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# Survival pessimism and the demand for annuities

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## Abstract

The “annuity puzzle” refers to the fact that annuities are rarely purchased despite the longevity insurance they provide. Most explanations for this puzzle assume that individuals have accurate expectations about their future survival. We provide evidence that individuals misperceive their mortality risk, and study the demand for annuities in a setting where annuities are priced by insurers on the basis of objectively-measured survival probabilities but in which individuals make purchasing decisions based on their own subjective survival probabilities. Subjective expectations have the capacity to explain significant rates of non-annuitization, yielding a quantitatively important explanation for the annuity puzzle.

**Keywords:** Annuity Puzzle, Subjective Expectations, Survival Probabilities

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# 1 Introduction

Annuities insure individuals against longevity risk by allowing them to exchange wealth for an income stream guaranteed until death. Theory predicts that under general conditions, risk-averse individuals will purchase a fairly-priced annuity (Yaari (1965); Davidoff et al. (2005)). Few households, however, ever purchase an annuity.<sup>1</sup> This divergence between theory and experience has become known as the “annuity puzzle”.

Most of the explanations that have been proposed for this puzzle<sup>2</sup> attempt to rationalize non-purchase by individuals who are assumed to have accurate perceptions of their survival probabilities. Individuals are not, however, well-informed about their survival probabilities (Hurd and McGarry (1995), Elder (2013), Wu et al. (2015)).

We study the demand for annuities in a setting where those annuities are priced by insurers on the basis of *objectively-assessed* survival probabilities but in which individuals make purchasing decisions on the basis of their own *subjective* survival probabilities. We estimate subjective survival curves for a sample of older individuals using directly-measured expectations. Consistent with an established literature (see, for example, Hurd and McGarry (1995); Elder (2013), Wu et al. (2015)), our study finds that, on average, individuals *under-*estimate their probability of survival through their 50s, 60s and 70s and *over-*estimate their chances of survival through their late 80s and beyond. Overall, pessimism dominates, and most respondents would perceive an annuity that is priced fairly from an actuarial point of view as one which is unfairly-priced.

As with all insurance products, individuals might, depending on their preferences, still purchase an annuity that is unfairly-priced as the longevity insurance provided by the annuity might be worth the low apparent (to them) ‘money’s worth’. Whether survival pessimism is

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<sup>1</sup>Lockwood (2012) reports that less than 5% of a sample of single retirees in the US own an annuity; Inkmann et al. (2011) show that only 6% of older households in the UK voluntarily purchase an annuity.

<sup>2</sup>These explanations including adverse selection (Brugiavini (1993); Finkelstein and Poterba (2004); Finkelstein and Poterba (2014)), bequest motives (Lockwood (2012); Gan et al. (2015)), precautionary saving for medical and long-term care expenses, (Reichling and Smetters (2015); Ameriks et al. (2011)), existing annuity provision from social security income (Pashchenko (2013)), and costs of administration (Mitchell et al. (1999)). See Brown (2007) for a general review of this literature.

an important driver of the demand for annuities by risk averse individuals is an open question. To assess the quantitative importance of survival pessimism for annuity purchases, we embed these subjective survival curves in a lifecycle model of consumption, saving and annuitization. We estimate the proportion of their wealth that individuals would choose to annuitize, given their idiosyncratic subjective survival curves. Parameterizing our model with plausible levels of risk aversion and patience, we are able to explain high rates of non-annuitization. In the setting of Yaari (1965), patient individuals with ‘objective’ expectations about their own survival will choose to annuitize their entire stock of wealth when offered an actuarially fair annuity. We find that, when behaving according to their ‘subjective’ expectations, the average rate of annuitization for such individuals would be between 42% and 64%, for a plausible range of levels of risk aversion. To benchmark the quantitative importance of this channel, we compare these results to those we obtain by introducing into our model actuarially unfair pricing caused by adverse selection (or transaction costs or other market imperfections), a leading rationalization of low annuity demand. In this case, the average share of wealth annuitized would range from 34% to 69% for the same range of levels of risk aversion. We therefore find that survival pessimism is quantitatively as important as the higher prices caused by adverse selection.

This result does not depend on other explanations that have been given for non-annuitization; in our model, individuals have only modest social security income, do not have bequest motives, face no medical cost risk, do not have access to means-tested income floors and annuities are priced fairly given objectively-measured survival rates. The difference between ‘objective’ and individual-specific ‘subjective’ survival curves is large enough, for many individuals, to outweigh the insurance value of annuitizing much of their retirement wealth.

Subjective expectations of survival have been shown to be empirically important in explaining a number of economic decisions. Hurd et al. (2004) find that those with particularly low expectations of survival are more likely to retire earlier and to claim Social Security benefits earlier; de Bresser (2020) studies a similar phenomenon and shows that bringing

individual-level variation in survival probabilities into a lifecycle model can help explain the timing of retirement and benefit claiming. Bloom et al. (2006) find that a higher subjective probability of survival is associated with higher wealth levels. Gan et al. (2015) finds that a model of wealth decumulation and bequests including subjective survival expectations better fits decumulation and bequest behavior than does one with life table survival probabilities. Heimer et al. (2019) solve a life-cycle model with subjective mortality beliefs and show that ‘pessimism’ about survival to older age, combined with ‘optimism’ at the oldest ages can explain both under-saving for retirement and slow decumulation of wealth at the end of life. None of these papers considers an annuitization choice, as we do.

There is evidence of a correlation between individual longevity – realized as well as expected – and decisions around annuitization. Examining the voluntary market for annuities in the United Kingdom, Finkelstein and Poterba (2004) find a positive association between ex-post survival and features of annuities purchased (for example those who buy back-loaded annuities are longer-lived). Teppa and Lafourcade (2013) find that stated optimism around survival is positively correlated with stated demand for annuities while Inkmann et al. (2011) find a similar link between subjective expectations and annuity purchases.<sup>3</sup> That paper also experiments with subjective survival probabilities in a life cycle model and shows that if subjective survival probabilities are reduced by 10% at each age (a quantity that is not empirically grounded), no households would demand an annuity. Wu et al. (2015) embed estimated subjective survival curves within a lifecycle model of consumption and savings. While they make calculations of the perceived ‘money’s worth’ of annuities (finding that annuities are perceived to offer less than actuarially fair value, on average), their model does not contain an annuitization choice. The contribution of our paper is to study the importance of the observed divergence between reported survival expectations and objective survival rates – the ‘survival pessimism’ discussed above – for the annuitization decision. Given that sufficiently risk averse agents may choose to buy unfairly priced insurance products in

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<sup>3</sup>However, Brown (2001) finds no evidence of this phenomenon.

preference to remaining uninsured, we do this by combining subjective expectations data with an economic model of consumption, savings and annuitization.

We proceed as follows. Section 2 outlines the data. Section 3 compares average reported survival expectations to official life tables, and sets out our method for constructing ‘subjective’ survival curves from stated beliefs. In Section 4 we outline the model of annuitization and the impact of introducing subjective survival curves on predicted rates of annuitization. Section 5 concludes.

## 2 Data

We draw on data from the English Longitudinal Study of Ageing (ELSA) (Marmot et al. (2017)), a biennial panel representative of the English household population aged 50 and above. ELSA is part of a network of longitudinal aging studies around the world, modeled on the US Health and Retirement Study (HRS). One module of the survey asks individuals about their expectations that certain events will happen in future, including whether or not they will leave an inheritance, whether they will still be in work at a certain age and whether at some point in the future they will not have enough resources to meet their financial needs. This battery of questions opens with the following statement:

*“Now I have some questions about how likely you think various events might be. When I ask a question I’d like you to give me a number from 0 to 100, where 0 means that you think there is absolutely no chance an event will happen, and 100 means that you think the event is absolutely certain to happen.”*

As part of this module, individuals are asked a question of the form “*What are the chances that you will live to be age  $X$  or more?*”, where the age  $X$  depends on the current age of the respondent. All individuals aged 65 and under are asked about survival to age 75. Those aged 66 or older are asked about the age which is between 11 and 15 years ahead of their current age and is a multiple of 5. For example, those aged 75-79 are asked about survival

to age 90. Additionally, from wave 3 onwards, all individuals aged under 70 were asked a second question about survival to age 85. We denote individual  $i$ 's reported probability of survival to age  $\alpha$  as  $R_i(\alpha)$ .

Over the first seven waves of ELSA, 16,345 unique individuals are asked one or more survival questions in 67,201 separate interviews. In all of our analysis, unless otherwise stated, we weight observations by the cross-sectional weights available in the ELSA data.

## 2.1 Evaluating the content of subjective reports

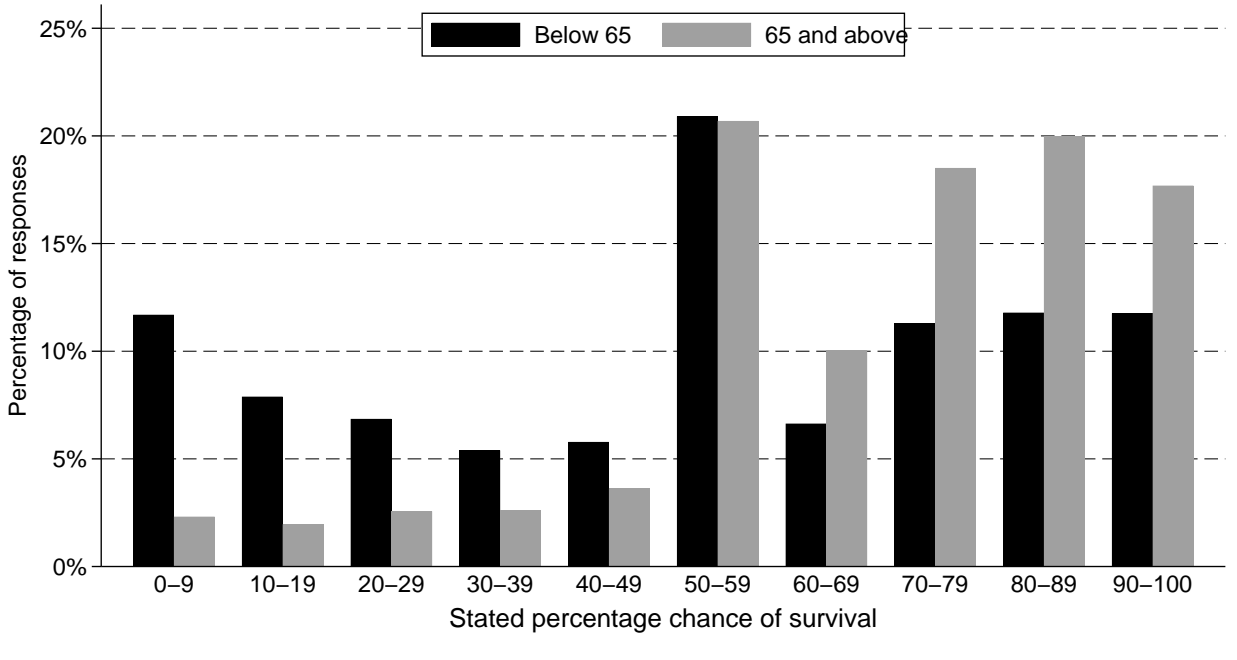
Before using individual responses to survival probability questions in analysis, we wish to assess whether individuals appear to understand the meaning of these questions and to be able to engage with the probabilistic concepts involved. Next, assuming that participants understand these questions, we would like to establish, as far as possible, whether answers constitute considered, reflective judgements that might plausibly guide behavior, or are instead picked with little prior thought.

In just 1.5% of interviews, individuals answer “Don’t know” to one or more survival probability questions, suggesting a willingness to answer in almost all cases. Figure 1 shows the distribution of reported survival probabilities for the full sample of first questions asked, in bins of 10 percentage points. We split the sample into those aged below 65 and those aged 65 and above, with the younger group much more likely to report high chances of survival.

Some individuals answer “0%” or “100%” to the survival questions (5.3% and 6.8% respectively). Both of these answers might be evidence of a lack of understanding of the question. However neither is conclusively so; respondents are asked to report probabilities on a discrete scale, and so rounding, combined with a terminal diagnosis or extreme optimism, respectively, could rationalize these answers. We include these individuals in the sample, but show in Appendix B.3 that our results change very little if we exclude either or both of these groups.

When individuals are asked two survival questions, they can report a higher chance of

Figure 1: Distribution of answers given to first survival question by age-group



Source: ELSA waves 1–7. 66,210 answers from 16,345 unique individuals

survival to the older age than to the younger age. This happens in 8.3% of interviews. Given that such responses indicate a fundamental misunderstanding of the question, we remove these individuals from all of the remaining analysis.

One may have reservations about the fact that a high proportion - 20.5% - of answers are “50%”. There could be a concern that individuals pick this focal answer when wanting to give a response but not understanding the question. We assess this by examining these individuals’ answers to other probability questions. Those individuals who answer “50%” almost always give a range of answers to other questions and are no more likely to answer “50%” to other probability questions than are the rest of the sample (Appendix A.1 gives further details). Of the 16,345 individuals who answered one or more survival questions, only 41 individuals (0.2%) answered “50%” to all survival questions in all waves. On the basis of this evidence, we retain answers of “50%” in our main sample but show in Appendix B.3 the (minimal) sensitivity of results to their removal.



Given that the overwhelming majority of individuals give answers that do not indicate a lack of understanding of probabilities, we perform four further tests aimed at assessing the informational content of responses and whether they relate to economic behaviour. Firstly, we find that responses are correlated with known mortality risk factors (e.g. smoking, drinking and health conditions) in a way that is consistent with existing evidence. For example, current smokers report a 6-8 percentage points lower probability of survival over the 11-15 year horizon, relative to current non-smokers.<sup>4</sup> Secondly, using the panel nature of the survey we find that reports ‘update’ over time in response to news relevant to mortality such as diagnoses of new health conditions. For example, a new cancer diagnosis was associated with a 4 percentage point reduction in the stated probability of surviving to an age 11 to 15 years ahead.<sup>5</sup> Thirdly, exploiting a link to administrative death records from the English National Health Service, we find that reported expectations are correlated with actual subsequent mortality over a 10 year horizon.<sup>6</sup> While it is possible that individuals’ answers to survival expectations questions could represent their ‘actual’ expectations even if there was no association with the above outcomes, these findings provide additional evidence that answers represent meaningful, reflective judgements.<sup>7</sup> Fourth, in line with expectations being drivers of economic behavior, we find that stated survival expectations are negatively correlated with purchases of life insurance, a product analogous to selling an annuity.<sup>8</sup>

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<sup>4</sup>This result is obtained using linear regression of reported probability on smoker status, controlling for a range of other risk factors, demographic variables, health conditions and self-reported health. Full details are given in in Appendix A.2.

<sup>5</sup>This result is from a linear fixed effects regression of reported survival probability on a range of variables for whether individuals have been diagnosed with particular health conditions, as well as other risk factors and demographic variables. Full details are given in in Appendix A.2.

<sup>6</sup>Full details are given in Appendix A.2.

<sup>7</sup>Hurd and McGarry (2002) make a similar point with respect to subjective expectations in the Health and Retirement Survey.

<sup>8</sup>This result is from a linear regression in which the outcome is the percentage of individuals’ total wealth portfolio held as life insurance. Stated expectations are negatively correlated with this outcome. Full details are given in Appendix A.2.

### 3 Assessing the accuracy of subjective expectations of survival

In this section, we describe the patterns in subjective reports, compare them to actual mortality rates and projections and derive idiosyncratic survival curves that will be used, together with our model, to evaluate the importance of these curves for the annuitization decision. The results that we find – that individuals are mostly pessimistic about survival to younger ages and optimistic regarding survival to older ages – are consistent with a well-established literature analyzing the accuracy of self-reported survival probabilities and self-reported life expectancy across a number of countries and in a variety of survey settings. We give a brief review of this literature before proceeding.

In the first studies comparing subjective expectations to an objective benchmark, Hurd and McGarry (1995) and Hurd and McGarry (2002) analysed the survival expectations data available in the first wave and first two waves, respectively, of the HRS and compared it to the period life tables available at that time. These studies concluded that while there was some evidence that men underestimated their chances of survival to age 75 relative to life tables (with women approximately accurate) and that women overestimated their chances of survival to age 85 relative to life tables (with men approximately accurate), mean expectations were broadly consistent with the period life tables. However, subsequent research using the HRS drew upon more expectations data and, due to the passage of time, was able to compare stated expectations to subsequent survival of the sample. Elder (2013) compared reported survival probabilities from the first waves of data in the HRS and AHEAD surveys to the respondents' *actual* subsequent survival to the age they were asked about. Doing so revealed a substantial (over 10 percentage points) underestimation of survival probabilities, on average, for those in their early 60s (who were asked about survival to age 75), a mild underestimation by those in their early 70s (who were asked about survival to age 85) and growing optimism about survival for those aged over 75 (who are asked about

ages between 11 and 15 years ahead of their current age). Ludwig and Zimper (2013) found the same patterns in waves 5 to 7 of the HRS, when using Human Mortality Database and Social Security Administration (SSA) life tables. Heimer et al. (2019) use the 2014 wave of the HRS, the Survey of Consumer Finances (SCF) and a survey of their own to document further evidence that US seniors underestimate their near-term survival up until the age of around 70 (by between 15 and 20 percentage points), after which they become gradually more optimistic about survival.

The pattern of substantial pessimism about survival through the 50s and 60s and early 70s, turning to relative optimism about survival to the oldest ages, that is found in US surveys is also present in a variety of surveys across a range of other countries. These include the Netherlands (Teppa and Lafourcade, 2013), Australia (Wu et al., 2015) and Germany (Bucher-Koenen and Kluth (2013)). Hurd et al. (2005) document similar patterns across a number of European countries using the Survey for Health, Ageing and Retirement in Europe.

### **3.1 Comparing reports to actual mortality data**

The UK Office for National Statistics (ONS) life tables contain actual and projected mortality data for the England and Wales population by sex and year-of-birth. These tables would be a natural benchmark against which to assess subjective expectations if the ELSA sample were representative of the whole English population.<sup>9</sup> As ELSA is representative only of the non-institutionalized population (meaning those in residential care, for example, are excluded), mortality rates for the ELSA population are slightly lower than given by the ONS life tables.<sup>10</sup> We use administrative death records linked to ELSA to “rescale” the

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<sup>9</sup>While ELSA includes only English residents, ONS cohort life tables are only available for England and Wales combined and not England-only, for the cohorts we analyse. However, this will make little difference to our analysis as Wales makes up around 6% of the England and Wales population and has very similar mortality patterns.

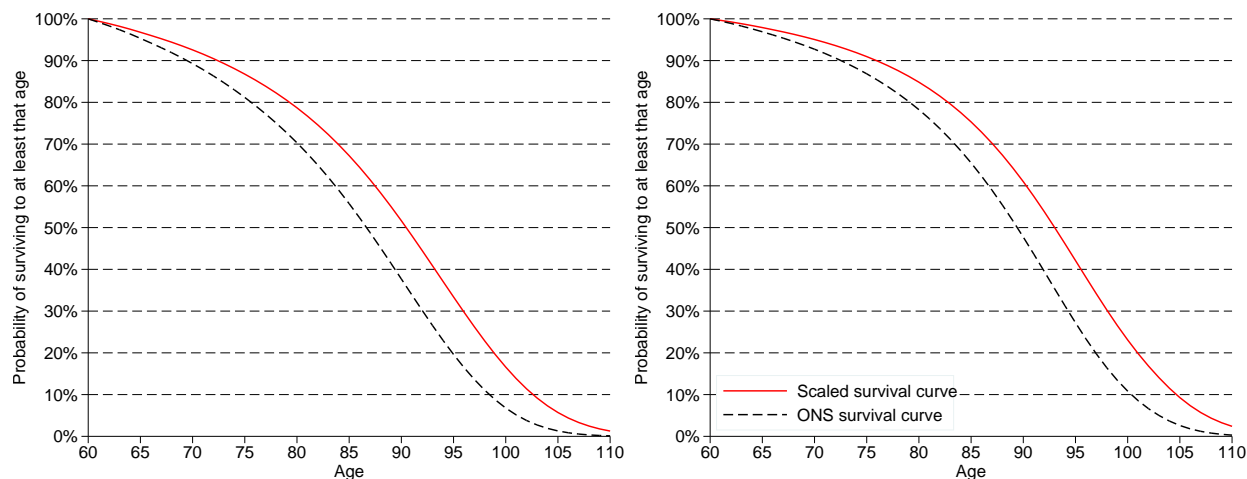
<sup>10</sup>The ELSA data is linked to administrative death records giving the date of death for any ELSA respondent who died on or before February 2013. Amongst those aged 50 to 70 in ELSA wave 1, the actual mortality rate over the period until February 2013 (approximately a 10-year period) was 10.5%. The average ONS life table implied death rate over that period for the same sample was 13.0%.

data in the ONS life tables by the observed difference in average mortality rates between the ELSA sample and those implied by the ONS life tables. In Appendix B.1 we show that our main results are somewhat attenuated, but qualitatively unchanged, if we use the ONS life tables without rescaling them.

ELSA is linked to administrative death records such that we know if any individual (including attriters) has died up until February 2013. We use this information to “rescale” the official life tables in the following way. We calculate, for each year of age, the actual mortality hazard rate observed in the ELSA sample and the expected mortality hazard rate if each individual faced the hazard rate implied by the ONS life table for their sex and year-of-birth. For each sex, we fit a cubic in age to actual hazard rates using OLS and calculate the ratio of the fitted hazard rate to the ONS hazard rate at each age. We take the mean ratio across ages and use this to rescale the hazard rates underlying the ONS survival curves for each sex and year-of-birth, yielding a set of “scaled” ONS survival curves. Our method yields hazard rates for men and women that are 71% and 69% of their original level, respectively – that is, those in the ELSA sample have lower mortality probabilities (and therefore higher life expectancies) than the population at large. Figure 2 shows a comparison of the original ONS survival curve with the “scaled” survival curve for 60-year old men and women, born in 1950. We see that, for example, men are estimated to have a 50% chance of survival to age 90, in comparison to the 38% figure in the ONS life table. The corresponding figures for women are 60% and 48% respectively.

We use the “scaled” ONS survival curves life tables as an “objective” benchmark to assess whether particular age-sex-cohort groups have positively biased (‘optimistic’) or negatively biased (‘pessimistic’) expectations of survival to various “target ages” (i.e. the age about which the individual is asked the question). We conduct this analysis at the most granular level for which these comparisons are possible: we calculate for each combination of age in years, year of birth, sex, and target age, the average reported survival probability, and compare this to the relevant scaled life table probability. A clear pattern emerges. Individuals

Figure 2: Comparison of ONS cohort survival curve and “scaled” survival curve for men (LHS) and women (RHS) aged 60 and born 1950



Source: ELSA waves 1–7 and ONS 2014-based cohort life tables for England and Wales.

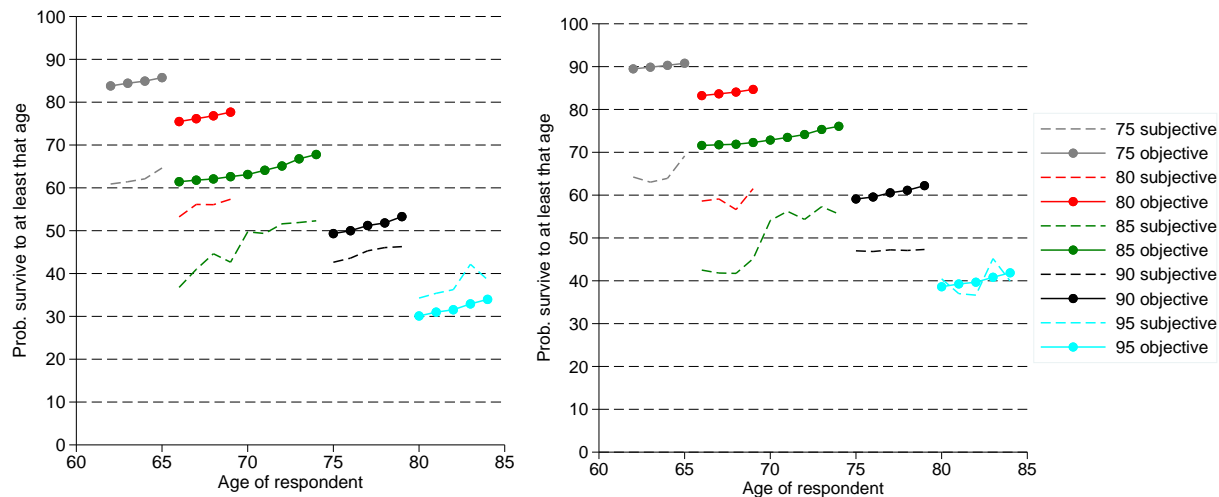
are, on average, ‘pessimistic’ about their chances of survival to ages 75, 80, 85 and 90 and then become increasingly optimistic as they get older and are asked about survival to older ages. While the degrees of ‘optimism’ and ‘pessimism’ vary slightly between cohorts, these patterns are consistently found across those born in the 1920s through to the 1950s. Comparing men and women, we see that women tend to be slightly more pessimistic on average, than men.

Figure 3 illustrates the comparison between mean subjective survival probabilities and scaled life tables for men and women born in the 1930s. We see that individuals in their early 60s under-estimate survival to age 75 by around 25 to 30 percentage points while those in their late 60s and early 70s under-estimate survival to age 85 by 15 to 20 percentage points. Turning to those in their late 80s, we see that they are close to accurate about their probability of survival, on average.<sup>11</sup>

These findings are in line with those in existing literature, including Elder (2013) and Heimer et al. (2019), which establish in a number of settings the pattern of over-estimation of mortality hazard rates at ages until around the mid-80s, with under-estimation of mortality

<sup>11</sup>Note that in Figure 3, each “subjective” data point corresponds to an average over respondents with different birth years. The “objective” data points are constructed by weighting the corresponding scaled life table survival probabilities according to the proportions of individuals with each birth year in the sample.

Figure 3: Comparison of mean ‘subjective’ reports and scaled ONS cohort survival rates/projections for men (LHS) and women (RHS) born 1930-39



Note: Different colored series correspond to different ages about which respondents are asked questions. Source: ELSA waves 1–7 and ONS 2014-based cohort life tables for England and Wales.

rates at older ages.

### 3.2 Constructing subjective survival curves

In this section, we describe how we use stated survival expectations to estimate the individual-specific subjective survival curves that will be used in our lifecycle model. There are an infinite number of possible survival curves consistent with the answers that individuals give to the survival questions. We are therefore required to make further assumptions if we are to infer individuals’ subjective survival curves from their reports about their survival to specific ages. We make an assumption on the functional form of individuals’ subjective survival curves. Specifically, we assume that subjective survival probabilities follow a Weibull distribution. The Weibull distribution is widely used in the epidemiological literature and for modeling aging processes generally.<sup>12</sup>

The Weibull distribution is a two-parameter  $(\lambda_i, k_i)$  distribution defined in the following

<sup>12</sup>See Bissonnette et al. (2017) for an example of use of the Weibull distribution to construct objective and subjective survival curves using data from the HRS. The authors report that their results are similar when using the Weibull or the (also widely-used) Gompertz distribution.

way. Person  $i$  with age  $z$ , has probability of surviving to at least age  $\alpha$ :

$$S_i(\alpha) = \exp \left[ - \left( \frac{\alpha - z}{\lambda_i} \right)^{k_i} \right] \quad : \quad \lambda_i, k_i > 0 \quad (1)$$

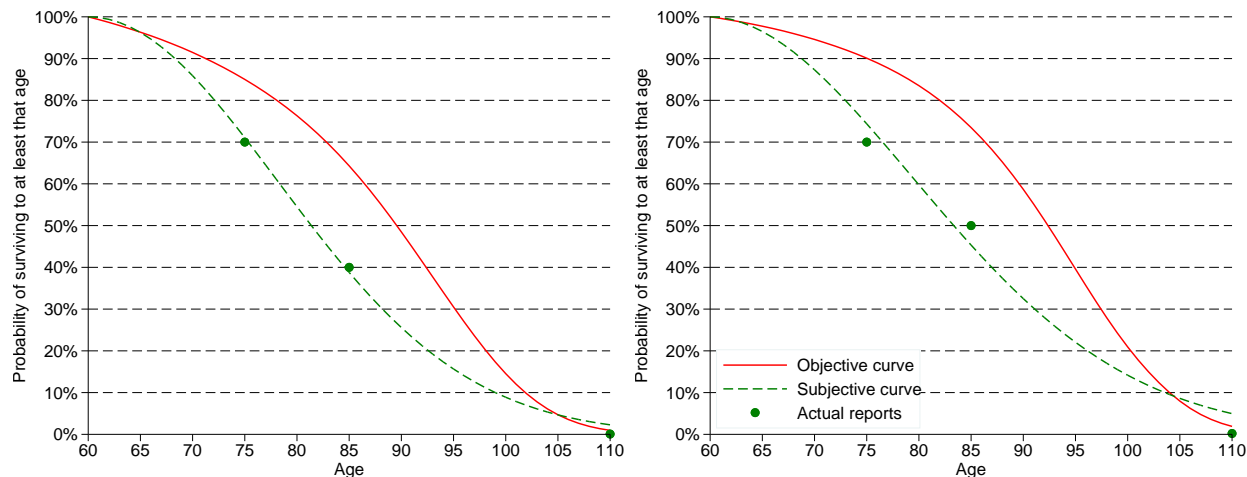
We estimate subjective survival curves for all individuals in our sample who answered two survival questions (i.e. all those aged under 70 who answered two questions and were not removed from the sample due to giving “impossible” answers). We make one additional weak assumption – that individuals believe that they are almost certain not to live beyond age 110 – by including the relevant scaled life table survival probability for each individual for target age 110 as a third “report” (we denote this third subjective “report” by  $R_i(110)$ ). We fit the individual’s Weibull-distributed subjective survival curve by estimating  $\lambda_i$  and  $k_i$  using these three reports and non-linear least squares. That is, denoting the set of 3 ages for which we have subjective reports by  $A_i$  we choose the parameter vector  $(\hat{\lambda}_i, \hat{k}_i)$  that satisfies

$$(\hat{\lambda}_i, \hat{k}_i) = \arg \min_{\lambda_i, k_i} \sum_{\alpha \in A_i} \left( R_i(\alpha) - \exp \left[ - \left( \frac{\alpha - z}{\lambda_i} \right)^{k_i} \right] \right)^2. \quad (2)$$

Figure 4 illustrates the curve-fitting procedure applied to the median responses from men and women born in the 1940s and compares these subjective survival curves to the relevant scaled life table survival curves. The subjective curve implies that at age 60, this group of individuals are pessimistic about survival to all ages up until around age 100 and optimistic about survival to ages beyond this. The subjective life expectancy measures implied by these curves are 8.6 and 9.6 years lower than the life expectancies calculated using life table survival curves for men and women, respectively. This is equivalent to life expectancies at the age of 60 that are 31% (for men) and 33% (for women) lower than those implied by life tables, on average.

Average overall pessimism of this magnitude is found across sexes and cohorts when interviewed in their 50s and 60s. Using the full sample of individuals for whom we have a subjective survival curve, we can compare ‘subjective’ and scaled life table life expectancy at

Figure 4: Comparison of median ‘subjective’ and ‘objective’ survival curves for men (LHS) and women (RHS) born 1940-49



Source: ELSA waves 1–7 and ONS 2014-based cohort life tables for England and Wales.

the individual level. Subjective life expectancy is lower than scaled life table life expectancy by 6.1 years, or 22%, amongst men, and 7.7 years, or 25%, amongst women, on average.

## 4 Subjective survival expectations and annuitization

Annuities provide insurance against longevity risk. The decision about how much of one’s wealth to annuitize at a given price ought to depend on the individual’s assessment of this longevity risk. Individuals who under-estimate their longevity may perceive an annuity as a worse deal than it truly is. This is a potential explanation for the unpopularity of annuities.

First, we can assess whether, given their subjective expectations, individuals would perceive an annuity as offering at least an actuarially ‘fair’ deal. An annuity rate is defined as actuarially fair with respect to a given discount rate and set of survival probabilities, if it enables the purchase of a guaranteed income stream until death that has expected discounted value equal to its price. For each individual for whom we have fitted a subjective survival curve, we calculate the actuarially fair annuity rate given their subjective survival curve and given their scaled life table survival curve. The actuarially fair annuity rate for an individual



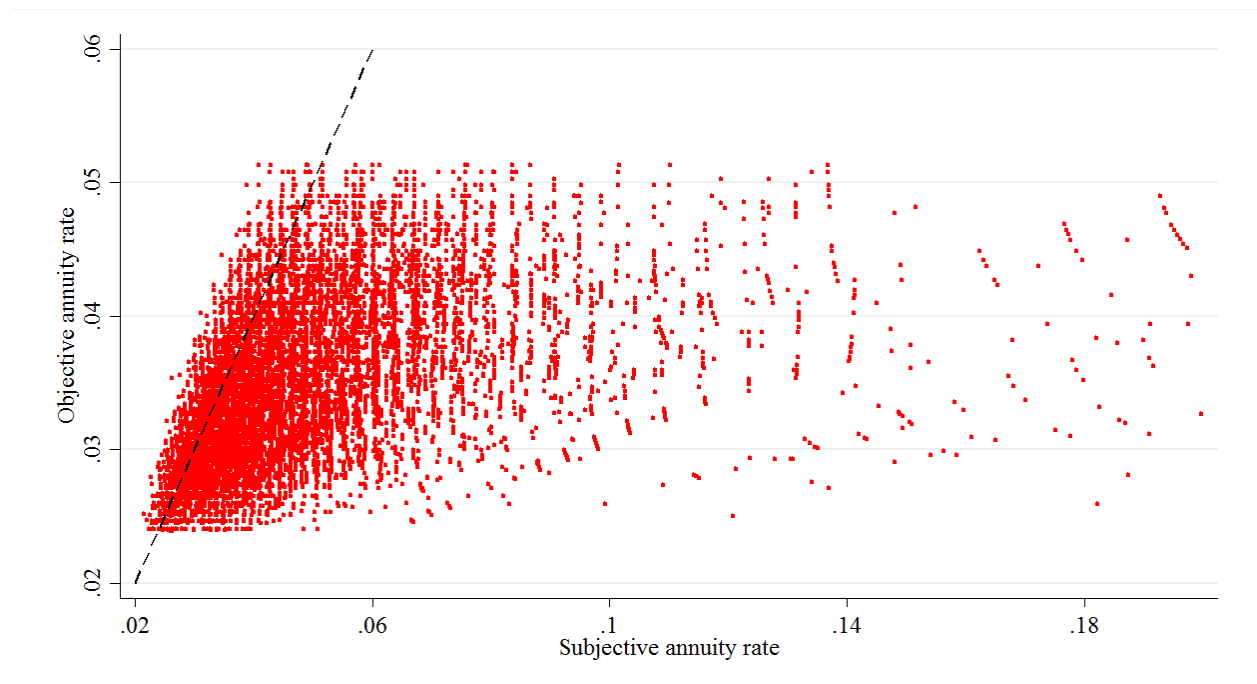
age  $z$ , given the survival curve  $S_i(\alpha)$  is given by:

$$\theta = \left[ \sum_{\alpha=z}^{110} \frac{S_i(\alpha)}{(1+r)^{\alpha-z}} \right]^{-1} \quad (3)$$

where  $r$  is the interest rate.

Figure 5 compares these ‘subjective’ and ‘objective’ annuity rates for our sample assuming a real interest rate of 0% in both cases. Variation in objective annuity rates comes from variation in gender, age and year of birth; variation in the subjective annuity rates comes from the estimated subjective survival curves. 88% of individuals would perceive an annuity that is priced fairly for the average person of their age, sex and cohort as offering a less than fair annuity rate.

Figure 5: Comparison of ‘objective’ and ‘subjective’ based annuity rates



Note: ‘Subjective’ annuity rates are the actuarially fair rate implied by the subjective survival curve constructed from the individuals responses to the survival expectations questions. ‘Objective’ annuity rates are the actuarially fair rate implied by the scaled ONS life table survival curve for the individuals sex, age and year of birth. Source: ELSA waves 3–7 and ONS 2014-based cohort life tables for England and Wales.

An individual who perceives an annuity as being unfairly priced may, of course, choose to annuitize some of their wealth if doing so offers sufficiently large insurance value. To examine whether survival ‘pessimism’, and the implied divergence between subjective and life table-based annuity rates, could lead to low rates of annuitization of retirement savings, we specify a model of consumption, saving and annuitization. We compare results for our sample in the case where individual survival expectations are consistent with scaled life tables to those where they are consistent with their subjective survival curve. We account for the fact that individuals have public pension entitlements, giving them some already annuitized income. We use data from ELSA on private pension and financial wealth and model the choice of how much (if any) of this wealth to annuitize at a rate that is actuarially fair given the individual’s age, sex and cohort.<sup>13</sup>

## 4.1 Model

In this section we outline the model of consumption, saving and annuitization. Just-retired agents (indexed by  $i$ ) have initial wealth,  $a_{i0}$ , and receive public pension income (state pension/social security),  $p_i$ , in each period. In period 0, agents choose to annuitize some fraction of their wealth. In each of the following periods, the agents choose how much of their resources (the sum of their public pension income, their annuity income and their un-annuitized wealth) to consume. Borrowing is not allowed.

Individuals make choices consistent with a survival curve – which we can specify to be either their subjective survival curve ( $S_i^s(\alpha)$ ) or their objective survival curve ( $S_i^o(\alpha)$ ). When beliefs are consistent with the *subjective* survival curve, an individual aged  $z$  believes that they have a probability of survival to each age  $\alpha \leq 110$  that is described by their estimated

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<sup>13</sup>We do not include housing wealth in the measure of wealth which may be annuitized – Appendix B.2 presents a version of our model in which we add a consumption flow coming from owner-occupied housing. The differences in annuitization rates between the ‘objective’ and ‘subjective’ cases in this version of the model are very similar to those in the baseline model.

Weibull survival curve:

$$S_i^s(\alpha) = \exp \left[ - \left( \frac{\alpha - z}{\hat{\lambda}_i} \right)^{\hat{k}_i} \right] \quad (4)$$

We assume that all individuals believe that they will die at the end of their 110th year at the latest. Individuals have a constant relative risk aversion utility function and they discount the future according to a geometric discount rate ( $\beta$ ). They perceive their expected lifetime utility to be:

$$U = \sum_{t=0}^{110-z} \beta^t S_i^x(z+t) \frac{c_{it}^{1-\gamma}}{1-\gamma} \quad (5)$$

for  $x \in \{s, o\}$  depending on whether they are assumed to have subjective or objective survival curves. Time is indexed by  $t$ , where  $t = 0$  is the year in which we observe a household in the survey, and  $z$  is their age in that year.

At time zero individuals can irreversibly annuitize any fraction their wealth,  $b_i \in [0, 1]$ , at rate  $\theta_i$ , which is the actuarially fair annuity rate given  $S_i^o(\alpha)$ , the *objectively-measured* survival curve for someone of their sex, year of birth, and age:

$$\theta_i = \left[ \sum_{\alpha=z}^{110} \frac{S_i^o(\alpha)}{(1+r)^{\alpha-z}} \right]^{-1} \quad (6)$$

Their annuity income in each future period is therefore defined as  $ann_i = \theta_i \cdot b_i \cdot a_{i0}$ . An individual's problem is therefore to choose their consumption in each remaining period of life  $\{c_{it}\}$  and the proportion of their initial wealth that they annuitize ( $b_i$ ):

$$\max_{\{c_{it}\}, b_i} \sum_{t=0}^{110-z} \beta^t S_i(z+t) \frac{c_{it}^{1-\gamma}}{1-\gamma} \quad (7)$$

$$s.t. \begin{cases} a_{it+1} = (a_{it} + p_i + ann_i - c_{it})(1+r) \\ a_{it+1} \geq 0 \end{cases} \quad (8)$$

Individuals' optimal choice of consumption is characterized by the Euler equation which will bind with equality whenever the no-borrowing constraint does not hold:  $c_t^{-\gamma} = \beta(1+r)s_i(z+$

$t)c_{t+1}^{-\gamma}$  where  $s_i(z+t) \equiv S_i(z+t+1)/S_i(z+t)$ . Individuals will be inclined to annuitize more of their wealth the higher is  $\beta$  (and so the fact that consumption from annuitized wealth cannot be front-loaded does not imply a substantial welfare cost) and the higher is  $\gamma$  (and so the longevity insurance provided by annuities is more valued).

We solve the model for each individual in waves 3, 4 and 5 of ELSA who has begun drawing their public pension, holds positive assets, and for whom we are able to construct a subjective survival curve.<sup>14</sup> We use the first observation for any individuals observed multiple times, yielding 2,848 observations. Each individual’s initial level of wealth ( $a_{i0}$ ) is taken as the sum of their household private pension wealth and gross financial wealth.<sup>15</sup> We take public pension income ( $p_i$ ) to be that level reported in the data. For each observation, we solve the model twice. In one case the individual’s expectations are consistent with the scaled life table survival curve for their sex, year of birth and age, and in the other, expectations reflect their fitted subjective survival curve estimated from their survey responses.

## 4.2 Results

We illustrate the impact of subjective survival expectations by comparing the mean rate of annuitization in the case where individuals behave according to the scaled life table survival curve for their age, sex and cohort with the case where they behave according to their own subjective survival curve. We define the mean rate of annuitization as the simple mean of individual rates,  $\frac{1}{N} \sum_{i=1}^N b_i$ . In Appendix B we show a qualitatively similar result holds when the outcome of interest is the share of aggregate initial wealth annuitized.

Figure 6 shows the mean rate of annuitization at various parameter combinations of patience ( $\beta$ ) and risk aversion ( $\gamma$ ). The real interest rate is set at 0%. Panel (a) shows the rate of annuitization when individuals have objectively-measured expectations. The annuity

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<sup>14</sup>In this period, the male public pension age was 65 and the female state pension age increased from age 60 to age 62. The public pension age is the age at which individuals can first claim their state pension. Over 99% of individuals begin to claim at this age. We drop 231 individuals over their public pension age who do not report deferring their state pension, but yet report a public pension income of zero.

<sup>15</sup>Pension wealth information is available up until wave 5. Wealth measurements are at the household level. For individuals in a couple, we therefore take half of this wealth level.

we consider is one which pays a constant (real) income. When households are fully patient ( $\beta = 1$ ), all households will fully annuitize – any risk-averse fully-informed individual will prefer a constant stream of income to self-insuring against longevity through a risk-free bond. As patience is decreased, the rate of annuitization falls.<sup>16</sup> At a coefficient of relative risk aversion of 3, the annuitization rate is 69% when the discount factor is set at 0.98, and 51% with  $\beta = 0.96$ .

For a fixed pair of preferences parameters, variation across individuals in the decision over how much of their wealth to annuitize is driven by the relative level of public pension entitlements ( $p_i$ ) and the level of wealth ( $a_{i0}$ ). Those who annuitize a smaller proportion of wealth are those who have little liquid wealth relative to the size of their accrued public pension entitlements. For these households, the value of the small amount of additional longevity insurance they would receive by annuitizing is less than the additional value of consuming that wealth sooner.

Panel (b) shows an equivalent set of results for the case when individuals are making decisions based on their *subjective* survival expectations. Comparing these rates to those reported in panel (a), it is clear that subjective expectations have the capacity to substantially reduce annuity demand. For fully patient individuals, the rate of annuitization falls from 100% to between 42% (log utility) and 64% (coefficient of relative risk aversion of 5). With modest impatience ( $\beta = 0.98$ ), rates of annuitization fall from 47% to 20% assuming log utility, and from 77% to 52% assuming a high rate of risk aversion ( $\gamma = 5$ ).

Appendix B shows that the difference between the proportion of wealth annuitized under ‘objective’ and ‘subjective’ expectations is qualitatively insensitive to (1) including a flow of consumption of housing services for homeowners and (2) using ONS life tables without

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<sup>16</sup>The annuity puzzle is less stark at lower levels of patience as we assume (realistically) that individuals cannot negotiate an actuarially-fair annuity with a bespoke payment schedule that fits their patience and risk aversion. If complete markets for annuities existed, the annuitization rate would be 100%, regardless of patience. Inkmann et al. (2011) show that this type of market incompleteness, combined with estimated risk aversion and patience can rationalize much of the lack of demand for annuities. To see the effects of our results in a complete markets context, the reader should focus only on the row with  $\beta = 1$ , the level of patience for which the offered contract is the preferred one.

“rescaling” and (3) various alternative choices on sample selection and weighting.

Figure 6: Percentage of individuals annuitizing at each parameter combination

(a) Objectively-measured expectations										(b) Subjectively-elicited expectations									
Discount factor	Coefficient of relative risk aversion									Discount factor	Coefficient of relative risk aversion								
	1	1.5	2	2.5	3	3.5	4	4.5	5		1	1.5	2	2.5	3	3.5	4	4.5	5
<b>0.960</b>	19%	31%	40%	46%	51%	54%	57%	59%	62%	<b>0.960</b>	8%	15%	21%	26%	31%	34%	37%	40%	42%
<b>0.965</b>	24%	37%	45%	51%	55%	58%	61%	63%	65%	<b>0.965</b>	10%	18%	24%	29%	33%	37%	40%	42%	44%
<b>0.970</b>	31%	43%	50%	56%	59%	62%	64%	67%	68%	<b>0.970</b>	13%	21%	27%	32%	36%	40%	42%	45%	47%
<b>0.975</b>	38%	49%	56%	61%	64%	67%	69%	71%	72%	<b>0.975</b>	16%	24%	30%	35%	39%	42%	45%	47%	49%
<b>0.980</b>	47%	57%	63%	66%	69%	72%	74%	75%	77%	<b>0.980</b>	20%	28%	34%	39%	43%	45%	48%	50%	52%
<b>0.985</b>	57%	65%	70%	73%	76%	78%	79%	81%	82%	<b>0.985</b>	25%	32%	38%	43%	46%	49%	51%	53%	55%
<b>0.990</b>	68%	74%	78%	81%	83%	84%	86%	86%	87%	<b>0.990</b>	30%	37%	43%	47%	50%	52%	54%	56%	58%
<b>0.995</b>	83%	86%	88%	90%	90%	91%	92%	92%	92%	<b>0.995</b>	36%	42%	47%	51%	53%	56%	58%	59%	61%
<b>1.000</b>	100%	100%	100%	100%	100%	100%	100%	100%	100%	<b>1.000</b>	42%	48%	52%	55%	58%	60%	62%	63%	64%

Source: Model predictions using ELSA waves 3–5 and ONS 2014-based cohort life tables for England and Wales. 2,848 observations.

To put the size of the falls in annuitization rates in context, Figure 7 shows the average rate of annuitization in a model where individuals have objectively-estimated survival expectations but are faced with an annuity rate which, due to adverse selection and other market imperfections, as well as transactions costs, is offered at 17.5% below the actuarially-fair rate.<sup>17</sup> In panel (b) of Figure 7, we show the model predictions in the case where individuals act according to their ‘objective’ survival curve, but face an annuity payout equal to 82.5% of the actuarially fair rate. We reproduce panel (a), where individuals have ‘objective’ expectations and face the actuarially fair annuity rate, for comparison. This is intended to illustrate the impact of the degree of actuarial unfairness observed in UK annuities markets which may be attributable to adverse selection or administrative loading.

At all parameter combinations, the actuarially-unfair pricing causes declines in annuitization rates comparable to those caused by misperceived survival probabilities. With moderate

<sup>17</sup>We use 17.5% based on the analysis of annuity rates available on the US market by Mitchell et al. (1999) who report that “the expected discounted value of annuity payouts per dollar of annuity premium averages between 80 and 85 cents for an individual chosen at random from the population”.

Figure 7: Percentage of individuals annuitizing at each parameter combination

(a) Objectively-measured expectations										(b) Objectively-measured expectations with 17.5% rate reduction									
Discount factor	Coefficient of relative risk aversion									Discount factor	Coefficient of relative risk aversion								
	1	1.5	2	2.5	3	3.5	4	4.5	5		1	1.5	2	2.5	3	3.5	4	4.5	5
<b>0.960</b>	19%	31%	40%	46%	51%	54%	57%	59%	62%	<b>0.960</b>	3%	8%	14%	19%	24%	28%	32%	35%	38%
<b>0.965</b>	24%	37%	45%	51%	55%	58%	61%	63%	65%	<b>0.965</b>	4%	10%	16%	22%	27%	31%	35%	38%	41%
<b>0.970</b>	31%	43%	50%	56%	59%	62%	64%	67%	68%	<b>0.970</b>	5%	12%	19%	25%	30%	34%	38%	42%	44%
<b>0.975</b>	38%	49%	56%	61%	64%	67%	69%	71%	72%	<b>0.975</b>	7%	15%	22%	29%	34%	38%	42%	45%	47%
<b>0.980</b>	47%	57%	63%	66%	69%	72%	74%	75%	77%	<b>0.980</b>	10%	19%	27%	33%	38%	42%	46%	49%	51%
<b>0.985</b>	57%	65%	70%	73%	76%	78%	79%	81%	82%	<b>0.985</b>	13%	23%	31%	38%	43%	47%	50%	53%	55%
<b>0.990</b>	68%	74%	78%	81%	83%	84%	86%	86%	87%	<b>0.990</b>	18%	29%	37%	43%	48%	51%	54%	57%	59%
<b>0.995</b>	83%	86%	88%	90%	90%	91%	92%	92%	92%	<b>0.995</b>	25%	36%	44%	49%	54%	57%	59%	62%	63%
<b>1.000</b>	100%	100%	100%	100%	100%	100%	100%	100%	100%	<b>1.000</b>	34%	45%	51%	56%	60%	63%	65%	67%	69%

Source: Model predictions using ELSA waves 3–5 and ONS 2014-based cohort life tables for England and Wales. 2,848 observations.

levels of impatience and risk aversion, the effects of actuarially-unfair pricing are marginally greater than the effects of misperceived survival probabilities: at  $(\beta, \gamma) = (0.98, 3)$ , the rate of annuitization with objectively-measured expectations is 69% but falls to 38% under the reduced annuity payouts and 43% when individuals make decisions based on their subjective survival expectations. The effect of subjective survival expectations has a slightly larger impact than adverse selection at higher levels of risk aversion and higher levels of patience – at  $(\beta, \gamma) = (1, 5)$ , the average proportion of wealth annuitized is 100% given objective expectations and actuarial fairness, is 69% given objective expectations and reduced payouts, and is 64% under subjective expectations and actuarial fairness.<sup>18</sup>

<sup>18</sup>The reason for these patterns can be understood by comparing the reasons in each case for annuities becoming less attractive products. With adverse selection raising annuity prices, all annuity payouts are discounted relative to the actuarially fair benchmark. With subjective survival rates, the annuity payouts late in life are not considered valuable – as individuals wrongly perceive that they will likely be dead by then. When individuals are less patient therefore, reducing annuity payments every period implies a greater reduction in welfare than does treating payments in the distant future as ‘wasted’. When risk aversion is high, the welfare cost of the higher price is low relative to the cost of insurance against the modest possibility of living a long time. This insurance is less valuable as the likelihood of that longevity appears to diminish. As a result the higher is risk aversion the greater is the effect of subjective survival probabilities relative to reduced annuity rates.

Overall, we take these results as indicating that the effect of individuals misperceiving their survival probabilities is as large as the effect of adverse selection.

Finally, we note that, as shown by Heimer et al. (2019), individuals at younger ages than in our sample are likely to also be pessimistic about their later-life survival and consequently accumulate less retirement wealth under subjective expectations than they would do if their expectations were unbiased. As the individual annuitization rate is increasing in wealth in our model, this implies that the effect of subjective expectations on the rate of annuitization would be shown to be even greater if this savings effect were taken into account.

## 5 Conclusion

Incorporating individual ‘subjective’ survival curves into a model of annuitization, consumption and saving has the capacity to explain part of the “annuity puzzle”. While market incompleteness and informational asymmetries play a role in rationalizing low annuity demand, we take our results as showing that misperceptions of survival probabilities are as important for explaining behavior.

Our results are important for government policy in relation to annuities and retirement provision more generally. While resources do exist to inform individuals about their life expectancy,<sup>19</sup> the divergence between self-assessed and objective life expectancies and the associated implications for annuity purchases leave a role for larger policy interventions to improve households’ understanding of the length of retirement that they might have to fund. As individuals approach retirement with increasingly large shares of their wealth in non-annuitized form, ensuring that individuals adequately understand their longevity in this way will become only more pressing.

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<sup>19</sup>For example, this online life expectancy calculator from the Social Security Administration: <https://www.ssa.gov/planners/lifeexpectancy.html>.



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## A Details of further analysis and tests from Section 2

### A.1 Analysis of “50%” answers

The following table details the distribution of individuals by the number of times they answered “50%” to questions in the expectations module other than the survival questions. The reporting patterns in the 20.5% of interviews in which individuals answered “50%” to the first survival question are very similar to distribution of responses amongst the whole sample.

Table 1: Distribution of number of expectations questions to which individuals answer “50%”

	Number of “50%” answers given					
	0	1	2	3	4	5
All individuals	55.42%	35.34%	8.07%	1.05%	0.12%	0.00%
Answered “50%” to 1st survival Q	52.09%	35.78%	10.20%	1.62%	0.30%	0.01%

Note: Other probability questions include those related to the probability of moving out of ones home in the future, of being in work in a number of years time, of having insufficient financial resources to meet needs at some point in the future, of it raining tomorrow and of giving and receiving an inheritance. Source: ELSA waves 17. 66,210 interviews from 16,345 unique individuals.

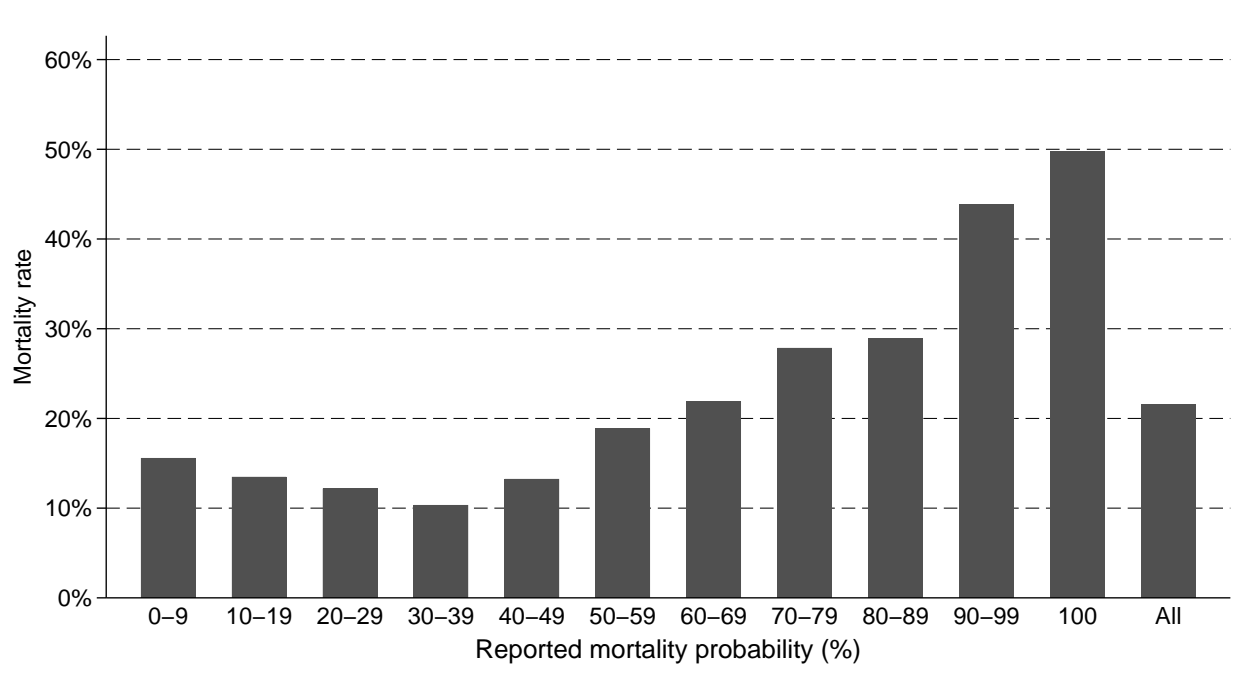
### A.2 Correlation of subjective reports with risk factors, new information, subsequent mortality and holdings of life insurance

Table 2 details the results of a regression of an individual’s answer to the first survival question they are asked on a range of risk factors as well as a full set of wave by single-year-of-age dummy interactions. Results are split by gender and by whether or not self-reported health is controlled for. The coefficients reported are in percentage point deviations. For example, a male current smoker reports 8.3 percentage points lower chance of survival to an age 11–15 years ahead of their current age, on average, when compared to current non-smokers (6.8 percentage points when controlling for self-reported health).

Table 3 reports the results of a fixed effects regression of individuals' answers to the survival expectations questions on a range of dummies for whether or not they have received a new diagnosis of a health condition since their last interview. We control linearly for age. We find that individuals do respond to new information by revising their survival expectations. For example, a new diagnosis of cancer or a case of a stroke cause large and statistically significant downward revisions in survival expectations of 4 and 6 percentage points, respectively.

Figure 8 shows the 10 year mortality rates of individuals according to their answer to the first survival question. We use the linked death records which give us a 10-year horizon for those interviewed in wave 1 of ELSA. We see clear differences in mortality rates according to stated expectations. These differences are statistically significant. The correlation between expected and actual mortality remains even when we control for age and sex-specific average mortality risk and the range of health factors controlled for in the previous regressions.

Figure 8: 10 year mortality rates by answer to survival question



Note: Reported probability of death is 100 minus the reported probability of survival in the first survival question the individual is asked. Source: ELSA wave 1 and linked death records. 11,502 individuals.

Finally, we examine the correlation of subjective expectations and purchases of life insurance, a product that is analogous to selling an annuity. In the ELSA questionnaire, individuals are asked whether they hold life insurance and the amount that would be paid out to others in the event of their death. We use this information, along with information about financial wealth, pension wealth and housing wealth, to generate a variable that puts the size of the (potential) insurance payout in the context of the net worth of the respondents. We call this variable the ‘share’ of wealth held as life insurance and define it as:

$$Share_i \equiv \frac{Payout_i \times Die10}{Payout_i \times Die10 + PW_i + FW_i + HW_i} \quad (9)$$

where  $Payout_i$  is the amount that will be paid out in the event of the individual’s death and  $PW_i$ ,  $FW_i$  and  $HW_i$  are the individual’s pension wealth, financial wealth and housing wealth, respectively.  $Die10$  is the individual’s ‘objective’ probability of dying within the next 10 years. The idea of multiplying the life insurance payout by this number is to capture the expected life insurance payout. In the absence of information about the term of individuals’ life insurance products, we assume a term of 10 years.<sup>20</sup> In the ELSA sample, 31% of individuals hold some life insurance and the mean portfolio share is 7% (23% amongst those who have some life insurance). Life insurance is more prevalent at younger ages.

We use these constructed variables to run a set of OLS regressions where the dependent variable is the share of wealth held in life insurance and the independent variable is the subjective report. We control for the the interaction of the respondent’s sex with a full set of age dummies, dummy variables for whether the respondent has a partner and whether the respondent has children, and the interaction of their partner’s sex and a full set of dummies for their partner’s age. We show results with and without controls for the individual’s total wealth (equal to the sum of their wealth and expected life insurance payout). We focus on

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<sup>20</sup>We obtain very similar results if we instead assume a remaining term of 5 or 15 years and if, instead of discounting the payout by a probability, we allow it to enter in equation (9) undiscounted. This last measure would be the share of wealth accounted for by a life insurance payout if the respondent were to die immediately.

ages where life insurance, which is ordinarily purchased to insure earnings, is most relevant. We select respondents aged between 50 and 69 (though we get very similar results if we further restrict to those aged 50 to 59).

Table 4 presents the results. We see that a 1 percentage point increase in the subjective belief about the probability of survival is associated with a 0.043 percentage points lower portfolio share held in life insurance (0.036 when controlling for wealth), significant at the 1% level. Columns 3 and 4 show an equivalent set of results but where the sample includes only those who hold life insurance.

## **B Robustness of results from Section 4.2**

### **B.1 Robustness of main results to using ONS life tables without rescaling**

We show here results of the model in the case where we use the ONS life table survival curves without rescaling as our measure of ‘objective’ survival probabilities. Annuity rates are calculated using the unscaled ONS survival curves. All other details of the model are as given in Section 4.1. Figure 9 shows the model predictions.

### **B.2 Definition of model including utility from housing consumption**

In Table 5 we show the results from various further robustness checks. We describe one of these checks in more detail here by defining the model used. We run an alternative version of the model where homeowners receive utility from the consumption of housing services. We assume that individuals receive a per-period flow of housing services,  $h$  equal to 4% of the value of their primary house, as reported in the ELSA data. This value is fixed in real

Figure 9: Percentage of individuals annuitizing at each parameter combination (unscaled ONS life tables)

(a) Objectively-measured expectations										(b) Subjectively-elicited expectations									
Discount factor	Coefficient of relative risk aversion									Discount factor	Coefficient of relative risk aversion								
	1	1.5	2	2.5	3	3.5	4	4.5	5		1	1.5	2	2.5	3	3.5	4	4.5	5
<b>0.960</b>	28%	40%	48%	54%	58%	61%	63%	65%	67%	<b>0.960</b>	20%	30%	36%	41%	45%	48%	51%	53%	55%
<b>0.965</b>	34%	46%	53%	58%	62%	64%	67%	68%	70%	<b>0.965</b>	24%	33%	40%	44%	48%	51%	54%	56%	58%
<b>0.970</b>	40%	51%	58%	62%	65%	68%	70%	72%	73%	<b>0.970</b>	29%	37%	43%	48%	51%	54%	56%	58%	60%
<b>0.975</b>	48%	57%	63%	67%	70%	72%	74%	76%	77%	<b>0.975</b>	33%	42%	47%	51%	55%	57%	59%	61%	63%
<b>0.980</b>	56%	64%	69%	72%	75%	77%	79%	80%	81%	<b>0.980</b>	38%	46%	51%	55%	58%	60%	62%	64%	66%
<b>0.985</b>	64%	71%	75%	78%	81%	82%	83%	85%	86%	<b>0.985</b>	44%	51%	55%	59%	61%	64%	66%	67%	69%
<b>0.990</b>	74%	80%	83%	85%	86%	88%	88%	89%	90%	<b>0.990</b>	50%	56%	60%	63%	65%	67%	69%	70%	71%
<b>0.995</b>	87%	89%	91%	92%	92%	93%	93%	94%	94%	<b>0.995</b>	55%	61%	64%	67%	69%	71%	72%	73%	74%
<b>1.000</b>	100%	100%	100%	100%	100%	100%	100%	100%	100%	<b>1.000</b>	61%	65%	68%	70%	72%	73%	75%	76%	77%

Source: Model predictions using ELSA waves 3–5 and ONS 2014-based cohort life tables for England and Wales. 2,848 observations.

terms in future periods. Their utility function is of the form:

$$U = \sum_{t=0}^{110-z} \beta^t S_i(z+t) \frac{(c_t + h)^{1-\gamma}}{1-\gamma} \quad (10)$$

All other details of the model are as given in Section 4.1.

### B.3 Further robustness of main results

We here show further robustness of our main results on the average annuitization rate under objective expectations, subjective expectations and objective expectations with a 17.5% rate reduction (i.e. the results in Figures 6 and 7). For brevity, we select one central parameter combination,  $\beta = 0.98$  and  $\gamma = 3$ . We show robustness to various sample selection choices: (1) removal of individuals who respond “100%” to one or more questions, (2) removal of individuals who respond “0%” to one or more questions, (3) removal of individuals who respond “50%” to one or more questions, (4) removal of individuals who respond “0%,”

“100%” or “50%” to one or more questions. We show robustness to (5) not weighting our results using the ELSA sample weights. We show results from (6) an alternative model in which individuals can choose only whether to annuitize the entirety of their wealth or none of it rather than being able to annuitize any fraction of their wealth. We show results (7) weighting the individual annuitization rates by individuals’ initial wealth i.e. showing the percentage of aggregate wealth that is annuitized. Finally, we show (8) the results from a model in which there is a utility flow from housing (as described in Section B.2). These robustness checks are shown in Table 5, with the baseline results from Figure 6 shown in the first row for comparison.



Table 2: Relationship between stated survival probabilities and risk factors

	Ex. self-reported health		Inc. self-reported health	
	Male	Female	Male	Female
<b>Smoking (relative to non-smoker)</b>				
Former occasional smoker	-3.1***	-0.6	-2.8***	-0.0
Former regular smoker	-1.6***	-0.2	-1.1**	0.2
Former smoker, DK frequency	-2.8**	-1.8	-2.2**	-1.3
Current smoker	-8.3***	-7.3***	-6.8***	-6.1***
<b>Alcohol consumption (relative to once or twice a month)</b>				
At least 3-4 days a week	-0.1	0.5	-0.9	0.1
Once or twice a week	0.0	0.2	-0.2	-0.1
A few times a year	-0.9	-1.1	-0.3	-0.7
Not at all	-2.2**	-1.4*	-1.2	-0.4
<b>Age mother died (relative to 60-64)</b>				
Under 50	2.4*	1.6	2.5*	2.0
50-59	1.2	-2.5*	1.0	-2.4*
60-64	0.0	0.0	0.0	0.0
65-69	3.4**	-1.7	3.3**	-1.3
70-74	2.2*	-0.5	2.1*	-0.4
75-79	2.5**	0.7	2.1*	0.8
80-84	3.6***	2.7***	3.2***	2.9***
85+	6.5***	7.4***	6.0***	7.5***
<b>Age father died (relative to 60-64)</b>				
Under 50	1.6	-0.0	1.9*	0.1
50-59	-0.1	0.5	0.3	0.7
60-64	0.0	0.0	0.0	0.0
65-69	0.7	-1.6*	0.7	-1.1
70-74	2.6***	1.7*	2.6***	1.7*
75-79	4.3***	3.0***	4.3***	3.0***
80-84	4.4***	2.5***	4.2***	2.5***
85+	7.7***	4.6***	7.6***	4.5***
<b>Health Conditions</b>				
Hypertension	-2.2***	-2.7***	-0.8*	-1.6***
Heart condition	-3.3***	-2.7***	-1.7**	-1.7**
Stroke	-2.4**	-0.3	-0.3	1.5
Cancer	-6.5***	-3.5***	-4.4***	-2.2***

Note: Coefficients represent percentage point deviations in mean response. Statistical significance at the 5%/1%/0.1% level is denoted by \*/\*\*/\*\*\*. Standard errors are clustered at the individual level. Other control variables, for which coefficients are not reported, are whether in a couple, income and wealth quintile, education level, whether working and dummy variables for whether diagnosed with Alzheimers, angina, arthritis, diabetes, lung disease, osteoporosis, Parkinsons and psychiatric disorders, whether the individual is white or non-white and a full set of dummy variables for each single year-of-age and wave interaction. Source: ELSA waves 17. 55,581 observations of 14,481 unique individuals.

Table 3: Revision to survival expectations following diagnosis with major health conditions

	1st survival question	2nd survival question
Alzheimer's Disease	-4.4	-18.5
Cancer	-4.4***	-3.2*
Dementia	3.1	5.4
Heart Attack	-3.5*	-3.1
Lung Disease	-1.5	1.0
Parkinson's Disease	-2.5	-6.8*
Psychiatric problems	-2.6*	-1.3
Stroke	-6.5***	-5.7**
Observations	48,917	22,926

Note: Coefficients represent percentage point deviations in mean response. Statistical significance at the 5%/1%/0.1% level is denoted by \*/\*\*/\*\*\*. Standard errors are clustered at the individual level. Source: ELSA waves 37. 48,917 observations of 13,811 unique individuals (22,926 observations of 8,135 unique individuals for the second question).

Table 4: Association of subjective expectations and life insurance holdings

	(1)	(2)	(3)	(4)
Subjective report	-0.0434*** (0.00893)	-0.0361*** (0.00888)	-0.119*** (0.0250)	-0.0984*** (0.0244)
Controls for wealth	No	Yes	No	Yes
Has life insurance	No	No	Yes	Yes
Observations	14915	14915	5103	5103

Note: Statistical significance at the 10%/5%/1% level is denoted by \*/\*\*/\*\*\*. Standard errors are clustered at the individual level. Source: ELSA waves 35.

Table 5: Annuitization rates under objectively-measured expectations, subjectively-elicited expectations and objective expectations with a rate reduction, under alternative sample selection, weighting and modelling assumptions, with model parameters  $\beta = 0.98$  and  $\gamma = 3$

Specification	Objective	Subjective	Objective + rate reduction
Baseline results	69	43	38
No “100%”s	70	45	38
No “0%”s	70	45	39
No “50%”s	70	45	39
No “0%”s, “50%”s or “100%”s	72	51	40
Unweighted	70	44	39
‘Discrete’ annuitization choice	76	44	36
Weighted by initial wealth	90	68	67
Utility flow from housing	56	26	17

Source: Model predictions using ELSA waves 3–5 and ONS 2014-based cohort life tables for England and Wales. 2,848 observations.

## C Computational Appendix

We solve the model numerically. For each individual  $i$ , the model outlined in equation (8) can be expressed recursively. We outline this below, first focusing on the periods after the age at which individuals are observed in the data. We denote this age as 0. In these periods the only decision they face is a consumption and saving choice. We then outline the problem at age 0 where individuals make an annuitization choice as well as consumption and saving choice.

### C.1 Recursive Form of the Model

#### C.1.1 Periods after annuitization decision has been made

At all ages  $t > 0$  the model can be expressed in recursive form as:

$$V_{it}(a_{it}, ann_i; p_i) = \max_{c_{it}} u(c_{it}) + \beta s_i(t+1) V_{it+1}(a_{it+1}, ann_i; p_i) \quad (11)$$

subject to the constraints given in equation (8) which we do not repeat here. Consumption in period  $t$  ( $c_{it}$ ) is a choice,  $u(\cdot)$  is a utility function,  $V(\cdot)$  is a value function, and assets ( $a_{it}$ ) and annuity income ( $ann_i$ ) are state variables.  $s_i(t+1)$  is the probability of individual  $i$  surviving to period  $t+1$  conditional on having survived to period  $t$  and could be either that calculated using life tables or could be that implied by their objective survival curve based on life tables or their subjective survival curve estimated from individual reports.  $p_i$  is the public pension income of individual  $i$ .

The fact that the public pension income stream and survival probabilities vary across individuals means that the value function differs across individuals. This implies that the value function must be solved for separately for each individual in our data. For notational convenience we suppress the  $i$  subscript for most of the rest of this Appendix.

By assuming that there is an age ( $T$ ) beyond which there is a zero probability of survival (110 in our application) we get  $s_{T+1} = 0$  and equation (11) for period  $T$  reduces to:

$$V_T(a_T, ann; p) = \max_{c_T} u(c_T) \quad (12)$$

and the function  $V_T$  can be obtained by maximizing the utility function subject to the budget constraint. Knowledge of  $V_T$  allows the maximization in the recursion for  $T - 1$  to be undertaken and for  $V_{T-1}$  to be obtained. This recursive procedure can be repeated for each period back to (and including) 1 which is the age immediately after the individual is observed in the data.

### C.1.2 Initial Period

The problem in period 0 differs from that in future periods as agents need to make an annuitization decision as well as well a consumption decision:

$$V_0(a_0; p) = \max_{c_0, b} u(c_0) + \beta s(1) V_1(a_1, ann; p) \quad (13)$$

where  $b$ , a choice variable, is the share of period 0 wealth annuitized and where the maximization is subject to the constraints given in equation (8). With knowledge of  $V_1(\cdot)$  in hand from the steps outlined in the previous subsection, the maximization in (13) can be undertaken. This allows us to obtain policy functions, in particular a function which relates initial wealth holdings to annuitization decisions for each individual. This is the function that yields the quantity of initial wealth annuitized for each individual in the data, which represent our central results. To be clear about what this object depends on we can write the policy function as  $b(a_{i0}; p_i, \mathbf{s}_i; \beta, \gamma)$  where we making explicit that the proportion annuitized depends on i) initial wealth  $a_{i0}$ , ii) individual circumstances: public pension income ( $p_i$ ) and the individual's entire survival curve ( $\mathbf{s}^i$ ), and iii) the values of  $\beta$  and  $\gamma$  which we vary in our application.

## C.2 Computational Implementation

In the absence of an analytical solution to the agents' problem, we solve for value functions in equation (1) and (2) numerically. We take a standard approach, by discretizing the state variables and solving for the value function at those discrete points. We define a grid of 50 points for assets from 0 to  $a_{i0}$  (initial assets for individual  $i$ ). Annuity income is placed on a grid of 10 points which are equally spaced from 0 to  $\theta_i a_{i0}$  where  $\theta_i$  is the annuity rate faced by individual  $i$ . We restrict the annuity choice to be on this grid of 10 shares (that is, individuals can annuitize 0% of their wealth, 100% of their wealth or can choose any of other 8 shares equally spaced between the two extremes). Consumption is a continuous choice, obtained at each point in the discretized state space using golden section search.<sup>21</sup>

We have confirmed that our results are not sensitive to the choices we have made over the manner of discretization and the number of grid points used. As an example, when expressed to two decimal places, our main result (from Figure 6) for the average share of wealth annuitized for  $\beta = 0.98, \gamma = 3$  is 69.33% under objectively-measured survival expectations and 42.51% under subjectively-measured survival expectations. When we quintuple each of the number of asset grid points, annuity income grid points and the grid of annuity share options to 250, 50 and 50 respectively, the shares would become 70.07% and 42.46%, respectively.

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<sup>21</sup>See, for example, Miranda and Fackler (2002).