Are Two Cheap, Noisy Measures Better Than One Expensive, Accurate One?

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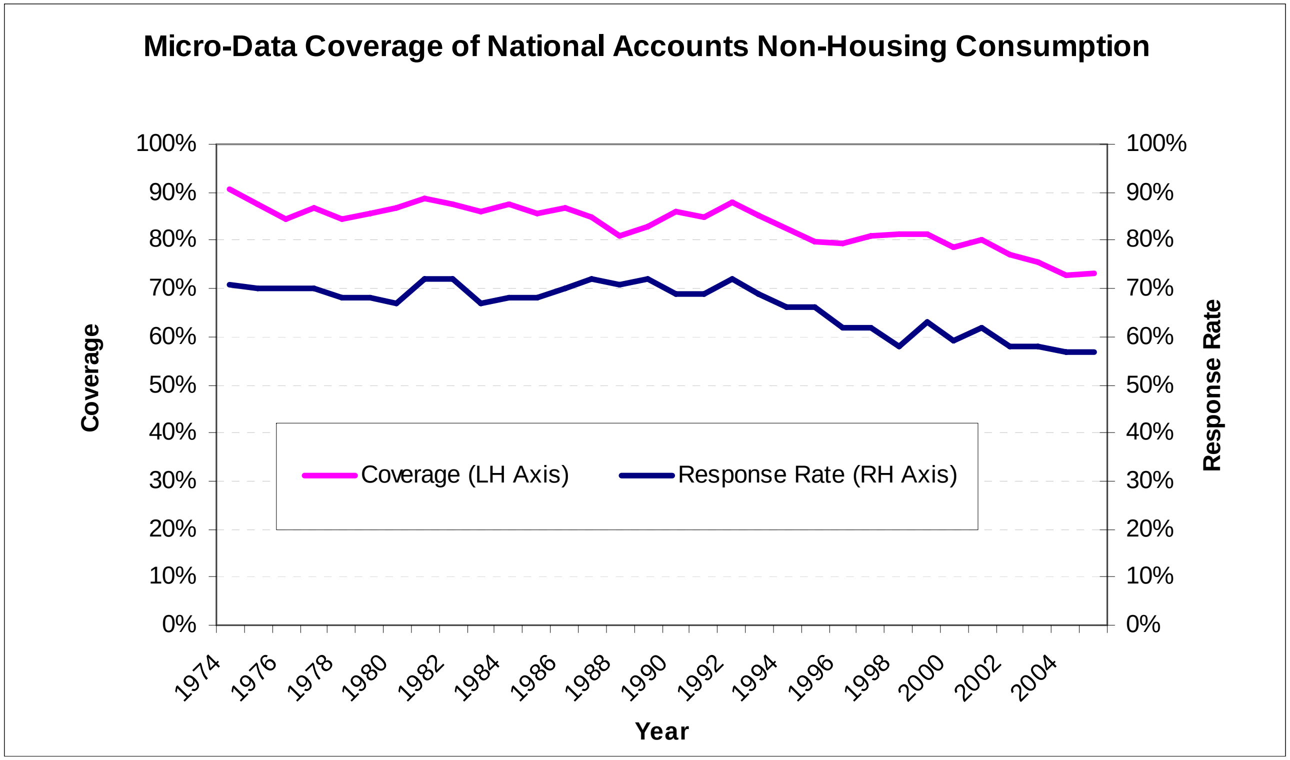
1. Motivation

Measurement error in surveys

- Survey responses are always subject to measurement error
- Even true for well defined quantities (eg., age, earnings, expenditure, not health)
- In general surveys (and especially longitudinal surveys), severe constraints on time that can be spent measuring any target variable

- Some important variables getting harder to measure, even by intensive methods



The value of multiple measures

- The value of multiple measures for means, regression coefficients, is familiar to most economists (eg., twin studies).
- The value of multiple measures is much more general.
- If the measurement error in two measures is mutual independent and independent of the true value, we can recover the entire distribution of the quantity of interest, up to location (Kotlarski).
- Basis for some recent econometrics.

Our Suggestion

- Rather than design estimators for given measurement error characteristics, design surveys to deliver measurement error with desirable properties.
- May sometimes be better to go for two `noisy' measures than attempting to increase the accuracy and precision of a single measure.

2. An Example

- To make things concrete, we consider a specific problem: estimating the population variance of the log of consumption.
- There is a large literature on consumption inequality (see for example Cutler and Katz (1992), Slesnick (1993), Blundell and Preston (1998), Krueger, D. and F. Perri, (2006).)
- The variance of the log is common measure of inequality.

- Let C be (true) log consumption with variance σ_c^2. (σ_c^2 is what we want to estimate.)
- We conceive of three measures: Z, X_1 and X_2. Z is the `expensive` measure and X_1 and X_2 are the `noisy' measures.
- Let c, z, x_1 and x_2 be de-meaned versions.

- *Define* the error associated with each method as (note no loss of generality – additivity here has no content, will be the independence etc. that matters):

(1) $u = z - c$

(2) $\varepsilon_i = x_i - c$

- These errors have variances σ_u^2, σ_1^2 and σ_2^2 and covariances with true consumption of σ_{cu}, σ_{c1} and σ_{c2}.

- While more expensive, z is also more precise ($\sigma_u^2 < \sigma_i^2, \quad i = 1,2$)
- We imagine that σ_u^2 (and possibly σ_{cu}) can be reduced, but at greater cost and respondent burden.
    - Longer diary period.
    - More (and finer) expenditure categories for recall questions.

- The `cheap' measures ($x_i$) might be a `one shot' recall question about total expenditure, or a recall question about a single category of expenditure (food, or other).
- The quantities $\frac{\sigma_c^2}{\sigma_z^2}$ and $\frac{\sigma_c^2}{\sigma_{x_i}^2}$ are the reliability of z and x_i respectively.

- The single measure z has variance:

(3) $$E[z^2] = \sigma_c^2 + 2\sigma_{cu} + \sigma_u^2.$$

- Thus if we use the sample variance of z as an estimate of σ_c^2, the asymptotic bias is $E[z^2] - \sigma_c^2 = 2\sigma_{cu} + \sigma_u^2$.

- The two measures (x_1, x_2) have covariance:

(4) $$E[x_1 x_2] = \sigma_c^2 + \sigma_{c1} + \sigma_{c2} + \sigma_{12}$$

- If we use the sample covariance of (x_1, x_2) as estimate of σ_c^2, the asymptotic bias is $E[x_1 x_2] - \sigma_c^2 = \sigma_{c1} + \sigma_{c2} + \sigma_{12}$.

- Note that this does not depend on σ_1^2 and σ_2^2 (and hence on the reliability of the noisy measures.)

- Suppose measurement errors are ***classical*** (independent of the true value and each other.) Equations (3) and (4) reduce to:

(5) $E[z^2] = \sigma_c^2 + \sigma_u^2$

(6) $E[x_1 x_2] = \sigma_c^2$

- $\frac{1}{N}\sum x_1 x_2$ is a consistent estimator of σ_c^2, *regardless* of the reliability of these measures. Of course, the precision of this estimator depends on the reliability of two measures.

- Measurement errors are typically ***non-classical.***

(7) $$E[z^2] = \sigma_c^2 + 2\sigma_{cu} + \sigma_u^2.$$

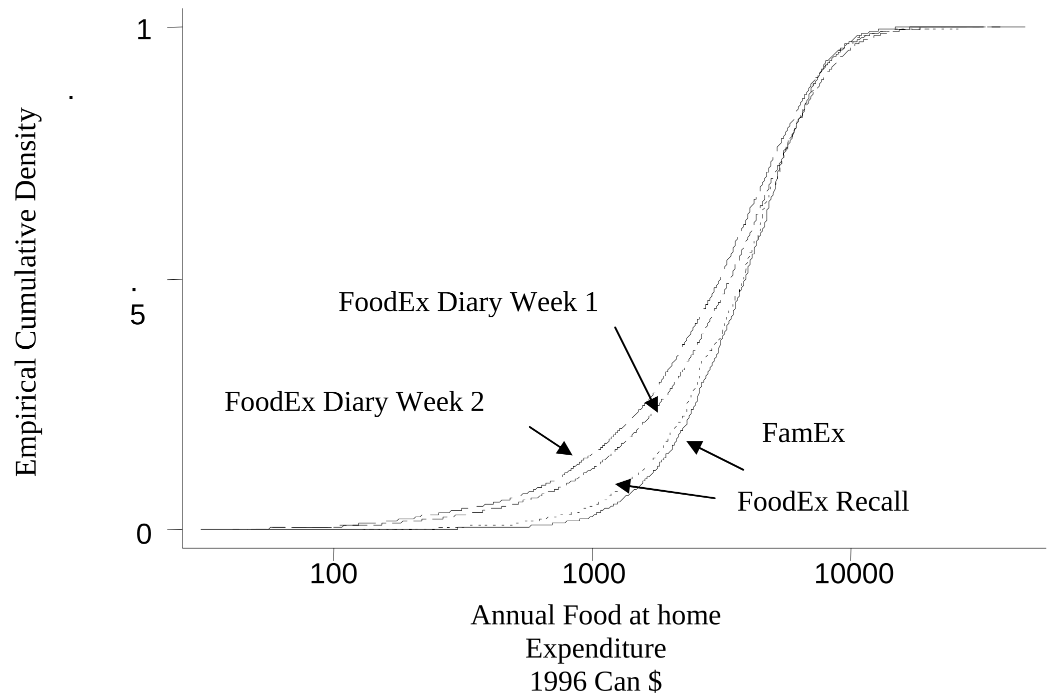
(8) $$E[x_1 x_2] = \sigma_c^2 + \sigma_{c1} + \sigma_{c2} + \sigma_{12}.$$

- Cognitive theories of response behaviour, economic theory, and pre-testing can be informative about the sources of bias (σ_{cu}, σ_u^2, σ_{c1}, σ_{c2} and σ_{12})
- Help us to design better questions/choose wisely from available measures

Measuring Consumption (Inequality) With a Subset of Goods

- Survey respondents seem able to answer recall questions about some categories of expenditure (eg. food.)
- If we can only ask about a subset of goods, which ones should we ask?

Food Expenditure, Empirical CDFs



Source: Ahmad, Brzozowski and Crossley, 2006

Here is what we said in 2003 (Browning, Crossley, Weber, *EJ*):

- *Always ask a 'total expenditure on non-durables and services' question…..there is a great deal of idiosyncratic behaviour in demand and sometimes households spend a good deal on sub-items that we would never think to ask about……*

- *Always ask a 'food at home' and a 'food outside the home' question with the same time period as for total expenditure….. respondents can report food at home accurately….being a large budget item, it is very useful in imputation…….*

- *Ask about utilities such as fuel and telephones…..*

- The idea here was that these could be measured reliably and contained variation that was orthogonal to food in/out. We very much had in mind to capture a large share of the total and/or a "prediction" approach (Skinner, 1987).

Some doubts:

- The `one-shot' total expenditure question performed poorly in recent experiments for the UKHLS.
  - Good response rate (>10%)
  - Significant under-reporting (~40%) relative to EFS (previous experience of about 30% in Canada and Italy).
  - Cognitive testing was particularly discouraging.

- Multiple measure framework suggests a different approach:
- Think of log consumption of specific goods (food, clothing, telephone, recreation) as our cheap error ridden measures (x_1, x_2)
- Choose goods so that the measurement errors have desirable properties.
- Use theoretical and empirical knowledge of consumer behaviour to choose.

- An Engel curve relates consumption of specific items to total consumption.

- Consider a linear in logs approximate Engel curve:

$$x_i = \alpha_i c + \eta_i(c) + e_i \qquad (9)$$

Heterogeneity

Approximation Error

Income Elasticity

- Now define the measurement error in each measure as:

$$\begin{aligned}\varepsilon_i &= x_i - c \\ &= \alpha_i c + \eta_i(c) + e_i - c \\ &= (a_i - 1)c + \eta_i(c) + e_i\end{aligned} \qquad (10)$$

- This allows us to relate properties of the measurement errors to theoretical and empirical knowledge of demands.

(8) $E[x_1 x_2] = \sigma_c^2 + \sigma_{c1} + \sigma_{c2} + \sigma_{12}$

(10) $\varepsilon_i = (a_i - 1)c + \eta_i(c) + e_i$

- Consider:
  - Income elasticities (luxury/necessity)
  - Substitute compliment
  - Adding up
  - Approximation error

- *Therefore*, choose goods that:

   i. Respondents can readily report,
   ii. Have close to unit income elasticities (or a luxury and a necessity), and not too much approximation error,
   iii. Are not strong complements or substitutes,

- Note that:
  - large shares not necessary (and perhaps undesirable), and
  - reliability helpful but not paramount (in contrast to a single measure approach.)

3. Simulation Study

- 1996 Canadian FAMEX:
- Intensive, high quality, annual recall (little infrequency)
- Sample of couples without children
- Take the log of total nondurable consumption as “true” target variable (c) for each household.
- Take the sample variance of log total nondurable consumption as the population parameter we with to estimate.

- For z we take total nondurable consumption ($z = c$), or total nondurable consumption with classical measurement error added ($z = c + \upsilon$).
- For (x_1, x_2) we take pairs of goods guided by the above advice.
- Following BPP 2004 we also consider consumption imputed by the (inverted) food Engel curve: $\widehat{\left(1/\alpha_f\right)} x_f$.

- We resample repeatedly from the Famex (1000 draws with replacement)
- Estimators:

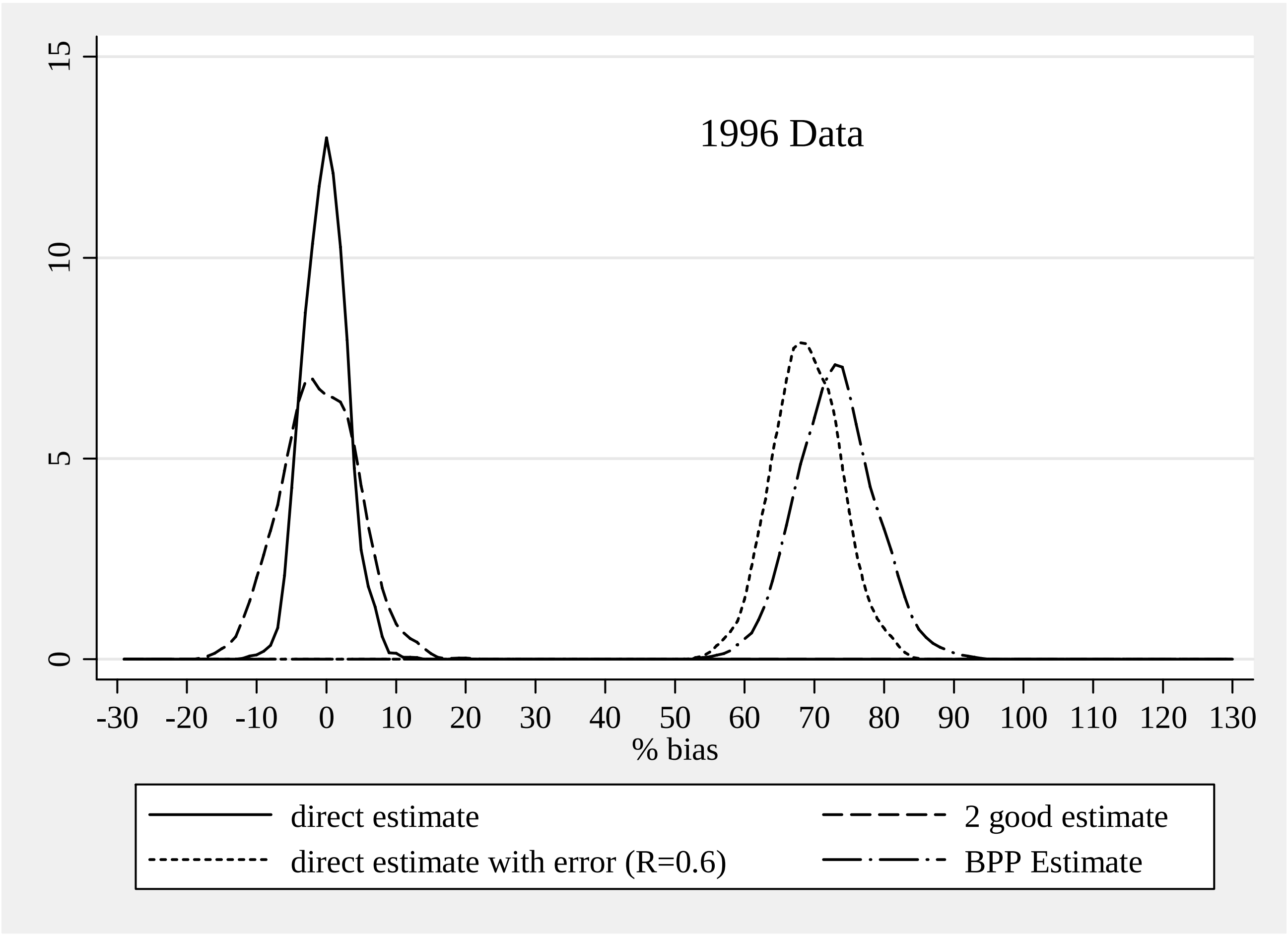
  i) The sample variance of of z

  ii) The sample variance of $\widehat{\left(1/\alpha_f \right) x_f}$

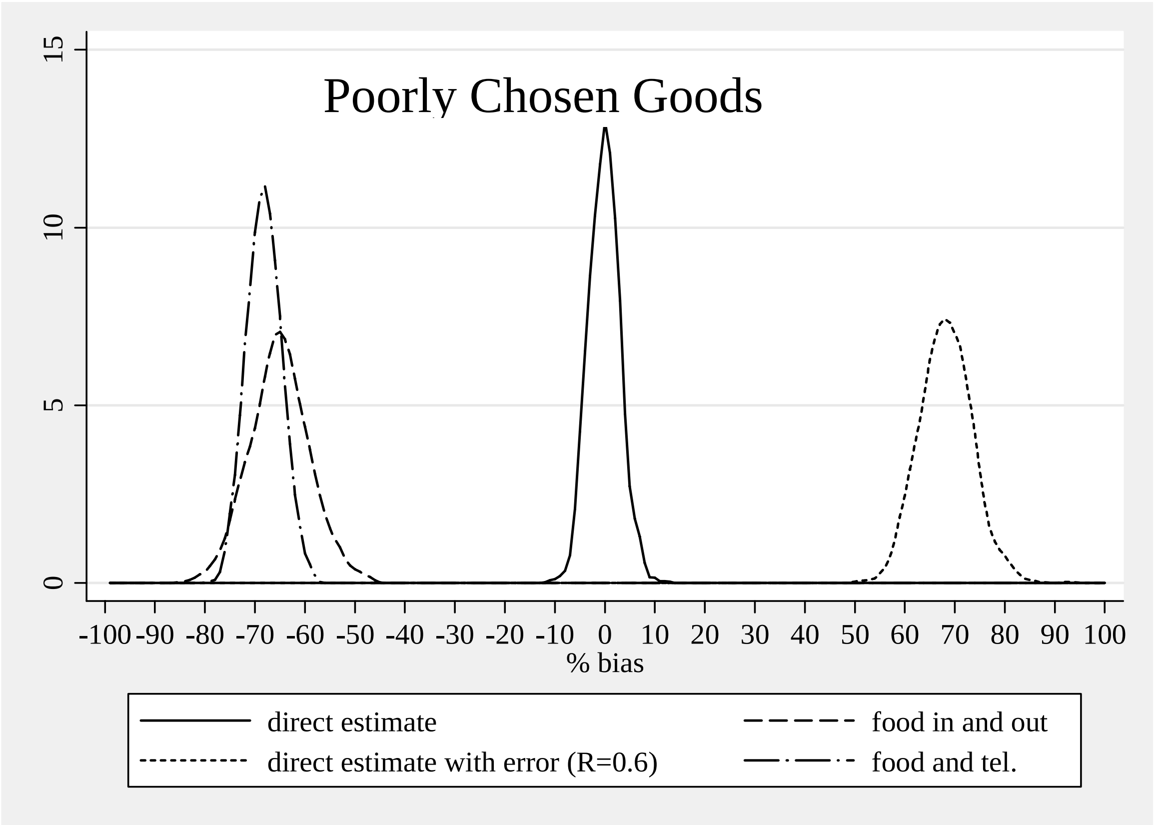
  iii) The sample covariance of (x_1, x_2)

- We study the bias and variance of our estimators.

- Use food as x_1 (seems well measured and likely always to be included in surveys that collect expenditure information.)
- Preliminary analysis: estimated Engel curves. Examine fit, income elasticities and error correlations.
- Suggests using recreation (or “leisure”) nondurable/semidurable goods and services as x_2.
- Danish evidence suggests that recall questions about recreation work well (Browning and Gortz, 2006).


1996 Data
0
5
10
15
-30 -20 -10 0 10 20 30 40 50 60 70 80 90 100 110 120 130
% bias
direct estimate
2 good estimate
direct estimate with error (R=0.6)
BPP Estimate

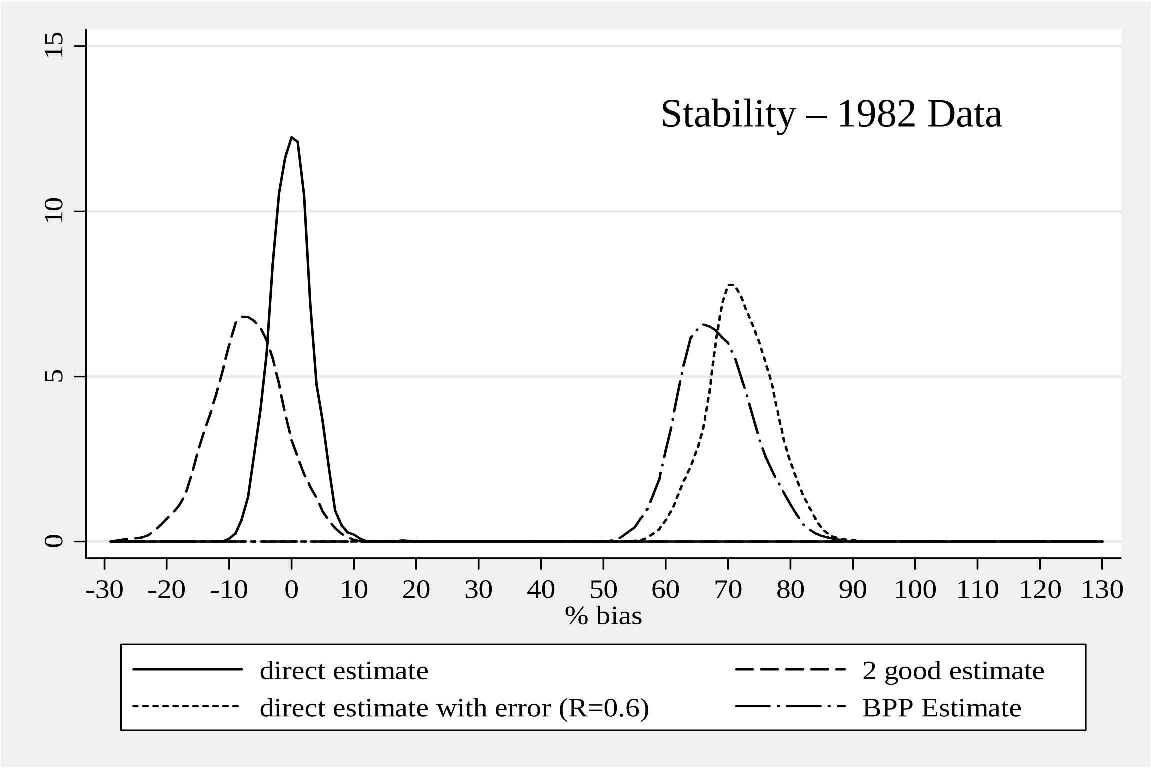
Estimator	Mean Estimate	Standard Deviation	95^{th} - 5^{th}	Mean % Bias
Direct, R=1	0.189	0.0058	0.199 - 0.180 = 0.019	-0.11
2 Good (food, recreation)	0.188	0.0102	0.204 - 0.171 = 0.033	-0. 87
Direct, R=0.6 (classical ME)	0.320	0.0092	0.335 - 0.305 = 0.030	64.4
BPP (inverse Engel curve imputation)	0.328	0.0107	0.345-0.310 = 0.035	73.1
True value = 0.189 1000 replications Population size = sample size = 2379				



(8) $E[x_1 x_2] = \sigma_c^2 + \sigma_{c1} + \sigma_{c2} + \sigma_{12}$

(10) $\varepsilon_i = (a_i - 1)c + \eta_i(c) + e_i$

- Note that approach is not completely general if $\alpha \neq 1$
- Also worry about stability in face of changing relative prices
- One solution is to combine the BPP approach (inverting Engel curves) with a 2 measure approach. But this requires more information.



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IV. Discussion

- Designing survey questions to eliminate measurement error is very difficult (impossible?)
- With the right kind of measurement errors, two error ridden measures can tell you a lot about the distribution of a quantity of interest – indeed one can recover the entire distribution.
- Maybe it is (relatively) easier to design survey questions to get close (or closer) to the right kinds of measurement error.

- Research agenda: How can we get multiple measures of quantities of interest in household surveys?
    - Multiple proxies (two goods as measures for total nondurable consumption)?
    - Ask the same thing two different ways (the sum of expenditure in different categories, then, income minus saving)?
    - Ask multiple household members?

Guidance from: survey response theory, economics (theory and empirical evidence) and pre-testing.