30-39

25%
20%
20%
20%
15%
5%

Figure 1: Distribution of answers given to first survival question by age-group

Source: ELSA waves 1–7. 66,210 answers from 16,345 unique individuals

40-49

50-59

Stated percentage chance of survival

60-69

70-79

80-89

90-100

Some individuals answer "0%" or "100%" to the survival questions (5.3% and 6.8% respectively). While answers of 0% chance of survival may be consistent with understanding of the question (in cases of terminal illness for example), an answer of 100% chance of survival over an 11-15 year period strongly suggests a lack of understanding of the question. When individuals are asked two survival questions, they can give an answer that is impossible if they report a higher chance of survival to the older age than to the younger age. This happens in 8.3% of interviews. Overall, individuals give answers that indicate a lack of understanding of the question in at least one of these two ways in 13.9% of interviews. We choose to remove these individuals from our sample in all of the following analysis.

We may have reservations about the fact that a high proportion - 20.5% - of answers are "50%". There could be a concern that individuals pick this focal answer when wanting

to give a response but not understanding the question. We assess this by examining these individuals' answers to other probability questions. Those individuals who answer "50%" almost always give a range of answers to other questions and are no more likely to answer "50%" to other probability questions than are the rest of the sample (Appendix A.1 gives further details). Of the 16,345 individuals who answered one or more survival questions, only 41 individuals (0.2%) answered "50%" to all survival questions in all waves. On the basis of this evidence, we retain answers of "50%" in our main sample but show in the Appendix the (minimal) sensitivity of results to their removal.

Given that the overwhelming majority of individuals give answers that do not indicate a lack of understanding of probabilities, we perform three further "tests" aimed at assessing the informational content of responses. Firstly, we find that responses are correlated with known mortality risk factors (e.g. smoking, drinking and health conditions) in the way that is consistent with existing evidence. For example, current smokers report a 6-7 percentage points lower probability of survival over the 11-15 year horizon, relative to current non-smokers.⁴ Secondly, using the panel nature of the survey we find that reports 'update' over time in response to news relevant to mortality such as diagnoses with new health conditions. For example, a new cancer diagnosis was associated with a 5 percentage point reduction in the stated probability of surviving to an age 11 to 15 years ahead.⁵ Third, exploiting a link to administrative death records from the National Health Service, we find that reported expectations are correlated with actual subsequent mortality over a 10 year horizon.⁶ While it is possible that individuals' answers to survival expectations questions could represent their 'actual' expectations even if there was no association with the above outcomes, these findings can be seen as additional evidence that answers represent meaningful, reflective

⁴This result is obtained using linear regression of reported probability on smoker status, controlling for a range of other risk factors, demographic variables, health conditions and self-reported health. Full details are given in Appendix A.2.

⁵This result is from a linear fixed effects regression of reported survival probability on a range of variables for whether individuals have been diagnosed with particular health conditions, as well as other risk factors and demographic variables. Full details are given in Appendix A.2.

⁶Full details are given in in Appendix A.2.

3 Assessing the accuracy of subjective expectations of survival

In this section, we describe the patterns in subjective reports, compare them to actual mortality rates and projections and derive idiosyncratic survival curves that will be used, together with our model, to evaluate the importance of these curves for the annuitization decision. The results that we find – that individuals are mostly pessimistic about survival to younger ages and optimistic regarding survival to older ages – are consistent with other recent work (Elder (2013), Wu et al. (2015), Heimer et al. (forthcoming)), so our treatment is brief.

3.1 Comparing reports to actual mortality data

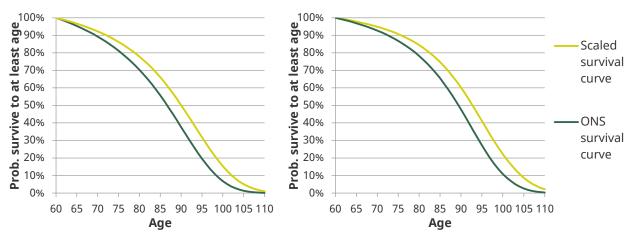
The UK Office for National Statistics (ONS) life tables contain actual and projected mortality data for the English population by sex and year-of-birth. These tables would be a natural benchmark against which to assess subjective expectations if the ELSA sample were representative of the whole English population. As ELSA is representative only of the non-institutionalised population (meaning those in residential care, for example, are excluded), we use linked administrative death records linked to ELSA to "rescale" the data in the ONS life tables.

ELSA is linked to administrative death records such that we know if any individual (including those who have attrited from the sample) has died up until February 2013. We use this information to "rescale" the official life tables in the following way. We calculate, for each year of age, the actual morality hazard rate observed in the ELSA sample and

⁷Hurd and McGarry (2002) make a similar point with respect to subjective expectations in the Health and Retirement Survey.

the expected mortality hazard rate if each individual faced the hazard rate implied by the ONS life table for their sex and year-of-birth. For each sex, we fit a cubic in age to actual hazard rates using OLS and calculate the ratio of the fitted hazard rate to the ONS hazard rate at each age. We take the mean ratio across ages and use this to rescale the hazard rates underlying the ONS survival curves for each sex and year-of-birth, yielding a set of "scaled" ONS survival curves. Our method yields hazard rates for men and women that are 71% and 69% of their original level, respectively – that is, those in the ELSA sample have lower mortality probabilities (and therefore higher life expectancies) than the population at large. Figure 2 shows a comparison of the original ONS survival curve with the "scaled" survival curve for 60-year old men and women, born in 1950. We see that, for example, the male group are estimated to have a 50% chance of survival to age 90, in comparison to the 38% figure in the ONS life table. The corresponding figures for women are 60% and 48% respectively.

Figure 2: Comparison of ONS cohort survival curve and "scaled" survival curve for men (LHS) and women (RHS) aged 60 and born 1950



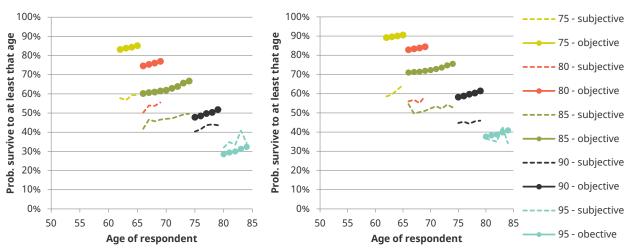
Source: ELSA waves 1–7 and ONS 2014-based cohort life tables for England and Wales.

We use the "scaled" ONS survival curves life tables as an "objective" benchmark to assess whether particular age-sex-cohort groups have positively biased ('optimistic') or negatively biased ('pessimistic') expectations of survival to various "target ages" (i.e. the age about which the individual is asked the question). We conduct this analysis at the most granular

level for which these comparisons are possible: we calculate for each combination of year of age, year of birth, sex, and target age, the average reported survival probability, and compare this to the relevant scaled life table probability. A clear pattern emerges: individuals are, on average, 'pessimistic' about their chances of survival to ages 75, 80, 85 and 90 and then become increasingly optimistic about survival at older ages. While the degrees of 'optimism' and 'pessimism' vary slightly between cohorts, these patterns are consistently found across those born in the 1920s through to the 1950s. Comparing men and women, we see that women tend to be slightly more pessimistic on average, than men.

Figure 3 illustrates the comparison between mean subjective survival probabilities and scaled life tables for men and women born in the 1930s. We see that individuals in their early 60s under-estimate survival to age 75 by around 25 to 30 percentage points while those in their late 60s and early 70s under-estimate survival to age 85 by 15 to 20 percentage points. Turning to those in their late 80s, we see that they are close to accurate about their probability of survival, on average.⁸

Figure 3: Comparison of mean 'subjective' reports and scaled ONS cohort survival rates/projections for men (LHS) and women (RHS) born 1930-39



Note: Different colored series correspond to different ages about which respondents are asked questions. Source: ELSA waves 1–7 and ONS 2014-based cohort life tables for England and Wales.

⁸Note that in Figure 3, each "subjective" data point corresponds to an average over respondents with different birth years. The "objective" data points are constructed by weighting the corresponding scaled life table survival probabilities according to the proportions of individuals with each birth year in the sample.

These findings are in line with those in existing literature, including Elder (2013) and Heimer et al. (forthcoming), which establish in a number of settings the pattern of overestimation of mortality hazard rates at ages until around the mid-80s, with under-estimation of mortality rates at older ages.

3.2 Constructing subjective survival curves

In this section, we describe how we use stated survival expectations to estimate the individual-specific subjective survival curves that will be used in our lifecycle model. There are an infinity of possible survival curves consistent with the answers that individuals give to the survival questions. We are therefore required to make further assumptions if we are to infer individuals' subjective survival curves from their reports about their survival to specific ages. We make an assumption on the functional form of individuals subjective survival curves. Specifically, we assume that subjective survival probabilities follow a Weibull distribution. The Weibull distribution is widely used in the epidemiological literature and for modelling ageing processes generally. 10

The Weibull distribution is a two-parameter (λ_i, k_i) distribution defined in the following way. Person i with age z, has prob of surviving to at least age α :

$$S_i(\alpha) = exp\left[-\left(\frac{\alpha - z}{\lambda_i}\right)^{k_i}\right] \quad : \quad \lambda_i, k_i > 0$$
 (1)

We estimate subjective survival curves for all individuals in our sample who answered two survival questions (i.e. all those aged under 70 who answered two questions and were not removed from the sample due to giving "impossible" answers). We make one additional

⁹This is true given that we exclude from our sample both those individuals who state that they have a 100% chance of survival to some older age and those individuals who are asked two survival questions and answer that they have a higher chance of survival to the older age. Survival curves are necessarily downward sloping and therefore inconsistent with both of these types of responses.

¹⁰See Bissonnette et al. (2017) for an example of use of the Weibull distribution to construct objective and subjective survival curves using data from the HRS. The authors report that their results are similar when using the Weibull or the (also widely-used) Gomertz distribution.

weak assumption – that individuals believe that they are almost certain not to live beyond age 110 – by including the relevant scaled life table survival probability for each individual for target age 110 as a third "report" (we denote this third subjective "report" by $R_i(110)$). We fit the individual's Weibull distributed subjective survival curve by estimating λ_i and k_i using these three reports and non-linear least squares. That is, denoting the set of 3 ages for which we have subjective reports by A_i we choose the parameter vector $(\hat{\lambda}_i, \hat{k}_i)$ that satisfies

$$(\hat{\lambda}_i, \hat{k}_i) = \underset{\lambda_i, k_i}{\operatorname{arg\,min}} \sum_{\alpha \in A_i} \left(R_i(\alpha) - exp \left[-\left(\frac{\alpha - z}{\lambda_i}\right)^{k_i} \right] \right)^2. \tag{2}$$

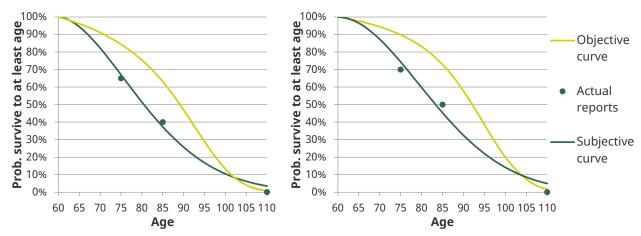
Figure 4 illustrates the curve-fitting procedure for the median responses from men and women born in the 1940s and compares these subjective survival curves to the relevant scaled life table survival curves. The subjective curve implies that at age 60, this group of individuals are pessimistic about survival to all ages up until around age 100 and optimistic about survival to ages beyond this. The subjective life expectancy measures implied by these curves are 8.6 and 9.6 years lower than the life expectancies implied by the life table survival curves for men and women, respectively. This is equivalent to life expectancies at the age of 60 that are 31% (for men) and 33% (for women) lower than those implied by life tables, on average.

Average overall pessimism of this magnitude is found across sexes and cohorts when interviewed in their 50s and 60s. Using the full sample of individuals for whom we have a subjective survival curve, we can compare 'subjective' and scaled life table life expectancy at the individual level. Subjective life expectancy is lower than scaled life table life expectancy by 5.2 years or 19% amongst men, and by 6.7 years, or 23%, amongst women, on average.

4 Subjective survival expectations and annuitization

Annuities provide insurance against longevity risk. The decision about whether or not to purchase an annuity at a given price ought to depend on the individual's assessment of this

Figure 4: Comparison of median 'subjective' and 'objective' survival curves for men (LHS) and women (RHS) born 1940-49



Note: Each curve shows, conditional on survival to age 60, the probability of survival—7 and ONS 2014-based cohort life tables for England and Wales.

longevity risk. Individuals who under-estimate their longevity may perceive an annuity as a worse deal than it 'truly' represents. This is a potential explanation for the unpopularity of annuities.

First, we can assess whether, given their subjective expectations, individuals would perceive an annuity as offering at least an actuarially 'fair' deal. An annuity rate is defined as actuarially fair with respect to a given discount rate and set of survival probabilities, if it enables the purchase of a guaranteed income stream until death that has expected discounted value equal to its price. For each individual for whom we have fitted a subjective survival curve, we calculate the actuarially fair annuity rate given their subjective survival curve and given their scaled life table survival curve. The actuarially fair annuity rate for an individual age z, given the survival curve $S_i(\alpha)$ is given by:

$$\theta = \left[\sum_{\alpha=z}^{110} \frac{S_i(\alpha)}{(1+r)^{\alpha-z}} \right]^{-1} \tag{3}$$

where r is the interest rate.

Figure 5 compares these 'subjective' and 'objective' annuity rates for our sample assuming

a real interest rate of 0% in both cases. Variation in objective annuity rates comes from variation in gender, age and year of birth; variation in the subjective annuity rates comes from the estimated subjective survival curves. 86% of individuals would perceive an annuity that is priced fairly for the average person of their age, sex and cohort as offering a less than fair annuity rate.

Figure 5: Comparison of 'objective' and 'subjective' based annuity rates

Note: 'Subjective annuity rates are the actuarially fair rate implied by the subjective survival curve constructed from the individuals responses to the survival expectations questions. 'Objective annuity rates are the actuarially fair rate implied by the scaled ONS life table survival curve for the individuals sex, age and year of birth. Source: ELSA waves 3–7 and ONS 2014-based cohort life tables for England and Wales.

An individual who perceives an annuity as being unfairly priced may, of course, choose to purchase if it offers sufficiently large insurance value. To examine whether survival 'pessimism', and the implied divergence between subjective and life table-based annuity rates would be sufficient, given plausible levels of risk aversion and discounting, to lead individuals to not annuitize their retirement savings we specify a model of annuity choice and wealth decumulation. We compare results for our sample in the case where individual survival ex-

pectations are consistent with scaled life tables to those where they are consistent with their subjective survival curve. We account for the fact that individuals have public pension entitlements, giving them some already annuitized income. We use data from ELSA on private pension and financial wealth and model the choice of whether to annuitize this at a rate that is actuarially fair given the individual's age, sex and cohort.¹¹

4.1 Model

In this section we outline the model of individual annuitization choice. Just-retired agents have initial wealth, a_0 , and receive public pension income (state pension/social security), p, in each period. In period 0, agents can choose whether or not to annuitize all their pension and financial wealth. If an individual chooses not to annuitize, they then decide how much of their wealth to consume in each period. Borrowing is not allowed.

Individuals make choices consistent with a survival curve – which we can specify to be their subjective survival curve $(S_i^s(\alpha))$ or their objective survival curve $(S_i^o(\alpha))$. When beliefs are consistent with the *subjective* survival curve, an individual aged z believes that they have a probability of survival to each age $\alpha \leq 110$ that is described by their estimated Weibull survival curve:

$$S_i^s(\alpha) = exp\left[-\left(\frac{\alpha - z}{\hat{\lambda}_i}\right)^{\hat{k}_i}\right] \tag{4}$$

All individuals believe that they will die at the end of their 110th year at the latest. Individuals have constant relative risk aversion utility within each period and they discount the future according to a geometric discount rate (β). They perceive their expected lifetime utility to be:

$$U = \sum_{t=0}^{110-z} \beta^t S_i^x(z+t) \frac{c_t^{1-\gamma}}{1-\gamma}$$
 (5)

for $x \in \{s, o\}$ depending on whether they are assumed to have subjective or objective survival curves.

¹¹We do not include housing wealth in the measure of wealth which may be annuitized – Appendix B.1 adds a consumption flow coming from owner-occupied housing and shows that results are similar.

At time zero (the year in which we observe a household in the survey), individuals can irreversibly annuitize their wealth at rate θ , which is the actuarially fair annuity rate given $S_i^o(\alpha)$, the *objectively-measured* survival curve for someone of their sex, year of birth, and age:

$$\theta = \left[\sum_{\alpha=z}^{110} \frac{S_i^o(\alpha)}{(1+r)^{\alpha-z}} \right]^{-1} \tag{6}$$

An individual's problem is therefore to choose $\{\mathbf{c}_t\}$ and b (whether to annuitize):

$$\max_{\{\mathbf{c}_t\},b} \sum_{t=0}^{110-z} \beta^t S_i(z+t) \frac{c_t^{1-\gamma}}{1-\gamma}$$
 (7)

$$s.t. \begin{cases} a_{t+1} = (a_t + p - c_t)(1+r) & \text{if } b = 0 \\ a_{t+1} = (a_t + p + \theta a_0 - c_t)(1+r) & \text{if } b = 1 \\ a_{t+1} \ge 0 \end{cases}$$

$$(8)$$

Individuals' optimal choice of consumption is characterized by the Euler equation which will bind with equality whenever the no-borrowing constraint does not hold: $c_t^{-\gamma} = \beta(1+r)s_i(z+t)c_{t+1}^{-\gamma}$ where $s_i(z+t) \equiv S_i(z+t+1)/S_i(z+t)$. Individuals will be more inclined to annuitize the higher is β (and so the fact that consumption from wealth cannot be front-loaded when annuitization is chosen does not imply a substantial welfare cost) and the higher is γ (and so the longevity insurance provided by annuities is more valued).

We solve the model for each individual in waves 4, 5 and 6 of ELSA who has begun drawing their public pension, holds positive assets, and for whom we are able to construct a subjective survival curve.¹² We use the first observation for any individuals observed multiple times, yielding 2,592 observations. Each individual's initial level of wealth (a_0) is taken as is the sum of their household private pension wealth and gross financial wealth.¹³ We take

¹²In this period, the male public pension age was 65 and the female state pension age increased from age 60 to age 62. The public pension age is the age at which individuals can first claim their state pension. Over 99% of individuals begin to claim at this age. We drop 453 observations where the individual is over their public pension age, does not report deferring their state pension, but yet reports a public pension income of zero.

¹³This information is available in waves 4 to 6. For individuals in a couple, we take half of this wealth

public pension income (p) to be that level reported in the data. For each observation, we solve the model twice. In one case the individual's expectations are consistent with the scaled life table survival curve for their sex, year of birth and age, and in the other expectations reflect their fitted subjective survival curve estimated from their survey responses.

4.2 Results

We illustrate the impact of subjective survival expectations by comparing the predicted proportion of individuals annuitizing in the case where they behave according to the scaled life table survival curve for their age, sex and cohort with the case where they behave according to their own subjective survival curve.

Figure 6 shows the proportion of individuals annuitizing at various parameter combinations of patience (β) and risk aversion (γ). The real interest rate is set at 0%. Panel (a) shows the proportion annuitizing when they have objectively-measured expectations. The annuity we consider is one which pays a constant (real) income. When households are fully patient ($\beta = 1$), all households will purchase an actuarially-fair annuity – any risk-averse fully-informed individual will prefer a constant stream of income to self-insuring against longevity through a risk-free bond. As patience is decreased, the proportion annuitizing falls.¹⁴ At a coefficient of relative risk aversion of 3, 98% of individuals would annuitize when the discount factor is set at 0.98, and 93% would annuitize with $\beta = 0.96$.

For a fixed pair of preferences parameters, variation across individuals in the decision over whether to annuitize is driven by the relative level of public pension entitlements (p) and the level of wealth (a_0) . Those who do not annuitize are those who have little liquid wealth relative to the size of their accrued public pension entitlements. For these households the level.

 $^{^{14}}$ The annuity puzzle is less stark at lower levels of patience as we assume (realistically) that individuals cannot negotiate an actuarially-fair annuity with a bespoke payment schedule that fits their patience and risk aversion. If complete markets for annuities existed, the proportion annuitizing would be 100%, regardless of patience. Inkmann et al. (2011) show the that this type of market incompleteness, combined with estimated risk aversion and patience can rationalize much of the lack of demand for annuities. To see the effects of our results in a complete markets context, the reader should focus only on the row with $\beta = 1$, the level of patience for which the offered contract is the preferred one.

value of the small amount of additional longevity insurance they would receive by annuitizing is less than the additional value of consuming that wealth sooner.

Panel (b) shows an equivalent set of results for the case when individuals are making decisions based on their subjective survival expectations. Comparing these rates to those reported in panel (b), it is clear that subjective expectations have the capacity to substantially reduce annuity demand. For fully patient individuals, rates of annuitization fall from 100% to between 62% (log utility) and 90% (coefficient of relative risk aversion of 5). With modest impatience ($\beta = 0.98$), rates of annuitization fall from 87% to 30% assuming log utility, and from 98% to 86% assuming a high rate of risk aversion ($\gamma = 5$).

Appendix B shows that are results are minimally sensitive to (1) including a flow of consumption of housing services for home-owners and (2) the removal of individuals answering "50%".

Figure 6: Percentage of individuals annuitising at each parameter combination

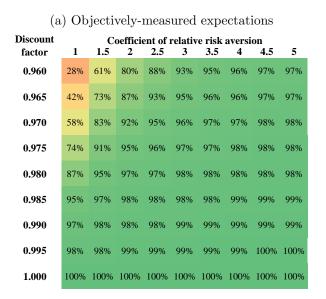
(a) Objectively-measured expectations								((b) Subjectively-elicted expectations										
Discount factor	1	C 1.5	oefficio 2	2.5	relativ 3	e risk 3.5	aversi 4	on 4.5	5	Discount factor	1	Co 1.5	oefficie 2	ent of 1 2.5	relativ 3	e risk : 3.5	aversi 4	on 4.5	5
0.960	28%	61%	80%	88%	93%	95%	96%	97%	97%	0.960	7%	21%	36%	49%	58%	66%	71%	75%	79%
0.965	42%	73%	87%	93%	95%	96%	96%	97%	97%	0.965	11%	28%	43%	55%	64%	70%	74%	78%	81%
0.970	58%	83%	92%	95%	96%	97%	97%	98%	98%	0.970	16%	34%	49%	60%	68%	73%	76%	81%	83%
0.975	74%	91%	95%	96%	97%	97%	98%	98%	98%	0.975	23%	41%	56%	65%	72%	75%	79%	83%	85%
0.980	87%	95%	97%	97%	98%	98%	98%	98%	98%	0.980	30%	49%	62%	70%	75%	78%	81%	84%	86%
0.985	95%	97%	98%	98%	98%	98%	99%	99%	99%	0.985	38%	57%	67%	73%	77%	80%	83%	86%	87%
0.990	97%	98%	98%	98%	99%	99%	99%	99%	99%	0.990	47%	63%	71%	76%	79%	82%	85%	87%	88%
0.995	98%	98%	99%	99%	99%	99%	99%	100%	100%	0.995	55%	68%	74%	78%	81%	84%	86%	88%	90%
1.000	100%	100%	100%	100%	100%	100%	100%	100%	100%	1.000	62%	72%	77%	81%	83%	85%	87%	89%	90%

Source: Model predictions using ELSA waves 4–6 and ONS 2014-based cohort life tables for England and Wales. 2,592 observations.

To put the size of the falls in annuitization rates in context, Figure 7 shows the proportion who would annuitize if they had objectively-estimated survival expectations but were faced with an annuity which, due to adverse selection and other market imperfections, as well

as transactions costs, is offered at 17.5% below the actuarially-fair rate.¹⁵ In panel (b) of Figure 7, we show the model predictions in the case where individuals act according to their 'objective' survival curve, but face an annuity payout equal to 82.5% of the actuarially fair rate. We reproduce panel (a), where individuals have 'objective' expectations and face the actuarially fair annuity rate, for comparison. This is intended to illustrate the impact of the degree of actuarial unfairness observed in UK annuities markets which may be attributable to adverse selection or administrative loading.

Figure 7: Percentage of individuals annuitizing at each parameter combination



(b) Objectively-measured expectations with 17.5% rate reduction

Discount factor	1	1.5	oefficio 2	ent of 1 2.5	relativ 3	e risk 3.5	aversi 4	on 4.5	5
0.960	0%	4%	17%	33%	49%	62%	72%	80%	86%
0.965	0%	9%	25%	44%	60%	72%	81%	87%	90%
0.970	1%	15%	35%	55%	71%	81%	88%	92%	94%
0.975	3%	24%	49%	68%	82%	89%	93%	95%	96%
0.980	10%	36%	63%	81%	90%	94%	96%	97%	97%
0.985	19%	54%	79%	91%	96%	97%	97%	98%	98%
0.990	34%	74%	92%	96%	97%	98%	98%	98%	98%
0.995	58%	91%	97%	98%	98%	98%	99%	99%	99%
1.000	87%	98%	99%	99%	99%	99%	99%	100%	100%

Source: Model predictions using ELSA waves 4–6 and ONS 2014-based cohort life tables for England and Wales. 2,592 observations.

For a majority of parameter combinations, the actuarially-unfair pricing causes comparable or smaller falls in annuitization rates than does the mis-perceived survival probabilities. With moderate levels of impatience and risk aversion, their effects are comparable: at $(\beta, \gamma) = (0.98, 2)$, the rate of annuitization with objectively-measured expectations is 97% but falls to 63% under the reduced annuity payouts and 62% when individuals make

 $^{^{15}}$ We use 17.5% based on the analysis of annuity rates available on the US market by Mitchell et al. (1999) who report that "the expected discounted value of annuity payouts per dollar of annuity premium averages between 80 and 85 cents for an individual chosen at random from the population".

decisions based on their subjective survival expectations. The effect of subjective survival expectations has a slightly larger impact than adverse selection at higher levels of risk aversion – at $(\beta, \gamma) = (0.98, 5)$, 99% would annuitize given objective expectations and actuarial fairness, 97% would annuitize given objective expectations and reduced payouts, while 86% would annuitize under subjective expectations and actuarial fairness. The one section of the parameter space that we consider in which the effect of adverse selection is larger than the effect of subjective expectations is when individuals are impatient and have extremely low risk aversion.¹⁶

Overall, we take these results as indicating the effect of individuals mis-perceiving their survival probabilities is as large if not larger than the effect of adverse selection.

5 Conclusion

Incorporating individual 'subjective' survival curves into a model of annuitization and use of wealth at older ages has the capacity to explain part of the "annuity puzzle". While market incompleteness and informational asymmetries play a role in rationalizing low annuity demand, we take our results as showing that misperceptions of survival probabilities are as important for explaining behavior.

Our results are important for government policy in relation to annuities and retirement provision more generally. While resources do exist to inform individuals about their life expectancy, ¹⁷ the divergence between self-assessed and objective life expectancies, and the

¹⁶The reason for these patterns can be understood by comparing the reasons in each case for annuities becoming less attractive products. With adverse selection raising annuity prices, all annuity payouts are discounted relative to the actuarially fair benchmark. With subjective survival rates, the annuity payouts late in life are not considered valuable – as individuals wrongly perceive that they will likely be dead by then. When individuals are less patient therefore, reducing annuity payments every period implies a greater reduction in welfare than does treating payments in the distant future as 'wasted'. When risk aversion is high, the welfare cost of the higher price is low relative to the cost of insurance against the modest possibly of living a long time. This insurance is less valuable as the likelihood of that longevity appears to diminish. As a result the higher is risk aversion the greater is the effect of subjective survival probabilities relative to reduced annuity rates.

¹⁷For example, this online life expectancy calculator from the Social Security Administration: https://www.ssa.gov/planners/lifeexpectancy.html.

implication of that divergence for annuity purchases leaves a role for larger policy interventions to inform households about the length of the retirement that they might have to fund. As individuals approach retirement with increasingly large shares of their wealth in non-annuitised form, ensuring that individuals are adequately informed about their longevity in this way will become only more pressing.

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A Details of further analysis and tests from Section 2

A.1 Analysis of "50%" answers

The following table details the distribution of individuals by the number of times they answered "50%" to questions in the expectations module other than the survival questions. The reporting patterns in the 20.5% of interviews in which individuals answered "50%" to the first survival question are very similar to distribution of responses amongst the whole sample.

Table 1: Distribution of number of expectations questions to which individuals answer "50%"

		Number	of "50%"	answers	given	
	0	1	2	3	4	5
All individuals	55.42%	35.34%	8.07%	1.05%	0.12%	0.00%
Answered "50%" to 1st survival Q	52.09%	35.78%	10.20%	1.62%	0.30%	0.01%

Note: Other probability questions include those related to the probability of moving out of ones home in the future, of being in work in a number of years time, of having insufficient financial resources to meet needs at some point in the future, of it raining tomorrow and of giving and receiving an inheritance. Source: ELSA waves 17. 66,210 interviews from 16,345 unique individuals.

A.2 Correlation of subjective reports with risk factors, new information and subsequent mortality

Table 2 details the results of a regression of an individual's answer to the first survival question they are asked on a range of risk factors as well as a full set of wave*single-year-of-age dummy interactions. Results are split by gender and by whether or not self-reported health is controlled for. The coefficients reported are in percentage point deviations. For example, a male current smoker reports 6.7 percentage points lower chance of survival to an age 11–15 years ahead of their current age, on average, when compared to current non-smokers.

Table 3 reports the results of a fixed effects regression of individuals' answers to the survival expectations questions on a range of dummies for whether or not they have received a new diagnosis of a health condition since their last interview. We control linearly for age. We find that individuals do respond to new information by revising their survival expectations. For example, a new diagnosis of cancer or a case of a stroke cause large and statistically significant downward revisions in survival expectations of 5 and 6 percentage

points respectively.

Finally, Figure 8 shows the 10 year mortality rates of individuals according to their answer to the first survival question. We use the linked death records which give us a 10-year horizon for those interviewed in wave 1 of ELSA. We see clear differences in mortality rates according to stated expectations. These differences are statistically significant. The correlation between expected and actual mortality remains even when we control for age and sex-specific average mortality risk and the range of health factors controlled for in the previous regressions.

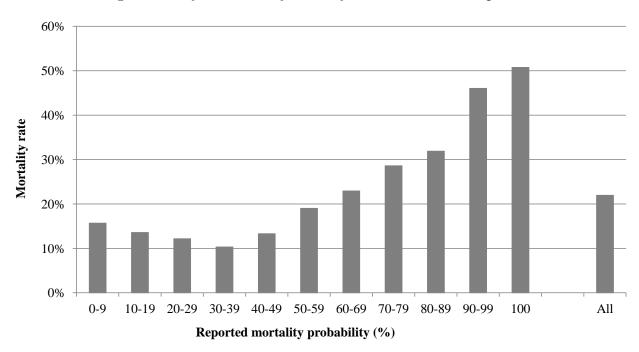


Figure 8: 10 year mortality rates by answer to survival question

Note: Reported probability of death is 100 minus the reported probability of survival in the first survival question the individual is asked. Source: ELSA wave 1 and linked death records. 11,502 individuals.

B Sensitivity of results from Section 4.2

B.1 Sensitivity of results to inclusion of housing consumption

We show here results of the model in the case where home-owners receive utility from the consumption of housing services. We assume that individuals receive a per-period flow of housing services, h equal to 4% of the value of their primary house. This value is fixed in

real terms in future periods. Their utility function is of the form:

$$U = \sum_{t=0}^{110-z} \beta^t S_i(z+t) \frac{(c_t+h)^{1-\gamma}}{1-\gamma}$$
 (9)

All other details of the model are as given in Section 4.1. Figure 9 shows the model predictions.

Figure 9: Percentage of individuals annuitising at each parameter combination (housing in the utility function)

	(a) Scaled life table expectations								(b) Subjective expectations										
Discount factor	1	Co 1.5	oefficio 2	ent of 1 2.5	relativ 3	e risk : 3.5	aversio 4	on 4.5	5	Discount factor	1	Co 1.5	oefficie 2	ent of 1 2.5	relativ 3	e risk : 3.5	aversio 4	on 4.5	5
0.960	6%	35%	63%	78%	86%	89%	91%	92%	93%	0.960	2%	10%	23%	36%	48%	58%	65%	72%	77%
0.965	13%	50%	75%	85%	88%	91%	92%	93%	94%	0.965	3%	14%	30%	42%	54%	63%	70%	76%	79%
0.970	26%	66%	82%	88%	91%	92%	93%	94%	95%	0.970	6%	20%	36%	48%	59%	67%	73%	79%	82%
0.975	46%	79%	88%	91%	93%	93%	94%	95%	95%	0.975	11%	27%	42%	55%	64%	71%	76%	81%	84%
0.980	69%	87%	91%	93%	94%	95%	95%	96%	96%	0.980	17%	35%	49%	60%	69%	75%	79%	83%	86%
0.985	85%	92%	93%	94%	95%	95%	96%	96%	96%	0.985	25%	41%	55%	65%	73%	78%	81%	85%	87%
0.990	92%	94%	95%	96%	96%	96%	97%	97%	97%	0.990	33%	48%	61%	70%	76%	80%	84%	87%	89%
0.995	95%	96%	97%	97%	97%	98%	98%	98%	98%	0.995	41%	56%	67%	74%	79%	83%	86%	89%	90%
1.000	100%	100%	100%	100%	100%	100%	100%	100%	100%	1.000	48%	62%	71%	77%	82%	85%	87%	90%	92%

Source: Model predictions using ELSA waves 4–6 and ONS 2014-based cohort life tables for England and Wales. 2,592 observations.

B.2 Sensitivity of results to inclusion of "50%" answers

We show here results from the model when we drop all observations where the individual answered "50%" to one of more survival question. There are 942 individuals dropped as a result, meaning that we have a sample size of 1,650 unique individuals in this case. Figure 10 shows the model predictions.

Figure 10: Percentage of individuals annuitising at each parameter combination (excluding individuals who answer 50%)

	(a) Scaled life table expectations								(b) Subjective expectations										
Discount factor	1	C 1.5	oefficio 2	ent of 1 2.5	relativ 3	e risk 3.5	aversi 4	on 4.5	5	Discount factor	1	Co 1.5	oefficio 2	ent of 1 2.5	relativ 3	e risk : 3.5	aversi 4	on 4.5	5
0.960	29%	63%	82%	91%	94%	96%	97%	97%	98%	0.960	12%	28%	45%	57%	64%	71%	74%	78%	81%
0.965	44%	76%	89%	94%	96%	97%	97%	98%	98%	0.965	16%	37%	52%	62%	69%	74%	77%	80%	83%
0.970	60%	86%	94%	96%	97%	98%	98%	98%	99%	0.970	23%	44%	58%	67%	73%	76%	79%	82%	84%
0.975	77%	92%	96%	97%	98%	98%	99%	99%	99%	0.975	32%	51%	64%	70%	76%	78%	81%	83%	85%
0.980	89%	96%	98%	98%	99%	99%	99%	99%	99%	0.980	42%	58%	68%	74%	77%	80%	83%	85%	86%
0.985	96%	98%	99%	99%	99%	99%	99%	99%	99%	0.985	50%	65%	71%	76%	79%	82%	84%	86%	87%
0.990	98%	99%	99%	99%	99%	99%	99%	99%	100%	0.990	58%	69%	74%	78%	81%	83%	85%	86%	88%
0.995	99%	99%	99%	99%	100%	100%	100%	100%	100%	0.995	64%	71%	77%	80%	82%	85%	86%	88%	89%
1.000	100%	100%	100%	100%	100%	100%	100%	100%	100%	1.000	68%	74%	78%	81%	84%	86%	87%	89%	89%

Source: Model predictions using ELSA waves 4-6 and ONS 2014-based cohort life tables for England and Wales. 1,650 observations.

Table 2: Relationship between stated survival probabilities and risk factors

	Ex. self-re	eported health	Inc. self-re	eported health
	Male	Female	Male	Female
Smoking (relative to non-smoker)				
Ex-occassional smoker	-3.0***	-0.7	-2.8**	-0.2
Ex-regular smoker	-1.9***	-0.4	-1.4**	-0.1
Ex-smoker (frequency: don't know)	-3.0***	-1.9	-2.6**	-1.4
Current smoker	-8.1***	-7.5^{***}	-6.7^{***}	-6.4^{***}
Alcohol consumption				
(relative to once or twice a month)				
At least 3–4 days a week	0.1	0.1	-0.6	-0.4
Once or twice a week	0.1	0.2	-0.2	-0.1
A few times a year	-1.2	-1.3^{**}	-0.6	-1.0^{*}
Not at all	-2.5^{**}	-2.1^{***}	-1.5	-1.2
Age mother died (relative to 60-64)				
Under 50	1.6	1.3	1.7	1.6
50-59	1.0	-2.6^{*}	0.7	-2.6*
65–69	3.3**	-1.8	3.1**	-1.5
70–74	2.2	-0.7	2.1	-0.5
75–79	1.9	0.2	1.4	0.3
80-84	3.1**	2.2^{*}	2.7^{**}	2.5**
85 and over	5.6***	6.9***	5.2***	7.0***
Age father died (relative to 60-64)				
Under 50	0.9	0.1	1.1	0.1
50-59	-0.1	0.3	0.2	0.5
65–69	0.9	-0.9	0.8	-0.6
70-74	2.7^{**}	1.8^{*}	2.6**	1.8*
75–79	4.5^{***}	2.9***	4.5^{***}	2.9***
80–84	4.4***	2.5***	4.2^{***}	2.4***
85 and over	7.1^{***}	5.0^{***}	7.0^{***}	4.9***
Health Conditions				
Cancer	-6.2***	-3.4***	-4.2***	-2.2^{***}
Heart condition	-3.1^{***}	-2.3***	-1.5^{*}	-1.4^{*}
Hypertension	2.1***	2.3***	-0.8^{*}	-1.3***
Stroke	-3.3***	-1.0	1.4	0.7

Coefficients represent percentage point deviations in mean response. Statistical significance at the 10%/5%/1% level is denoted by */**/***. Standard errors are clustered at the individual level. Other control variables, for which coefficients are not reported, are whether in a couple, income and wealth quintile, education level, whether working and dummy variables for whether diagnosed with Alzheimers, angina, arthritis, diabetes, lung disease, osteoporosis, Parkinsons and psychiatric disorders, whether the individual is white or non-white and a full set of dummy variables for each single year-of-age and wave interaction. Source: ELSA waves 17. 52,170 observations of 14,092 unique individuals.

Table 3: Revision to survival expectations following diagnosis with major health conditions

	1st surival question	2nd survival question
Alzheimers disease	-7.8^*	-1.9
Cancer	-4.5^{***}	-3.0^{*}
Dementia	-7.4^{**}	-2.6
Heart attack	-2.5	-1.3
Lung disease	-2.2	1.4
Parkinson's disease	-2.9	-8.6
Psychiatric problems	-2.0	-1.2
Stroke	-5.7^{***}	-5.0^{*}

Note: Coefficients represent percentage point deviations in mean response. Statistical significance at the 10%/5%/1% level is denoted by */**/***. Standard errors are clustered at the individual level. Source: ELSA waves 37. 37,760 observations of 12,027 unique individuals.