DEMOGRAPHICS IN DEMAND SYSTEMS

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Abstract

Household composition can be expected to affect the allocation of household expenditure among goods, at the very least because of economies of scale as household size increases and because different people have different needs (adults versus children, for example). Specifying demographic effects correctly in demand analysis is important both in order to estimate correct price and expenditure elasticities and for the purpose of making household welfare comparisons. A common way of including demographics is as a function that scales total expenditure, and to make this scaling function independent of the level of total expenditure. A popular method in the parametric estimation of demand systems is to estimate share equations that are quadratic in the logarithm of total expenditure, but there is also a substantial literature on the semi-parametric estimation of Engel curves. We employ some of these semi-parametric techniques to show that, for some goods, further terms are likely to be required in the Engel curve addition to quadratic terms. We use this to identify the parameters of a scaling function that varies with total expenditure.

Key Words: Demand analysis, demographics, base independence, semi-parametric.

JEL Classification: C14, D11, D12.

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Summary

- Specifying demographic effects correctly in demand analysis is important both in order to estimate correct price and expenditure elasticities and for the purpose of making household welfare comparisons.

- A common way of including demographics is as a function that scales total expenditure, and to make this scaling function independent of the level of total expenditure. A popular method in the parametric estimation of demand systems is to estimate share equations that are quadratic in the logarithm of total expenditure. In addition the parameters of a base independent scale can be identified with quadratic Engel curves.

- We find evidence that both the quadratic specification and the base independent restriction may be overly restrictive, and use further terms to identify the parameters of a scale that varies with total expenditure.

- The parameter that shows how the scale varies with total expenditure is well determined and is less than one, which means the scale decreases as total expenditure increases. The base independent scale estimated on the same data is generally hard to identify precisely and the point estimate is often implausibly low.

1 Introduction

Demand analysis invariably takes place at the level of the household, not the individual, and so how to include household structure in demand analysis is a crucial question. At the very least we would expect young children to have different needs to adults, and for economies of scale in the provision of some household goods and services to mean that a couple living together, for example, do not have the same demands as two single adult households with half the couple's budget each.

There are at least two reasons why specifying demographic effects correctly is important. The first is in order to estimate correct price and expenditure elasticities. Given enough data, one approach would be simply to estimate the responses of demands to changes in prices and total budget separately for each household type. Scarcity of data usually prevents this, and, in addition, if there is some relationship between demand across household types, not pooling the data leads to a loss in estimation efficiency. The second is for the purpose of making household welfare comparisons.

In share equation demand systems such as the Almost Ideal Demand System (AIDS) of Deaton and Muellbauer (1980) and its quadratic extension the QUAIDS (Banks, Blundell and Lewbel (1997)) a popular way of including demographics is as a function that scales total expenditure. For reasons expanded on below it is also common to make this scaling function independent of the level of total expenditure (or 'base independent'). Figure 1 below shows various budget share Engel curves for couples without children and with one child which have been estimated in different ways. One is a quadratic logarithmic (in total expenditure) specification (as in QUAIDS) and the other two are an unrestricted semi-parametric regression (using spline smoothing) and a semi-parametric regression imposing base independence. These regressions (and the data) will be discussed in more detail below, but the reason for introducing them at this early stage is to illustrate firstly that the quadratic logarithmic specification appears overly restrictive when compared to the unrestricted semi-parametric regression (for at least some of the goods); and secondly that the base independence restricted regressions look quite different from the unrestricted regressions for many of the goods. Both these points are important motivations for the analysis in this paper.

In section 2 we review some important previous work on incorporating demographics in demand systems. We discuss the notion of expenditure scaling and introduce a scale which is dependent on the level of expenditure. In section 3 we discuss the data we use in the empirical part of the paper and do some preliminary analysis. In section 4 we discuss the theoretical and empirical identification of the parameters of the expenditure scaling functions and then move on to the main empirical application in section 5, where we concentrate on semi-parametric estimation of Engel curves.

2 Demographically extended demand systems

If we are simply interested in allowing price and expenditure elasticities to vary across household types then it seems natural to incorporate demographics by just letting some parameters of the particular demand system being estimated vary by household composition. For example, in recent years, semi-parametric estimation of budget share Engel curves has become quite common, and a popular approach, because of its simplicity, has been to estimate a partially linear form, where demographics only enter as intercept shifters. The problem with this specification is that the restrictions imposed by consumer theory have implications for the way in which demographics can enter the demand system. As shown by Blundell, Duncan and Pendakur (1998), consideration of the integrability conditions means that including demographics via the partially linear specification imposes strong restrictions – for example, if one good has a budget share that is linear in log expenditure (linearity is usually a good approximation for food, for example) then all goods are restricted to have linear budget shares.

In order to ensure integrability of a demographically extended demand system it is most straightforward to start from the underlying cost function for a reference household (a single adult household, say) and introduce demographics in a way that guarantees that the demographically modified cost function is still a valid cost function[1]. For example, in the demographic translating procedure of Pollak and Wales (1980), fixed costs are added to or deducted from the operations of the household. If the cost function of the reference household is $\overline{c}(u,p)$ then translating replaces this by $c(u,p,z) = \overline{c}(u,p) + \sum\limits_i d_i p_i$ for a household with characteristics z, where the d's are the translation parameters, which are functions of z (for example, $d_i(z) = \sum_k \eta_{ik} z_k$ or $\prod_k z_k^{\eta_{ik}}$). If the reference demands are $\overline{q}_i(M, \mathbf{p})$ then the demographically translated demands, $q_i(M, \mathbf{p}, z)$, are equal to $d_i + \overline{q}_i\left(M - \sum\limits_j d_j p_j, \mathbf{p}\right)$.

This is a specific example of a general method for letting demographics enter the cost function explored by Lewbel (1985). Following his analysis

$$c(u,p,z) = f\left[\overline{c}(u, h[p,z]), p, z\right]$$

or

$$M = f\left[\overline{M^*}, p, z\right]$$

[1] i.e. concave, homogenous of degree one and non-decreasing in prices (and increasing in at least one price) and increasing in u.

where $\overline{M^*} = \overline{c}(u, h[p, z])$. For $c(u, p, z)$ to be a valid cost function, f must be monotonically increasing in $\overline{M^*}$ and so can be inverted to give $\overline{M^*} = g[M, p, z]$. Hence, if $\overline{V}\left(\overline{M}, p\right)$ is the indirect utility function associated with the inversion of $\overline{c}(u, p)$ and $V(M, p, z)$ is that associated with $c(u, p, z)$ then we have the relationship

$$V(M, p, z) = \overline{V}(g[M, p, z], p^*)$$

where $p^* = h[p, z]$.

Since (compensated) budget shares are derived from the differentiation of the log of the cost function with respect to log prices we obtain the following relationship between the budget shares of the reference household and the demographically modified budget shares:

$$\begin{aligned} w_i(u, p, z) &= \frac{\partial \ln f[\overline{c}(u, p^*), p, z]}{\partial \ln \overline{c}(u, p^*)} \sum_k \frac{\partial h_k[p, z]}{\partial \ln p_i} \frac{\partial \ln \overline{c}(u, p^*)}{\partial \ln p_i} + \frac{\partial \ln f[\overline{c}(u, p^*), p, z]}{\partial \ln p_i} \\ &= \frac{\partial \ln f[\overline{c}(u, p^*), p, z]}{\partial \ln \overline{c}(u, p^*)} \sum_k \frac{\partial h_k[p, z]}{\partial \ln p_i} \overline{w}_i(u, p^*) + \frac{\partial \ln f[\overline{c}(u, p^*), p, z]}{\partial \ln p_i} \end{aligned}$$

or in Marshallian form

$$w_i(M, p, z) = \frac{\partial \ln f\left[\overline{M^*}, p, z\right]}{\partial \ln \overline{M}} \sum_k \frac{\partial h_k[p, z]}{\partial \ln p_i} \overline{w}_i\left(\overline{M^*}, p^*\right) + \frac{\partial \ln f\left[\overline{M^*}, p, z\right]}{\partial \ln p_i}$$

2.1 Expenditure scaling

2.1.1 Expenditure invariant scales

In budget share analysis it is natural to let demographics simply scale total expenditure (equivalently translate log total expenditure), and a scaling function that is commonly used is $\overline{M} = M/s(p, z)$ so that $\ln c(u, p, z) = \ln \overline{c}(u, p) + \ln s(p, z)$ – i.e. the scale does not vary with the expenditure level. Thus we have that

$$\begin{aligned} w_i(M, p, z) &= \overline{w}_i\left(\overline{M}, p\right) + \frac{\partial \ln s(z, p)}{\partial \ln p_i} \\ &= \overline{w}_i(\ln M - \ln s(z, p), p) + \frac{\partial \ln s(z, p)}{\partial \ln p_i} \end{aligned} \qquad (1)$$

which illustrates the implications that this form of expenditure scaling has for the relationship of share equations across different demographic groups – namely that w_i cannot just have demographic terms in the intercept unless $\overline{w}_i$ is linear in $\ln M$ so that w_i is linear in $(\ln M - \ln s(z, p))$. If $\overline{w}_i$ were to be quadratic in $\ln M$, for example, then w_i would involve $(\ln M - \ln s(z, p))^2$ which introduces interactions between $\ln s(z, p)$ and $\ln M$ in w_i.

2.1.2 Expenditure dependent scales

Since the studies of Working (1943) and Leser (1963) it has been common to model budget shares as functions of log total expenditure – for example, the AIDS model and Jorgenson, Lau and Stoker's (1982) Translog model are linear in log total expenditure, and Banks, Blundell and Lewbel (1997) extended the AIDS model to include quadratic terms following evidence that linearity was insufficiently flexible for some goods. Focusing on specifying Engel curves as functions of log total expenditure, then, if we want the modified Engel curves to be made up of the same functions of log expenditure as the reference Engel curves, $\ln \overline{M}$ must be a linear function of $\ln M$, and so, apart from the expenditure invariant scale, the only other possible transformation is of the form

$$\begin{aligned} \ln M &= \Phi_1(p,z)\ln \overline{M} + \ln \Phi_0(p,z) \\ &\Rightarrow \ln \overline{M} = \frac{\ln M - \ln \Phi_0}{\Phi_1} \end{aligned}$$

This gives

$$w_i(\ln M, p, z) = \Phi_1 \overline{w}_i\left(\frac{\ln M - \ln \Phi_0}{\Phi_1}, p\right) + \frac{\partial \Phi_1}{\partial \ln p_i}\left[\frac{\ln M - \ln \Phi_0}{\Phi_1}\right] + \frac{\partial \ln \Phi_0}{\partial \ln p_i}$$

which means that w_i and $\overline{w}_i$ will usually be functions of the same basic functions of log expenditure as long as $\overline{w}_i$ has a linear term in log expenditure. With this modifying function, the scaling term will now depend on utility (or expenditure) and is given by

$$\ln s(u,p,z) = \ln s(\overline{c}(u,p), p, z) = \ln \Phi_0(p,z) + (\Phi_1(p,z) - 1)\ln \overline{c}(u,p)$$

Degree-one-homogeneity of the cost function with respect to prices places some restrictions on the parameters of the scaling function – adding up over the share equations, it can be seen that

$$\begin{aligned} 1 &= \Phi_1 + \left[\frac{\ln M - \ln \Phi_0}{\Phi_1}\right]\sum_i \frac{\partial \Phi_1}{\partial \ln p_i} + \sum_i \frac{\partial \ln \Phi_0}{\partial \ln p_i} \\ \Rightarrow \Phi_1 &= 1 - \left[\frac{\ln M - \ln \Phi_0}{\Phi_1}\right]\sum_i \frac{\partial \Phi_1}{\partial \ln p_i} - \sum_i \frac{\partial \ln \Phi_0}{\partial \ln p_i} \end{aligned}$$

Since Φ_1 does not depend on $\ln M$, this implies that $\sum_i \partial \Phi_1 / \partial \ln p_i = 0$, i.e. Φ_1 is homogeneous of degree zero in prices, and we obtain

$$\Phi_1 = 1 - \sum_i \frac{\partial \ln \Phi_0}{\partial \ln p_i}$$

This method of including demographic effects into a demand system is, in fact, used in a paper by Ray (1983), although the precise model and implications are not really explicitly worked through. Ray uses an AIDS model[2] so that budget shares are given by

$$w_i(\ln M, p, z) = \frac{\partial \ln a\,(p,z)}{\partial \ln p_i} + \frac{\partial \ln b\,(p,z)}{\partial \ln p_i}\,(\ln M - \ln a\,(p,z))$$

(since the AIDS cost function is $\ln c\,(u, \mathbf{p}, \mathbf{z}) = \ln a(\mathbf{p}, \mathbf{z}) + ub(\mathbf{p}, \mathbf{z})$) and it turns out that in the parameterisation that Ray chooses, since $\ln a(p,z)$ is specified such that the effect of characteristics are price independent, and $\Phi_1\,(p,z)$ is specified as $\prod_i p_i^{\eta_i z}$ so that $\partial \ln \Phi_1 / \partial \ln p_i = \eta_i$, all the parameters can be identified in the linear specification. If this was not the case, then it would not be possible to identify all the parameters of $\ln a\,(p,z)$. Indeed, this would be so even under utility independence of the scaling function, as can be seen from the following (and is discussed in Dickens, Fry and Pashardes (1993)). Denoting $\partial \ln s / \partial \ln p_i$ by η_i, recall from equation 1 that

$$w_i\,(\ln M, \mathbf{p}, \mathbf{z}) = \overline{w}_i\,(\ln M - \ln s, \mathbf{p}) + \eta_i$$

Denoting the reference household's Engel curves by

$$\overline{w}_i = a_i + b_i \ln M$$

and the Engel curves for a household with characteristics z by

$$w_i^Z = a_i^Z + b_i^Z \ln M$$

then

$$\begin{aligned} w_i^Z &= a_i^Z + b_i^Z \ln M \\ &= a_i + b_i\,(\ln M - \ln s) + \eta_i \\ &= [a_i - b_i \ln s + \eta_i] + b_i \ln M \end{aligned}$$

so that $\ln s$ and η_i cannot be identified separately. With quadratic Engel curves, identification is possible, since

$$\overline{w}_i = a_i + b_i \ln M + c_i\,(\ln M)^2$$

[2] The cost function for the AIDS model is $\ln \overline{c}\,(u,p) = \ln \overline{a}(p) + u\overline{b}(p)$. Ray presents his 'general scale' as (using his notation) $\overline{m}_0\,(z)\,\phi\,(p,z,u)$, where $\overline{m}_0$ is a 'basic' component and ϕ the price and utility varying component. As is common in AIDS modelling, $\ln \overline{a}(p)$ is specified as $\alpha_0 + \sum_i \alpha_i \ln p_i + \frac{1}{2}\sum_i \sum_j \gamma_{ij} \ln p_i \ln p_j$ and $\overline{b}(p)$ as $\prod_i p_i^{\beta_i}$. Ray then parameterises $\ln \phi\,(p,z,u)$ as $u \prod_i p_i^{\beta_i}\left(\prod_i p_i^{\eta_i z} - 1\right) = u\overline{b}(p)\,[g(p,z) - 1]$ (denoting $\prod_i p_i^{\eta_i z}$ by $g(p,z)$) which can be seen to equal $[\ln \overline{c}\,(u,p) - \ln \overline{a}(p)]\,(g(p,z) - 1)$, and so $\ln c\,(u,p,z) = \Phi_1\,(p,z) \ln \overline{c}\,(u,p) + \ln \Phi_0\,(p,z)$ with $\ln \Phi_0\,(p,z) = \ln \overline{m}_0\,(z) - \ln \overline{a}\,(p)\,(g(p,z) - 1)$ and $\Phi_1\,(p,z) = g(p,z)$.

and

$$\begin{aligned}
w_i^Z &= a_i^Z + b_i^Z \ln M + c_i^Z (\ln M)^2 \\
&= a_i + b_i (\ln M - \ln s) + c_i (\ln M - \ln s)^2 + \eta_i \\
&= a_i - b_i \ln s + c_i (\ln s)^2 + \eta_i \\
&\quad + (b_i - c_i \ln s) \ln M \\
&\quad + c_i (\ln M)^2
\end{aligned}$$

So

$$\begin{aligned}
c_i^Z &= c_i \\
b_i^Z &= b_i - c_i \ln s \\
a_i^Z &= a_i - b_i \ln s + c_i (\ln s)^2 + \eta_i
\end{aligned} \tag{2}$$

and we have two equations and two unknowns with 1 good, or $2N$ equations and $N+1$ unknowns with N goods.

With the base dependent scaling function, quadratic Engel curves no longer identify all the parameters, since (denoting $\partial \ln \Phi_0 / \partial \ln p_i$ by κ_{0i} and $\partial \Phi_1 / \partial \ln p_i$ by κ_{1i})

$$\begin{aligned}
w_i^Z &= a_i^Z + b_i^Z \ln M + c_i^Z (\ln M)^2 \\
&= \Phi_1 \left[a_i + b_i \left(\frac{\ln M - \ln \Phi_0}{\Phi_1} \right) + c_i \left(\frac{\ln M - \ln \Phi_0}{\Phi_1} \right)^2 \right] + \kappa_{1i} \left(\frac{\ln M - \ln \Phi_0}{\Phi_1} \right) + \kappa_{0i} \\
&= \Phi_1 a_i - \ln \Phi_0 b_i + \frac{c_i (\ln \Phi_0)^2}{\Phi_1} - \frac{\kappa_{1i} \ln \Phi_0}{\Phi_1} + \kappa_{0i} \\
&\quad + \left(b_i - 2 \frac{c_i \ln \Phi_0}{\Phi_1} + \frac{\kappa_{1i}}{\Phi_1} \right) \ln M \\
&\quad + \frac{c_i}{\Phi_1} (\ln M)^2
\end{aligned}$$

which gives

$$\begin{aligned}
c_i^Z &= \frac{c_i}{\Phi_1} \\
b_i^Z &= b_i - 2 \frac{c_i \ln \Phi_0}{\Phi_1} + \frac{\kappa_{1i}}{\Phi_1} \\
a_i^Z &= \Phi_1 a_i - \ln \Phi_0 b_i + \frac{c_i (\ln \Phi_0)^2}{\Phi_1} - \frac{\kappa_{1i} \ln \Phi_0}{\Phi_1} + \kappa_{0i}
\end{aligned} \tag{3}$$

So c_i^Z can identify Φ_1, but then (with one good) we have two other equations but three unknowns, so we cannot identify the remaining parameters, most importantly the other component of the scale, $\ln \Phi_0$, cannot be identified. If we add another good, we get two more equations (since c_j^Z just identifies Φ_1 again) and two more unknowns, and adding another good again gives two more equations and two more unknowns, and so on – so that, in fact, in the quadratic model, there are no restrictions between b_i^Z and b_i or a_i^Z and a_i. So quadratic Engel curves cannot give identification of all the parameters in this model. Although all the parameters cannot be identified, quadratic Engel curves can still be used to test the restriction that $c_i^Z = c_i/\Phi_1$ – obviously this is only a restriction if we are estimating shares for more than one good.

3 Data and preliminary analysis

Our data are taken from the UK Family Expenditure Survey (FES). The FES is a continuous household survey which began in 1957 and is carried out by the Office for National Statistics. Approximately 7,000 households are interviewed each year. The survey consists of a comprehensive household questionnaire which asks about regular household bills and expenditure on major but infrequent purchases (e.g. rent, gas and electricity bills), an individual questionnaire for each adult (aged 16 or over) which asks detailed questions about their income, including details about economic activity (primary and secondary) and sources of income (including wages, pensions and benefits), and a diary of all personal expenditure kept by each adult for two weeks.

Our reference household group is working age couples (married or cohabiting) where the household head is employed and our comparison group is working age couples, head employed with one dependent child. We look at the pattern of non-housing expenditure. We estimate a system of budget shares for food in, food out, alcohol, fuel, clothing, transport and other goods and services. In order to prevent the number of observations we have becoming too small we pool data across two years.

We use three pairs of years spaced over nearly fifteen years – from 1985/86 to the most up-to-date years we have available, which are 1998/99. We begin with a brief look at the quadratic logarithmic specification since the QUAIDS is widely used in empirical demand analysis. We test $c_i^Z = c_i/\Phi_1$ as implied by the base dependent scale and the restrictions in equation 2 implied by base independence against the unrestricted model using a chi-squared test.

The models are estimated by arranging the data as $\mathbf{w} = \begin{pmatrix} \overline{\mathbf{w}} \\ \mathbf{w}^Z \end{pmatrix}$ and $\mathbf{x} = \begin{pmatrix} \overline{\mathbf{x}} & 0 \\ 0 & \mathbf{x}^Z \end{pmatrix}$ (where $x = \left\{1, \ln M, (\ln M)^2\right\}$) and estimating

$$\mathbf{w} = \mathbf{x}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

where $\boldsymbol{\beta} = \begin{pmatrix} \overline{\boldsymbol{\beta}} \\ \boldsymbol{\beta}^Z \end{pmatrix}$ ($\overline{\boldsymbol{\beta}} = \{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$, $\boldsymbol{\beta}^Z = \left\{\mathbf{a}^Z, \mathbf{b}^Z, \mathbf{c}^Z\right\}$) and the relevant restrictions between $\overline{\boldsymbol{\beta}}$ and $\boldsymbol{\beta}^Z$ are imposed. We estimate the budget share equations as a system allowing for correlations in the error terms across equations and using the weighted sum of squared residuals as our criterion function (i.e. weighted by a consistent estimator of the cross-equation residual covariance matrix). The results of the tests are shown in table 1 where BD denotes the base dependent scale model and BI the base independent scale model.

Table 1: Chi-squared tests for base dependent (BD) and independent (BI) models.

	BD			BI			
	χ^2_5	p value	estimate of Φ_1	χ^2_{11}	p value	$\ln s$	scale estimate
Year							
1998-99	2.424	0.788	0.857 (0.209)	13.912	0.237	−0.013 (0.074)	0.986
1992-93	3.068	0.689	0.841 (0.177)	17.952	0.082	0.035 (0.073)	1.035
1980-81	4.218	0.519	0.849 (0.196)	15.807	0.148	0.012 (0.071)	1.012

While base independence is not strongly rejected by the data under the quadratic model, $\ln s$ is not very well identified and the point estimates of the scale parameter are rather implausible. Of course, these estimates will be biased if the quadratic model is not the correct model. As mentioned in the introduction, and illustrated in figure 1, comparisons of the quadratic regressions with semi-parametric regressions where little prior structure is placed on the shape of the Engel curves indicate that the quadratic specification may be overly restrictive. This is important here since we have seen that the quadratic model cannot identify all the parameters of the scale, and we now turn to this subject in the next section.

4 Identification and interpretation of scaling parameters

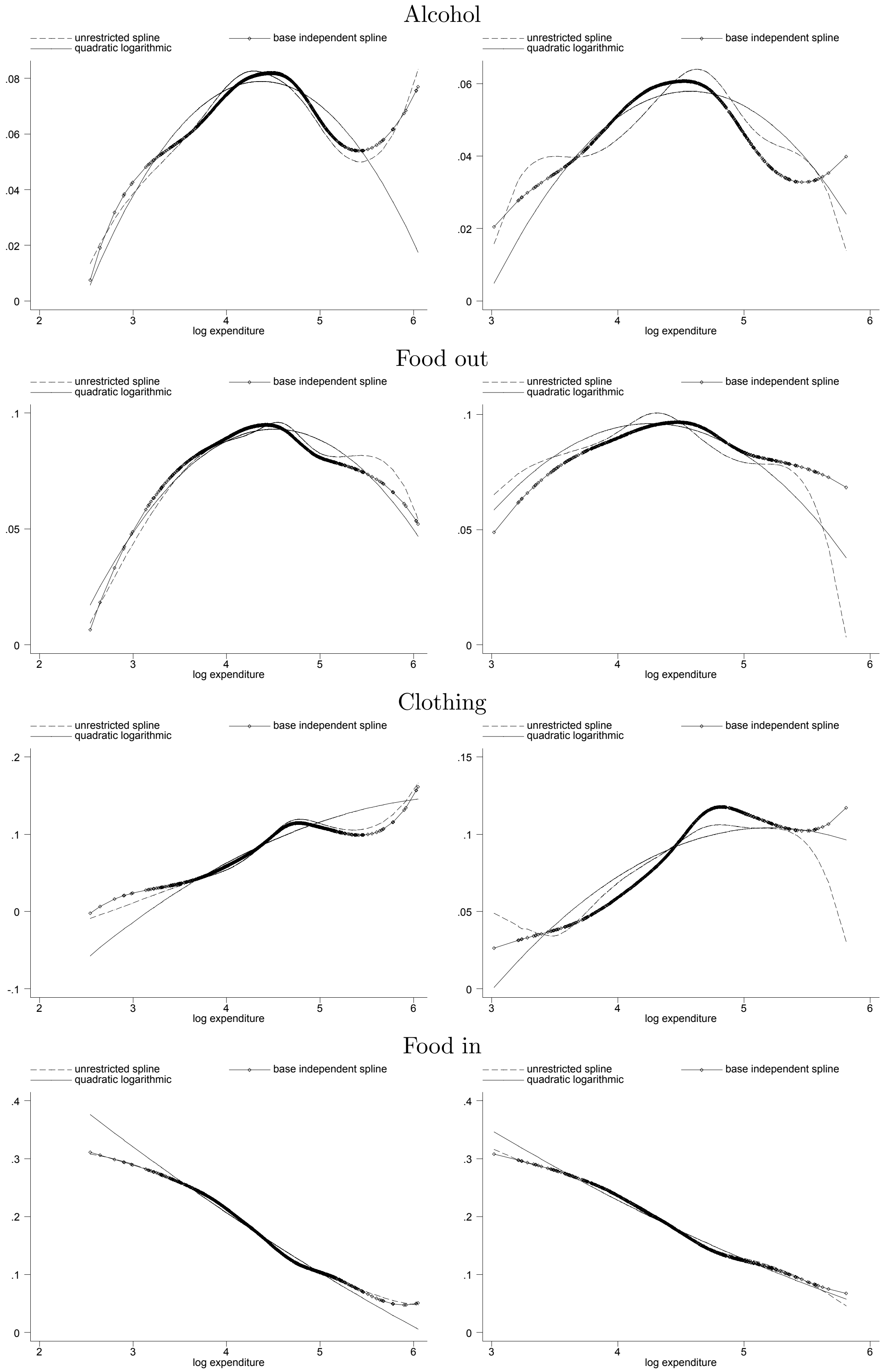
4.1 Empirical identification

If we move from the quadratic specification to Engel curves that include further functions of log expenditure then we may get identification of all the parameters of the base dependent scaling function. For example, budget shares that are cubic[3] in log total expenditure give full identification, since:

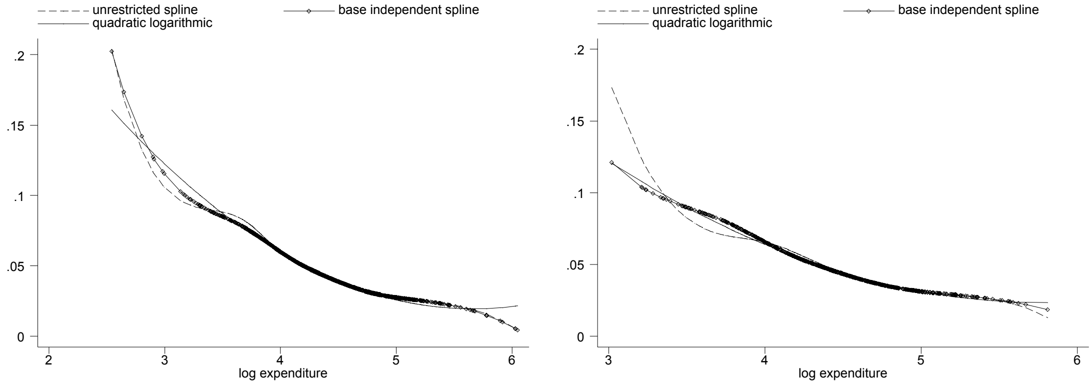
$$\overline{w}_i = a_i + b_i \ln M + c_i (\ln M)^2 + d_i (\ln M)^3$$

[3]Note that we do not require the Engel curves to be restricted to have a maximum rank of three since we do not require them to be exactly aggregable (see Gorman (1981) and Lewbel (1991)).

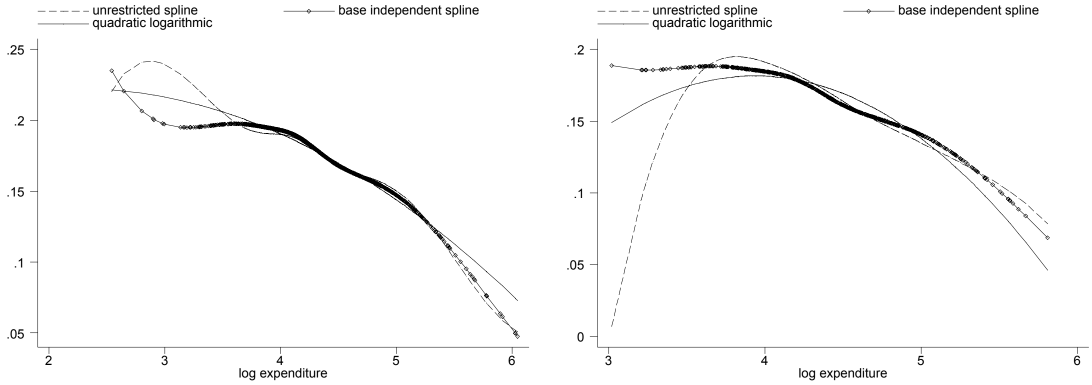
Figure 1: Quadratic regression, and unrestricted and base independent spline regressions.



Fuel



Travel



and

$$
\begin{aligned}
w_i^Z &= a_i^Z + b_i^Z \ln M + c_i^Z (\ln M)^2 + d_i^Z (\ln M)^3 \\
&= \Phi_1 \left[a_i + b_i \left(\frac{\ln M - \ln \Phi_0}{\Phi_1} \right) + c_i \left(\frac{\ln M - \ln \Phi_0}{\Phi_1} \right)^2 + d_i \left(\frac{\ln M - \ln \Phi_0}{\Phi_1} \right)^3 \right] \\
&\quad + \kappa_{1i} \left(\frac{\ln M - \ln \Phi_0}{\Phi_1} \right) + \kappa_{0i} \\
&= \Phi_1 a_i - \ln \Phi_0 b_i + \frac{c_i (\ln \Phi_0)^2}{\Phi_1} - \frac{d_i (\ln \Phi_0)^3}{\Phi_1^2} - \frac{\kappa_{1i} \ln \Phi_0}{\Phi_1} + \kappa_{0i} \\
&\quad + \left(b_i - 2 \frac{c_i \ln \Phi_0}{\Phi_1} + 3 \frac{d_i (\ln \Phi_0)^2}{\Phi_1^2} + \frac{\kappa_{1i}}{\Phi_1} \right) \ln M \\
&\quad + \left(\frac{c_i}{\Phi_1} - 3 \frac{d_i \ln \Phi_0}{\Phi_1^2} \right) (\ln M)^2 \\
&\quad + \frac{d_i}{\Phi_1^2} (\ln M)^3
\end{aligned}
$$

so

$$
\begin{aligned}
d_i^Z &= \frac{d_i}{\Phi_1^2} \\
c_i^Z &= \frac{c_i}{\Phi_1} - 3\frac{d_i \ln \Phi_0}{\Phi_1^2} \\
b_i^Z &= b_i - 2\frac{c_i \ln \Phi_0}{\Phi_1} + 3\frac{d_i (\ln \Phi_0)^2}{\Phi_1^2} + \frac{\kappa_{1i}}{\Phi_1} \\
a_i^Z &= \Phi_1 a_i - \ln \Phi_0 b_i + \frac{c_i (\ln \Phi_0)^2}{\Phi_1} - \frac{d_i (\ln \Phi_0)^3}{\Phi_1^2} - \frac{\kappa_{1i} \ln \Phi_0}{\Phi_1} + \kappa_{0i}
\end{aligned}
$$

and all the parameters can be identified (for example, even with one good d_i^Z allows Φ_1 to be identified, then c_i^Z gives identification of $\ln \Phi_0$, then b_i^Z gives κ_{1i} and a_i^Z gives κ_{0i}.

Another way of identifying all the parameters would be to further parameterise the quadratic regression[4]. For example, it is common in the base independent model to estimate a QUAIDS specification over varying prices with $\ln s$ linear in $\ln \mathbf{p}$ so that η_i $(= \partial \ln s / \partial \ln p_i)$ is constant across price regimes. However, our preliminary quadratic analysis did not support the hypothesis of a constant η_i over price regimes, and this, coupled with the implication from the semi-parametric regressions that the quadratic specification may be overly restrictive, means that we do not pursue that route here but move to non-quadratic Engel curves.

4.2 Implications for the Engel curves for children's goods

Further help in identification may be obtained from the fact that, as well as the general restrictions on the shape of the Engel curves between demographic groups, these scaling models have strong implications for the shape of the comparison group Engel curves for goods that the reference group does not buy. In the two groups we are looking at, for instance, the reference household is without children, and so the two models have implications for the shape of the Engel curve for children's goods for households with children. Since $\overline{w}_c = 0$ (where the c subscript denotes children's goods), then under base independence

$$w_c (\ln M, p, z) = \eta_c$$

i.e. spending on children's goods is a constant share of total expenditure, and under the base dependent model

$$w_c (\ln M, p, z) = \kappa_{1c} \left[\frac{\ln M - \ln \Phi_0}{\Phi_1} \right] + \kappa_{0c}$$

[4]This approach is taken in Donaldson and Pendakur (1999), who look at relaxing base independence for both relative scales and absolute scales (i.e. the fixed cost type model as in the translating procedure of Pollak and Wales (1980) discussed above). Ray (1996) also tests, and rejects, a base independent equivalence scale in favour of one that varies with utility using a non-linear preference cost function proposed in Blundell and Ray (1984).

i.e. the share of spending on children's goods is linear in log total expenditure. The constant budget share for children's goods implied by the base independent model is clearly a very strong restriction and one we would expect to be quite unrealistic empirically. This turns out to be the case in our data, where the hypothesis of constant shares for children's clothing is rejected.

This is clearly an extra restriction, but it does not, in the base dependent model, identify $\ln \Phi_0$ on its own since we have

$$\begin{aligned} a_c &= \kappa_{0c} - \frac{\kappa_{1c} \ln \Phi_0}{\Phi_1} \\ b_c &= \frac{\kappa_{1c}}{\Phi_1} \ln M \end{aligned}$$

i.e. two equations and three unknowns (given that Φ_1 can be identified even in the quadratic model).

4.3 Semi-parametric estimation

We want to estimate the following general form for the budget shares

$$w_i = f_i (\ln M) + \epsilon_i$$

There are a variety of smoothing techniques available for semi-parametric estimation – we choose to estimate the share equations via cubic spline smoothing using a power-basis for the splines, i.e.

$$f_i (\ln M; \boldsymbol{\beta}) = \beta_0 + \beta_1 \ln M + \beta_2 (\ln M)^2 + \beta_3 (\ln M)^3 + \sum_{k=1}^{K} \beta_{3+k} (\ln M - t_k)_+^3$$

where $(y)_+ = yI(y > 0)$ and $t_1 < ... < t_K$ are the knot points. The parameters $\{\beta_k\}_{k=0}^{3+K}$ are estimated using penalised least squares (with the roughness penalty on $\{\beta_{3+k}\}_{k=1}^{K}$) – so that $\widehat{\boldsymbol{\beta}}(\lambda)$ is the minimiser of

$$\sum_{j=1}^{N} (y_j - f_i (x_j; \boldsymbol{\beta}))^2 + \lambda \sum_{k=1}^{K} R(\beta_{3+k})$$

where $R(u)$ is the roughness penalty function and λ controls the trade-off between smoothness and fit (as λ becomes large, the regression approaches a cubic specification). We use a simple quadratic penalty function, $R(u) = u^2$ as in Ruppert and Carrol (1997), so that

$$\widehat{\boldsymbol{\beta}}(\lambda) = (\mathbf{X}'\mathbf{X} + \lambda \mathbf{S})^{-1} \mathbf{X}'\mathbf{Y}$$

where $\mathbf{S}$ is a $(K+4) \times (K+4)$ diagonal matrix with the first 4 diagonal elements being 0 and the remaining K diagonal elements being 1. The smoothing weight, λ, is chosen by visual inspection – there is a point at which the curve goes fairly quickly from being quite rough to being reasonably smooth. We experimented with an automated procedure for choosing λ by using the cross-validation method (refs), but the resulting Engel curves looked quite under-smoothed. As a check, an adaptive local linear regression was run alongside the spline model to verify that the two looked reasonably similar at the chosen smoothing parameter.

For the Engel curves shown in figure 1, a J-test (Davidson and MacKinnon (1981)) showed that the estimates from the spline regression had explanatory power when added to the quadratic regression.

Under semi-parametric estimation, the parameters of the scaling function are chosen following a generalised least squares regression approach suggested in Ai and Chen (2000). For example, in the base dependent model the following approach is adopted. Starting from the shape restrictions, we find

$$\begin{aligned} w_i^Z(\ln M) &= \Phi_1 \overline{w}_i \left(\frac{\ln M - \ln \Phi_0}{\Phi_1} \right) + \kappa_{1i} \left[\frac{\ln M - \ln \Phi_0}{\Phi_1} \right] + \kappa_{0i} + \varepsilon_i^Z \\ &\Longrightarrow \\ \frac{w_i^Z(\ln M)}{\Phi_1} &\equiv \widetilde{w}_i^Z = \overline{w}_i \left(\ln \widetilde{M} \right) + \frac{\kappa_{1i}}{\Phi_1} \ln \widetilde{M} + \frac{\kappa_{0i}}{\Phi_1} + \frac{\varepsilon_i^Z}{\Phi_1} \end{aligned}$$

where $\ln \widetilde{M} = (\ln M - \ln \Phi_0)/\Phi_1$. So transforming w_i^Z into $\widetilde{w}_i^Z$ and $\ln M^Z$ into $\ln \widetilde{M}$ given the values chosen for $\ln \Phi_0$ and Φ_1, the data can be pooled and the model estimated using the cubic spline method. Then $\ln \Phi_0$ and Φ_1 are chosen (using a grid search) to minimise the weighted sum of squared residuals. Note that both $\ln \widetilde{M}$ and a constant interacted with a dummy for the Z type household must also be added, and the coefficients on these will give κ_{1i}/Φ_1 and κ_{0i}/Φ_1 respectively – hence, we only need to search over $\ln \Phi_0$ and Φ_1, since, given these parameters, κ_{1i} and κ_{0i} are determined by the model. In the data transformation, the error term for the Z-type households is divided by Φ_1, and so, as in generalised-least-squares estimation, when pooling the transformed data for estimation, the observations for the Z-type households must be weighted by Φ_1.

4.4 Theoretical identification

We might not actually care about knowing all the parameters of the scale, but one circumstance where we will care is if we want to give the scale an "equivalence scale" interpretation, where, when we write $c(u, p, z) = \overline{c}(u, p)\, s(u, p, z)$, we describe $s(u, p, z)$ as the amount by which a reference-type household's budget needs to be multiplied for a household with characteristics z to enjoy the same level of welfare that the reference household achieves. Whereas the inclusion of demographics in demand systems to account for the fact that spending patterns vary across households is fairly innocuous (provided that the way they are included accords with the restrictions of consumer theory), this is not the case when the demographic parameters estimated from the demand analysis are given the equivalence scale interpretation. This is because of the well documented problem of identifying equivalence scales from demand patterns alone (Pollak and Wales (1979)). Specifically, the demands generated from the utility function $U(q,z)$ are indistinguishable from those generated by $\Psi(U(q,z),z)$ (where Ψ is monotonically increasing in U). If the cost function for $U(q,z)$ is $c(u,p,z)$ and that for $\Psi(U(q,z),z)$ is $\widetilde{c}(u,p,z)$ then we have that $c(u,p,z) = \widetilde{c}(\Psi(u,z),p,z)$. In the first case the equivalence scale $s(u,p,z)$ is equal to

$$\frac{c(u,p,z)}{\overline{c}(u,p)}$$

but

$$\frac{c(u,p,z)}{c(u,p)} = \frac{\widetilde{c}(\Psi(u,z),p,z)}{\widetilde{c}(\Psi(u),p)}$$

and in general it will not be the case that

$$\frac{\widetilde{c}(\Phi(u,z),p,z)}{\widetilde{c}(\Phi(u),p)} = \frac{\widetilde{c}(u^*,p,z)}{\widetilde{c}(u^*,p)} \equiv \widetilde{s}(u^*,p,z)$$

where $u^* = \Phi(u,z)$ and $\widetilde{s}(u,p,z)$ is the equivalence scale associated with $\widetilde{c}(u,p,z)$.

The appeal of the base-independence restriction on the equivalence scale is that, in general, the restriction allows identification of the equivalence scale from demand behaviour alone (Lewbel (1989), Blackorby and Donaldson (1991)). Suppose there were two base independent equivalence scales associated with a given demand behaviour so that

$$\ln c(u,p,z) = \ln \overline{c}(u,p) + \ln s(p,z)$$

and

$$\begin{aligned} \ln \widetilde{c}(u,p,z) &= \ln c(\Phi(u,z),p,z) = \ln \overline{c}(\Phi(u,z),p) + \ln s(p,z) \\ &= \ln \overline{c}(u,p) + \ln \widetilde{s}(p,z) \end{aligned}$$

where $\ln \widetilde{s}(p,z) = \ln s(p,z) + \ln g(p,z)$ say. This gives

$$w_i(\ln M,p,z) = \overline{w}_i\left(\ln\left[\frac{M}{s(z,p)}\right],p\right) + \frac{\partial \ln s(z,p)}{\partial \ln p_i}$$

and

$$\widetilde{w}_i(\ln M,p,z) = \overline{w}_i\left(\ln\left[\frac{M}{s(z,p)}\right] - \ln g(p,z),p\right) + \frac{\partial \ln s(z,p)}{\partial \ln p_i} + \frac{\partial \ln g(z,p)}{\partial \ln p_i}$$

But since, by assumption, $w_i(\ln M,p,z) = \widetilde{w}_i(\ln M,p,z)$, then

$$\overline{w}_i\left(\ln\left[\frac{M}{s(z,p)}\right] - \ln g(p,z),p\right) + \frac{\partial \ln g(z,p)}{\partial \ln p_i} = \overline{w}_i\left(\ln\left[\frac{M}{s(z,p)}\right],p\right) \tag{4}$$

This is of the form $\rho(r+t) = \rho(r) + \lambda(t)$ which implies that $\rho(r+t) - \rho(r)$ depends only on t, and if ρ is continuous, the solution can only be of the form $\rho(r) = a + br$ for some constants a and b (Aczel (1966)). Hence $\overline{w}_i$ must be linear in log expenditure, which implies that $\ln \overline{c}(u,p)$ must be of the form $\ln \alpha(p) + \beta(p) h(u)$ giving $\overline{w}_i = a_i + b_i [\ln M - \ln \alpha]$ where $a_i = \partial \ln \alpha(p) / \partial \ln p_i$ and $b_i = \partial \ln \beta(p) / \partial \ln p_i$. Thus equation 4 gives

$$\begin{aligned} a_i + b_i \left[\ln \frac{M}{s} - \ln \alpha - \ln g\right] + \frac{\partial \ln g}{\partial \ln p_i} &= a_i + b_i \left[\ln \frac{M}{s} - \ln \alpha\right] \\ &\Rightarrow \frac{\partial \ln g}{\partial \ln p_i} = b_i \ln g \\ &\Rightarrow \frac{\partial \ln g}{\partial \ln p_i} = \frac{\partial \ln \beta}{\partial \ln p_i} \ln g \\ &\Rightarrow \ln g(p,z) = \beta(p)\sigma(z) \end{aligned}$$

for some $\sigma(z)$. So, as long as budget shares are not affine functions of log expenditure, then a unique base independent equivalence scale is associated with a given demand behaviour.

Not surprisingly, it turns out that the extended scaling function also, generally, allows unique identification of the equivalence scale from demand behaviour alone. Again suppose there were two scales associated with the same behaviour, say $\ln \Phi_0, \Phi_1$ and $\ln \widetilde{\Phi}_0 = \ln f_0(p,z) + \ln \Phi_0, \widetilde{\Phi}_1 = f_1(p,z)\Phi_1$, then (suppressing $\mathbf{p}$ and z for brevity)

$$w_i(\ln M, p, z) = \Phi_1 \overline{w}_i \left(\frac{\ln(M/\Phi_0)}{\Phi_1}\right) + \frac{\partial \ln \Phi_1}{\partial \ln p_i} \ln\left(\frac{M}{\Phi_0}\right) + \frac{\partial \ln \Phi_0}{\partial \ln p_i}$$

and

$$w_i(\ln M, p, z) = f_1 \Phi_1 \overline{w}_i \left(\frac{\ln(M/\Phi_0) - \ln f_0}{f_1 \Phi_1}\right) + \left[\frac{\partial \ln \Phi_1}{\partial \ln p_i} + \frac{\partial \ln f_1}{\partial \ln p_i}\right] \ln\left(\frac{M}{\Phi_0} - \ln f_0\right) + \frac{\partial \ln \Phi_0}{\partial \ln p_i} + \frac{\partial \ln f_0}{\partial \ln p_i}$$

which implies

$$\Phi_1 \overline{w}_i \left(\frac{\ln(M/\Phi_0)}{\Phi_1}\right) = f_1 \Phi_1 \overline{w}_i \left(\frac{\ln(M/\Phi_0) - \ln f_0}{f_1 \Phi_1}\right) + \frac{\partial f_1}{f_1 \partial \ln p_i} \ln\left(\frac{M}{\Phi_0} - \ln f_0\right) - \frac{\partial \ln \Phi_1}{\partial \ln p_i} \ln f_0 + \frac{\partial \ln f_0}{\partial \ln p_i} \tag{5}$$

.

Since the left hand side of equation 5 obviously can involve no interactions between f_1 and $\ln(M/\Phi_0)$ then we must have $\partial f_1 / \partial \ln p_i = 0$ so $f_1(p,z) = f_1(z)$. Then equation 5 becomes

$$\Phi_1 \overline{w}_i \left(\frac{\ln(M/\Phi_0)}{\Phi_1}\right) = f_1 \Phi_1 \overline{w}_i \left(\frac{\ln(M/\Phi_0) - \ln f_0}{f_1 \Phi_1}\right) + \frac{\partial \ln f_0}{\partial \ln p_i}$$

which again implies that $\overline{w}_i$ must be linear in log expenditure, giving

$$\begin{aligned}
\Phi_1 a_i + b_i \left[\ln \frac{M}{\Phi_0} - \Phi_1 \ln \alpha\right] &= f_1 \Phi_1 a_i + b_i \left[\ln \frac{M}{\Phi_0} - \ln f_0 - f_1 \Phi_1 \ln \alpha\right] - \frac{\partial \ln \Phi_1}{\partial \ln p_i} \ln f_0 + \frac{\partial \ln f_0}{\partial \ln p_i} \\
&\Rightarrow \\
\frac{\partial \ln f_0}{\partial \ln p_i} &= \frac{\partial \ln \Phi_1}{\partial \ln p_i} \ln f_0 + \frac{\partial \ln \beta}{\partial \ln p_i} \left[\ln f_0 - \Phi_1 \ln \alpha \left(1 - f_1\right)\right] + \Phi_1 \frac{\partial \ln \alpha}{\partial \ln p_i} \left(1 - f_1\right) \\
&\Rightarrow \\
\ln f_0 &= \Phi_1 \left[\beta \tau \left(z\right) + \ln \alpha \left(1 - f_1\right)\right]
\end{aligned}$$

for some $\tau(z)$.

5 Results

We split clothing into its adult and child components and use the restrictions implied for children's goods by the two scaling models. As a check that the grid search procedures were working we ran them for a cubic specification and checked that the criterion function was minimised at the parameters estimated from running restricted generalised least squares on the cubic model. As before, we estimate all the goods as a system of equations and minimise the weighted sum of squared residuals. We choose the initial values to search around from the results of a cubic regression. The parameters from the grid search were always very similar to those from the simple cubic regression, and so we present these results in tables 2 and 3 below. The semi-parametric Engel curves lie well within the confidence bands of the cubic regressions as is shown for alcohol and food out for couples with children in figure 2 (for the unrestricted estimates).

Figure 2: Some semi-parametric and cubic regressions with confidence bands.

Alcohol | Food out

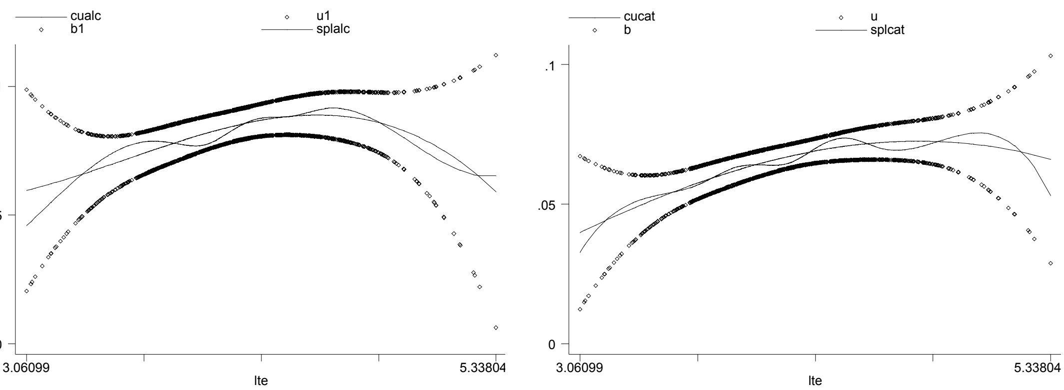


Table 2: Parameter estimates for base dependent model – cubic regression with clothing restrictions.

	$\ln \Phi_0$	Φ_1	scale at: mean	med	min	Q1	Q3	max
Year								
1998-99	0.529 (0.662)	0.909 (0.154)	1.131	1.144	1.335	1.176	1.113	0.982
1992-93	0.339 (0.690)	0.969 (0.163)	1.228	1.234	1.291	1.244	1.221	1.157
1985-86	0.402 (0.719)	0.947 (0.194)	1.249	1.257	1.327	1.275	1.239	1.144
Note: standard errors in parentheses								

Table 3: Parameter estimates for base independent model – cubic regression with clothing restrictions.

	$\ln s$	scale
Year		
1998-99	−0.021 (0.066)	0.979
1992-93	0.091 (0.066)	1.094
1985-86	0.118 (0.075)	1.125
Note: standard errors in parentheses		

The point estimates of the base independent scale are rather low, and, as with the quadratic regressions, are imprecisely determined. Remember that the scale is the amount of expenditure a couple with a child needs compared to a childless couple – so a value of 1.05, for example, means that the child 'costs' only 5% of two adults. The scale implied by the base dependent model seems more reasonable – although the scale is less that one at maximum expenditure for 1998-99, which does not really make much sense. This could be due to the fact that $\ln \Phi_0$ is not very precisely determined. As with the quadratic regressions, though, Φ_1 *is* determined quite precisely, and Φ_1 is, perhaps, really the more interesting parameter here anyway if we are interested in how the scale varies across the expenditure distribution. In all years Φ_1 is less than one, which means the scale decreases as total expenditure increases. This makes sense – one quite implausible feature of a base independent scale is that the absolute cost of a child can rise a great deal as expenditure rises. For example, if a couple with £20,000 requires an extra £4,000 (so the scale is 1.2), then a couple with £ 200,000 requires £ 40,000. Although, of course, all family members are required to be better off in the second, higher expenditure household, it seems implausible, particularly for very young children whose needs are mainly food and clothing, that expenditure would need to increase this much. Again, the value of the scale we obtain at the minimum expenditure level is quite interesting – it is quite close to 1.3, which is almost exactly the value of the

OECD scale comparing a couple with one child to a childless couple[5].

For comparison, tables 4 and 5 show the results of the analysis when clothing is not split into its adult and child related components. The point estimates of Φ_1 are, again, below one – a little higher than the previous estimates for 1992-93 and 1985-86 and a little lower for 1998-99. Again $\ln \Phi_0$ is not very precisely determined, in fact even less so than before, and the overall scales are lower than the previous estimates – indeed for 1992-93, they are always below one. The point estimates for the base independent scales are also slightly lower than before.

Table 4: Parameter estimates for base dependent model – cubic regression with aggregate clothing .

			scale at:					
	$\ln \Phi_0$	Φ_1	mean	med	min	Q1	Q3	max
Year								
1998-99	0.541 (0.736)	0.889 (0.154)	1.045	1.061	1.309	1.099	1.025	0.876
1992-93	−0.108 (0.755)	0.982 (0.147)	0.830	0.832	0.854	0.836	0.827	0.801
1985-86	0.344 (0.832)	0.951 (0.191)	1.174	1.181	1.241	1.196	1.165	1.082
Note: standard errors in parentheses								

Table 5: Parameter estimates for base independent model – cubic regression with aggregate clothing .

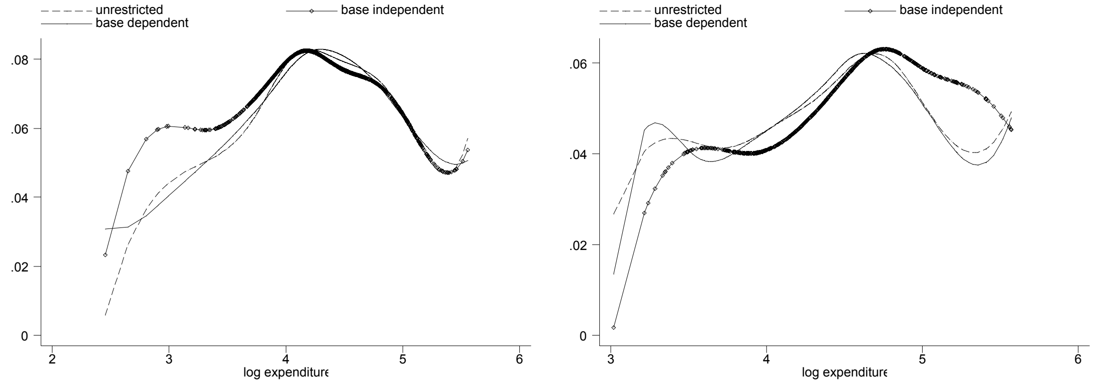
	$\ln s$	scale
Year		
1998-99	−0.025 (0.075)	0.974
1992-93	0.053 (0.066)	1.054
1985-86	0.104 (0.076)	1.110
Note: standard errors in parentheses		

The nonparametric regression curves for the unrestricted, base dependent and base independent models (both at the parameters resulting from the grid search) are shown below in figure 3.

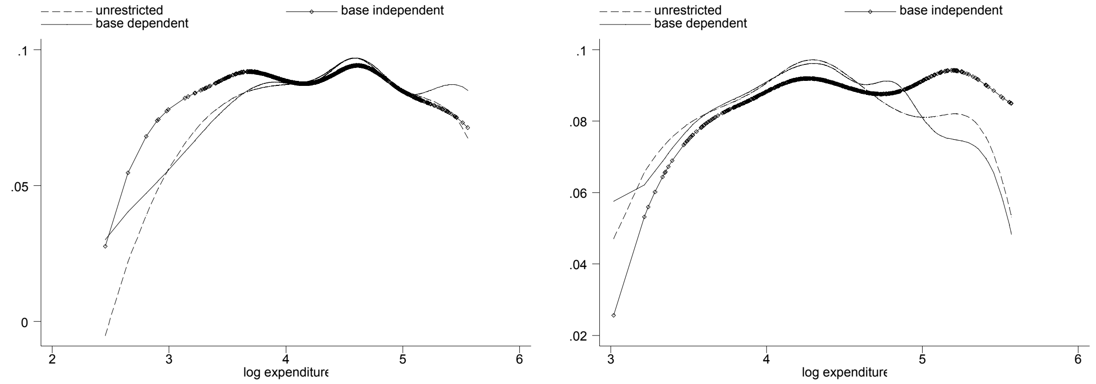
[5] The OECD scale normalised at one for a single adult gives a value of 0.7 to each additional adult and 0.5 to each child. So the scale comparing a couple with one child to a childless couple is $2.2/1.7 \simeq 1.29$.

Figure 3: Spline regression - unrestricted, base dependent and base independent models.

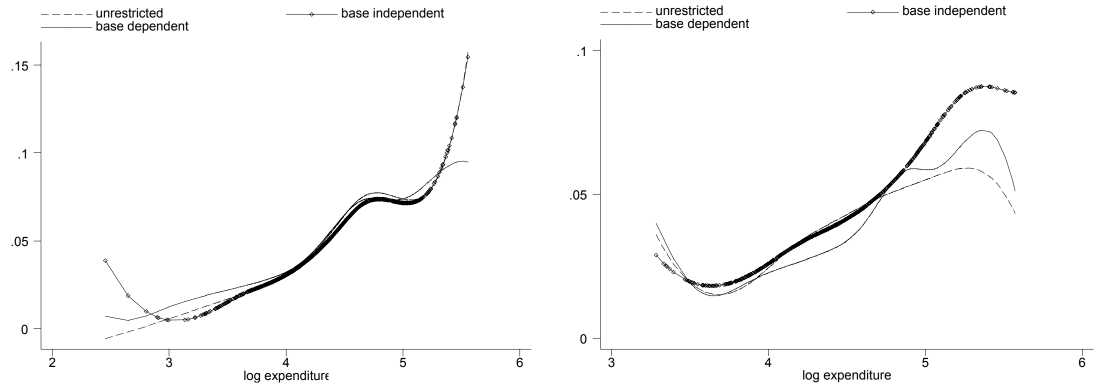
Alcohol



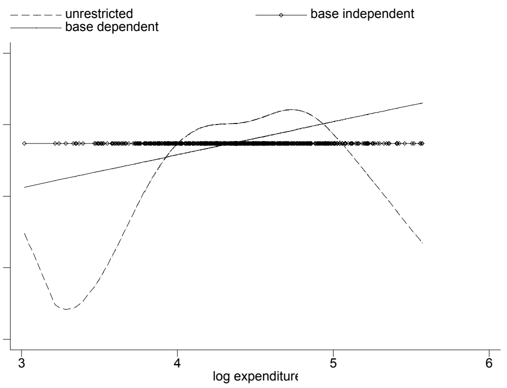
Food out

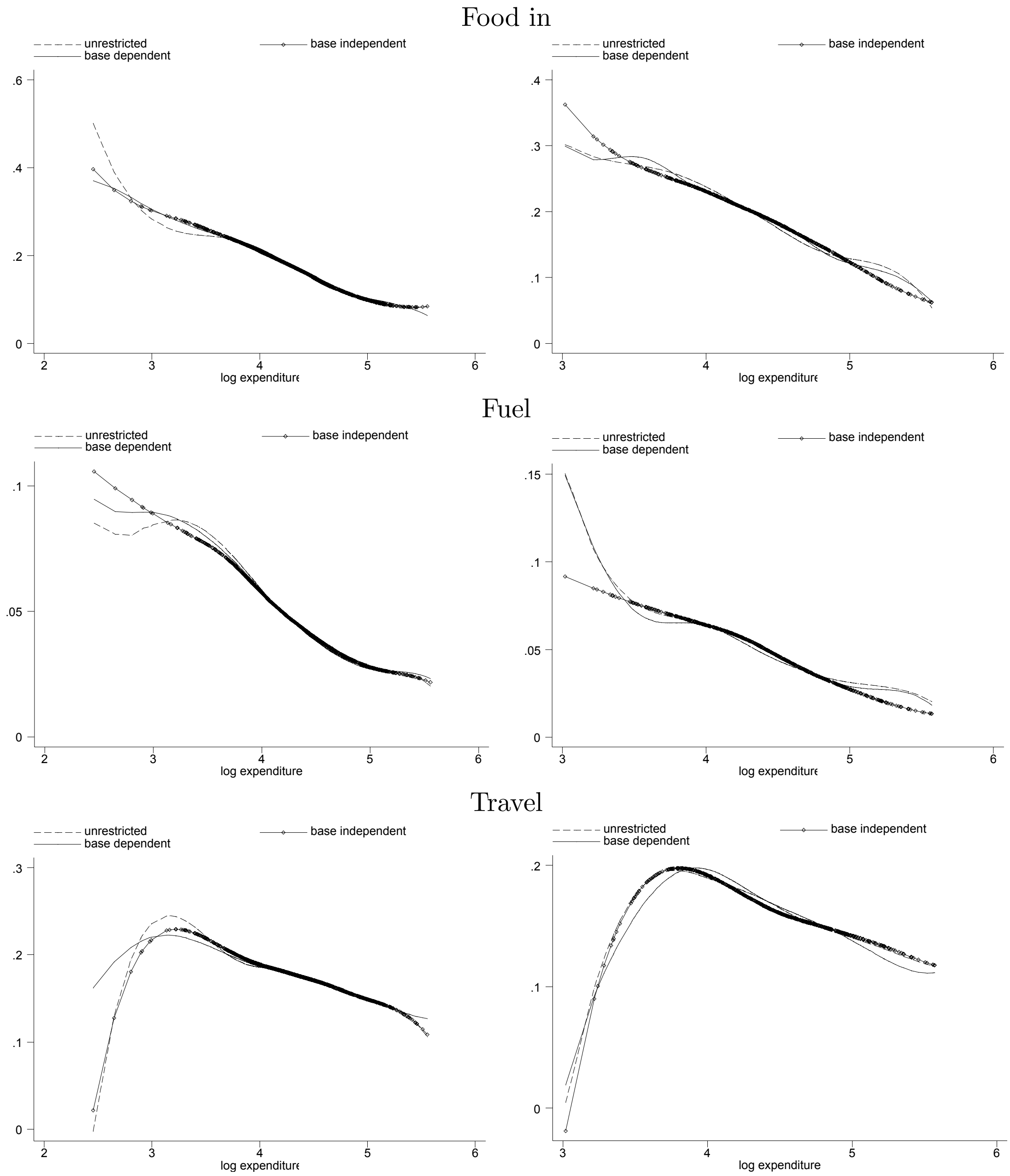


Adult Clothing



Children's Clothing





6 Conclusions

Specifying demographic effects correctly in demand analysis is important both in order to estimate correct price and expenditure elasticities and for the purpose of making household welfare comparisons. A common way of including demographics is as a function that scales total expenditure, and to make this scaling function independent of the level of total expenditure. A popular method in the parametric estimation of demand systems is to estimate share equations that are quadratic in the logarithm of total expenditure, but there is also a substantial literature on the semi-parametric estimation of Engel curves. We have used some of these semi-parametric techniques to show that, for some goods, it is likely that further terms are required in addition to quadratic terms in the Engel curves. We have used this to identify the parameters of a scaling function that varies with total expenditure. Although the 'intercept' of this scale is not very precisely determined, the term that shows how the scale varies with total expenditure *is* well determined and is less than one, which means the scale decreases as total expenditure increases. The base independent scale estimated on the same data is generally hard to identify precisely and the point estimate is often implausibly low.

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