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Is the Elasticity of Intertemporal Substitution Constant?

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Motivation

- Inter-temporal consumption models are the workhorses of modern macroeconomics and public finance.

$$\max E\left[\sum_{t=1}^{T}\beta^{t}u(c_{t})\right]$$

- Usually assume the power/isoelastic/CRRA form for the \`felicity' function:

$$u(c_{t})=\frac{c^{1-(1/\theta)}}{1-(1/\theta)}$$

- The power parameter θ determines responses to interest rates:

$$\theta = \frac{u'(c_t)}{u''(c_t)c_t} = EIS = \frac{\partial \ln c_t}{\partial \ln r}$$

- The elasticity of inter-temporal substitution (EIS) is the inverse of fluctuation aversion.

- With expected utility, it is the inverse of the coefficient of relative risk aversion.

- The power form is convenient for the study of dynamic models (esp. steady states): additivity + power = homothetic preferences over periods (states).

- This linearity allows consumption and marginal utility to both grow at constant rates (see eg., King, Plosser and Rebelo, 2002).

- However, it is not innocuous.

- For example, standard macro arguments against capital taxation rest on the homotheticity of preferences over consumption at different dates (Chari and Kehoe, 2006)

- More generally, assuming that fluctuation aversion is independent of level of wealth may have important consequences for DSGE welfare calculations, costs of business cycles, etc.

- The constant EIS assumption is hard to test, but evidence is against
  - Blundell, Browning and Meghir (1994), Atkeson and Ogaki (1996), Attanasio and Browning (1996)
  - Also macro arguments: eg. Guvenen, 2006)

- This paper poses two questions:

  1. Is the EIS constant? (*answer: no*)

  2. Is there an easier way to test the constant EIS assumption? (*answer: yes, moreover, there is a much easier way to estimate the EIS if it is constant…..)*

- Perhaps surprisingly, the answers come from thinking about *intra*-temporal allocation.

Outline of the Rest of the Talk:

1. Background
   a. Estimating the EIS
   b. What is $u(c_t)$?
   c. The EIS in a multi-good world
2. Remark
3. Proposition
   a. Proof
   b. Intuition
   c. Implications
4. Empirical Illustration
5. Extensions
6. Conclusion

Background (a): Estimating the EIS

The exact Euler Equation is estimable by nonlinear IV (GMM):

$$\left(\frac{c_{t+1}}{c_t}\right)^{-1/\theta} \beta(1+r_{t+1}) = \varepsilon_{t+1}$$

$$E\left[\varepsilon_{t+1}\right]=1, \quad E\left[\left(\left(\frac{c_{t+1}}{c_t}\right)^{-1/\theta} \beta(1+r_{t+1})-1\right)z_t\right]=0$$

However, nonlinear IV is not consistent if c_t measured with error.

- A linear (in logs) approximation gives an equation that can be estimated by linear IV:

$$\Delta \ln c_{t+1} = \alpha + \theta r_{t+1} + u_{t+1}$$

- However, the error term, u_{t+1}, now contains approximation error – including higher order terms in consumption growth and interest rates.

- These may be correlated with lagged consumption, income, interest rates: theory no longer provides instruments.

- The essence of the problem is that we have short panels, noisy data and little variation in the inter-temporal price (the interest rate.)

- There is a large literature about this problem: Ludvigson and Paxon (2001), Chris Carroll (2001), Attanasio and Low (2004), Alan and Browning (2003).

- An alternative is structural estimation as in Gourinchas and Parker (2002). This requires fully specifying the income process (in which case we could find instruments: there are no free lunches.)

- All of these papers assume the isoelastic (constant EIS) form.

- Testing the isoelastic form is even harder: we need not only to estimate the EIS but in addition how it varies with the level of wealth (or consumption).

Background (b): What is *u(c_t)*?

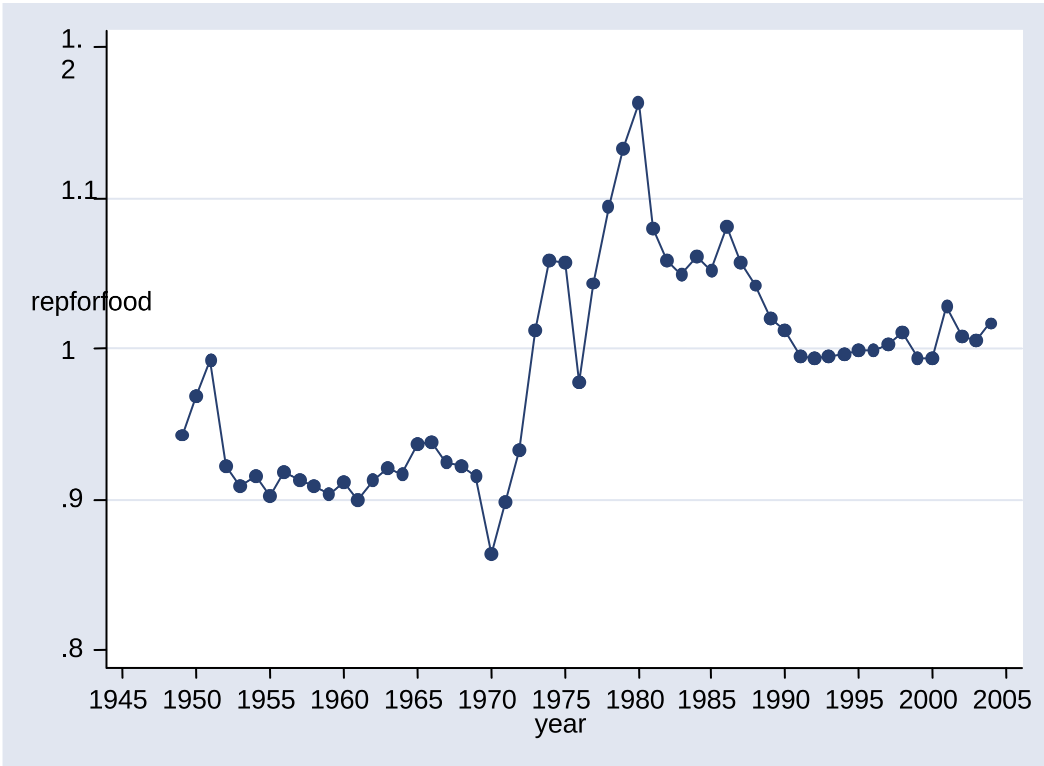
1. A direct utility function over a composite commodity
   a) Hick's composite commodity arguments– constant relative prices. ✗
   b) Homothetic preferences – no luxuries or necessities ✗

2. An indirect utility function over within period expenditure and relative prices (two stage budgeting):

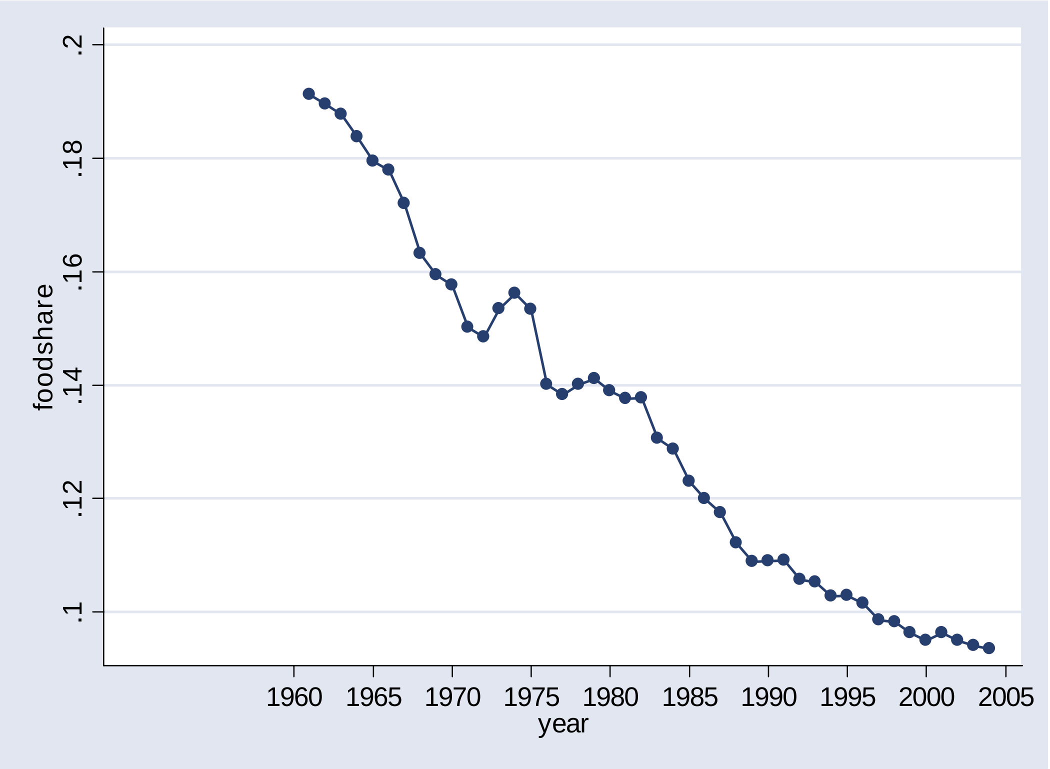
$$u(x_t; p_t)$$

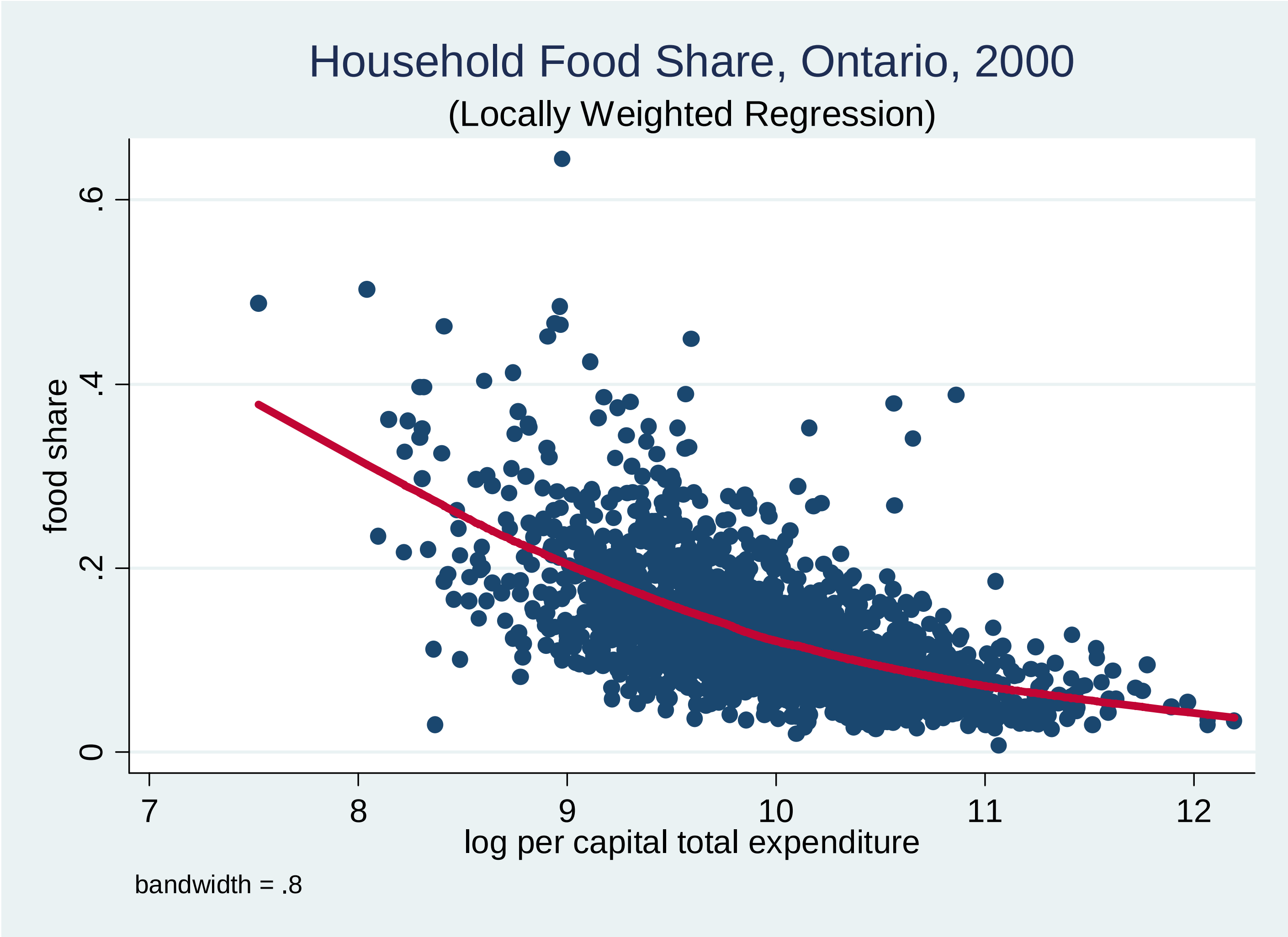
(Blundell, Browning and Meghir, 1994; Attanasio and Weber, 1995; others).

Relative Price of Food, Canada, Historical



Aggregate Food Share, Canada, Historical




Household Food Share, Ontario, 2000
(Locally Weighted Regression)
food share
0
.2
.4
.6
7
8
9
10
11
12
log per capital total expenditure
bandwidth = .8

Background (c): EIS with multiple goods

- An EIS can be defined for individual goods (Deaton, 1992; Browning and Crossley, 2000); The overall EIS is the share-weighted average of good specific EIS: $EIS = s_1 EIS_1 + s_2 EIS_2$

- Goods with high income elasticities will also have high EIS: "Luxuries are easier to postpone."

- The overall EIS is unlikely to be constant because:
  a. Individual goods have different EIS (luxuries, necessities)
  b. The share of luxuries and necessities will change with the level of consumption.

Remark

We can be more precise. *The indirect utility function defined over total expenditure within period has a power utility representation (constant EIS) if and only if within period preferences take one of the following two forms:*

$$u = a(p)\frac{x^{1-1/\theta}}{1-1/\theta} + b(p) \quad \theta \neq 1$$

$$u = a(p)\log x + b(p) \qquad \theta = 1$$

- These are PIGL/PIGLOG preferences (Muellbauer, 1975,1976).

- Sufficiency follows from repeated differentiation.

- To show necessity, integrate up from $-\frac{u_{xx}}{u_x} = \frac{k}{x}$, allowing constants of integration to depend on relative prices.

- Browning (1985), Muellbauer (1987) contain the result but do not develop the implications.

- PIGLOG/PIGL Preferences are at most rank 2.

- The rank of demands refers to the space spanned by the Engel curves (it is a measure of flexibility over income).
  - Homothetic preferences are rank 1.
  - An Almost Ideal Demand System (AIDS) is PIGL, and rank 2.
  - A Quadratic Almost Ideal Demand System is rank 3.

Proposition 1:

Suppose that:

1. *The felicity function has either canonical PIGL or canonical PIGLOG forms above (so the at the EIS is independent of prices and income).*
2. *Within period preferences are homothetic.*

There is no transformation $v = F(u(x;p))$ *such that:*

- i. *Within period demands are unchanged by* $F(\cdot)$
- ii. *The (constant) power parameter associated with v is different from the (constant) power parameters associated with u.*

Proof: by contradiction (see paper). Demands are unchanged by $F(\cdot)$ if $F_p = 0$. We show that any transformation satisfying *ii* depends on prices.

- Obvious question: *why can't we take monotonic transformations of the demand system:*

$$v = F(u(x; p))$$
$$\max E\left[\sum_{t=1}^{T} \beta^t v(x_t; p_t)\right]$$

and control the EIS with the transformation?

Answer: *you can, but you can't preserve the constant EIS*

Intuition: Suppose F is a power transformation:

$$v = \frac{1}{1-\gamma}\left(a(p)\frac{x^{1-1/\theta}}{1-\theta} + b(p) \right)^{1-\gamma}$$

Then the EIS associated with *v* is defined by:

$$\frac{1}{EIS} = \gamma\left[\frac{a(p)x^{1-1/\theta}}{a(p)x^{1-1/\theta} + b(p)} \right] + \frac{1}{\theta}$$

In general, the EIS depends on *x* and *p*.

There are two special cases:

1) If $b(p)=0$ (homotheticity) then $\frac{1}{EIS}=\gamma+\frac{1}{\theta}$

2) If within period preferences are PIGLOG, we can write:

$$v=\frac{1}{1-\gamma}\left(\exp\left[a(p)\ln x+b(p)\right]\right)^{1-\gamma}$$

And the EIS is given by: $\gamma a(p)-a(p)+1$. The EIS depends on relative prices but not the level of consumption. This is the functional form used by Attanasio and Weber (1995).

Outside of these special cases, nonlinear transformations lead to an EIS that depends on *x* and *p*.

Implicatoin: *If intertemporal preferences are additive, the EIS is constant, and within period preferences are not homothetic, then the intertemporal preference parameter is identified by the curvature of Engel curves.*

Using Roy's identity:

$$w_k = -\frac{a_k}{a(p)(1-1/\theta)} - \frac{b_k}{a(p)} x^{\frac{1}{\theta}-1} \quad \theta \neq 1$$

$$w_k = -\frac{a_k}{a(p)} \log x - \frac{b_k}{a(p)} \quad \theta = 1$$

- If preferences are not homothetic, and the EIS is independent of x and p, then there is a one-to-one relationship between the EIS and the curvature of Engel curves.

- Engel curves are much easier to estimate than Euler equations. We need only cross-sectional data, and we have lots of variation in x.

- We can use demand data to test the constant EIS assumption.

- The rank 2 condition imposes two kinds of restrictions:

1) The curvature of each Engel curve must be adequately captured by two terms.

$$w_k = -\frac{a_k}{a(p)(1-1/\theta)} - \frac{b_k}{a(p)} x^{\frac{1}{\theta}-1} \quad \theta \neq 1$$

2) θ must be the same for each Engel curve.

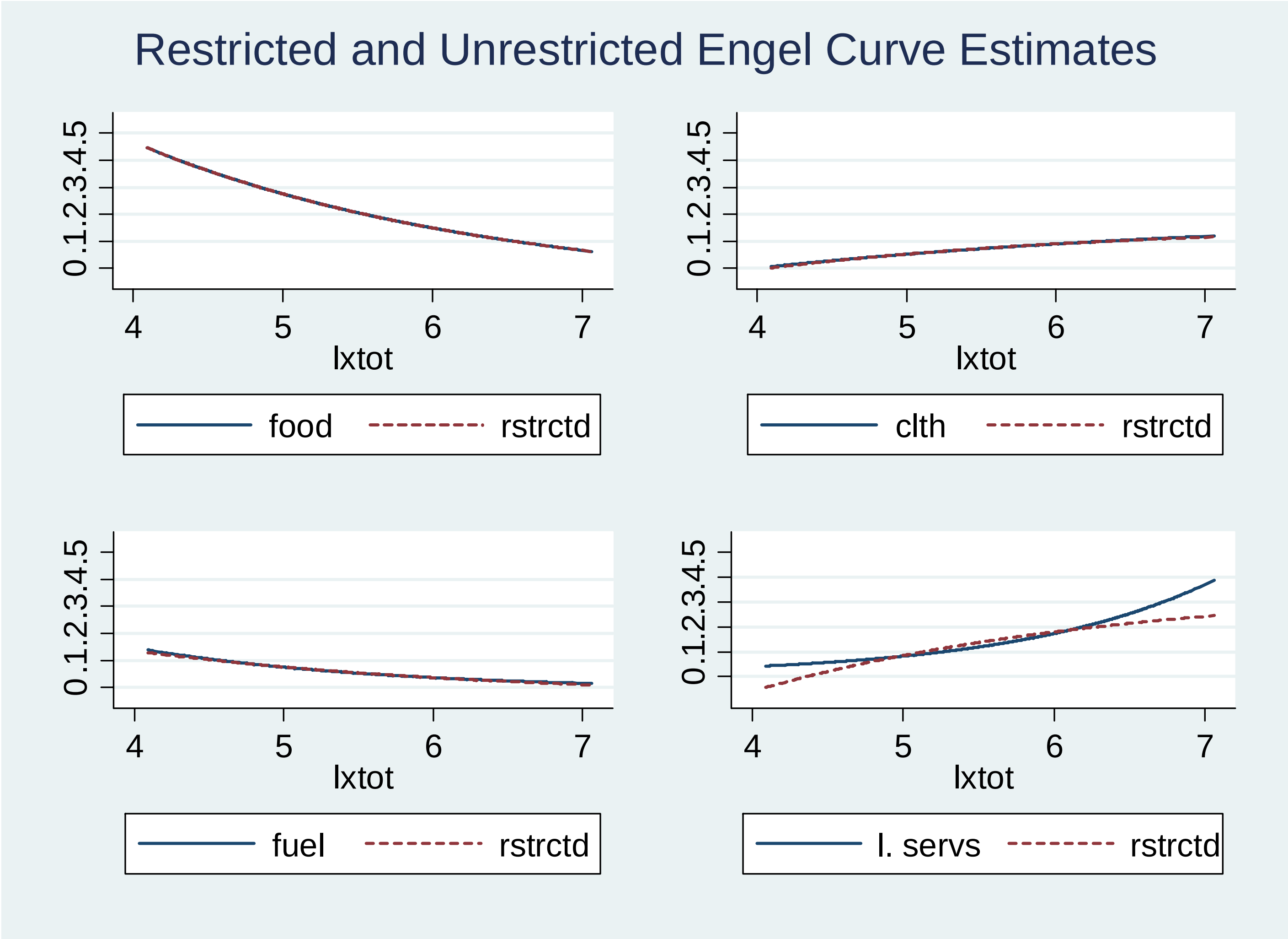
- The literature contains numerous test of demand system rank: Lewbel (1991), Banks, Blundell and Lewbel (1997), Donald (1997), Lewbel (2004).

- They find rank 3 or higher.

- Tests of constant EIS bases on demand system rank will have much more power than tests bases on intertemporal responses (such as Blundell, Browning and Meghir, 1994; Atkeson and Ogaki, 1996; Attanasio and Browning, 1996).

Empirical Illustration:
(UK FES, homogenous sample):

Commodity	θ (EIS)	Het. Robust Confidence Interval	Test of Functional Form (Het. Consistent T-test)
Food	1.72	[1.31,2.14]	-0.23
Clothing	1.43	[0.47,2.40,]	-1.31
Fuel and Light	2.69	[1.50,3.89]	-1.91
Leisure Services	0.56	[0.47,0.65]	-0.27
Test of Equality	$\chi^2_3 =$	43.74	($p<0.0001$)


Restricted and Unrestricted Engel Curve Estimates
0 .1 .2 .3 .4 .5
4 5 6 7
lxtot
food
rstrctd
0 .1 .2 .3 .4 .5
4 5 6 7
lxtot
clth
rstrctd
0 .1 .2 .3 .4 .5
4 5 6 7
lxtot
fuel
rstrctd
0 .1 .2 .3 .4 .5
4 5 6 7
lxtot
l. servs
rstrctd

Extension (1): HARA Utility

- The HARA class of utility functions are defined by linear risk tolerance: power (isoelastic/CRRA), translated power, quadratic, negative exponential.
- HARA + additivity = quasi-homotheticity over states/periods (Pollak, 1971).
- Many key results in Finance depend on the assumption of HARA preferences (for example, fund separation theorems); There are very few tests of HARA in the literature.
- Our results for power/isoelastic generalize to HARA.

- Share equations derived from indirect utility functions in the HARA class have the form:

$$w_k = \frac{\theta}{1-\theta}\frac{a_k}{a(p)} + \frac{1}{1-\theta}\gamma\frac{a_k}{a(p)}x^{-1} + \frac{b_k}{a(p)}\frac{(\gamma+\theta x)^{1/\theta}}{x}$$

- These demands are at most rank 3: translated power, quadratic; If we rule out quadratic, translated power is DRRA.

- The term that gives the extra flexibility goes to zero as total consumption grows, implying rank 2 for the rich.

- A solution to this might be to relate γ to an external reference point.

Extension (2): Epstein-Zin preferences

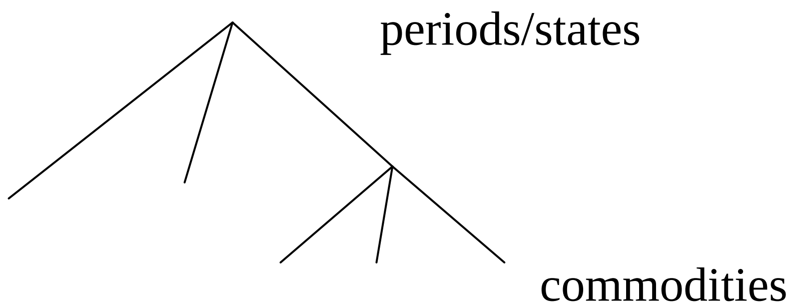
$$U_t = \beta v(c_t) + (1-\beta) v(z_{t+1})$$
$$z_{t+1} = g^{-1}\left(E\left[g(U_{t+1})\right]\right)$$

- Breaks the link between risk aversion and the EIS (fluctuation aversion).

- Our results hold for a multiple good version of this.

Summing up:

- Curvature of the (indirect) utility function affects both inter- and intra-temporal allocation. Assumptions about the latter can place restrictions on the former.



- Given a constant EIS, the power parameter is much easier to estimate than the literature suggests.

- However, the assumption of a constant EIS is can be tested with demand patterns and is strongly rejected.

- Issue is more general than CRRA.

- A result in the same vein as Deaton (1974), Browning and Crossley (2000), Chetty (2005)……"Curvature" shows up in multiple places…

- This has important implications for endogenous inequality (poverty traps – Bliss, 2006); studying dynamic models; testing full insurance (Ogaki and Zhang, 2001); and more…