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Focal pricing constraints and pass-through of input cost changes

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Focal Pricing Constraints and Pass-Through of Input Cost Changes

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Abstract

Pass-through rates are relevant in a variety of contexts, such as estimating antitrust damages. It is often asserted that focal pricing, the practice of charging only special prices, e.g. ending in 9s, reduces the degree of pass-through in an industry. This claim has had serious consequences; for example, it has contributed to the dismissal of high-profile antitrust cases. However, it is not grounded in economic theory or evidence. I prove that, in a simple but general framework, expected pass-through is unchanged by the presence of focal pricing constraints. Therefore, the fact that an industry is characterised by focal pricing constraints does not entail that it will also be characterised by a low pass-through rate.

Keywords: pass-through; focal pricing; discrete pricing; antitrust damages

1 Introduction

A firm sells goods to consumers and incurs costs in the form of input costs and sales taxes. The degree to which changes in these costs are passed through to changes in the prices paid by consumers is relevant in a variety of contexts, including tax incidence, merger control,

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and antitrust cases.[1] The degree of pass-through is affected by numerous industry-specific characteristics, including the nature of competition between firms, the curvature of demand, returns to scale, and any frictions in setting prices. There is a well-developed literature on this topic, e.g. Bulow and Pfleiderer (1983) discuss the theoretical impact of different demand curvatures on pass-through rates in the context of tax incidence.

This paper aims to address an issue that has received very little academic attention to date: the impact of focal pricing constraints on pass-through rates. This question is of substantial importance, especially in antitrust cases, where there is a widespread misconception that focal pricing leads to little or no pass-through. This misconception has been deployed in a variety of antitrust cases in multiple jurisdictions and has contributed to the dismissal of high-profile cases.[2] This paper aims to fill the gap in the literature that has allowed this misconception to be deployed unchecked in cases worth hundreds of millions of dollars. I formally prove that the expected pass-through rate is unchanged by the presence of focal pricing constraints, under a simple but general framework. I refer to this as the Irrelevance Theorem. The Irrelevance Theorem holds exactly under several standard models of competition between firms, and different demand functions, and as an approximation in other cases.

Focal pricing is a widely observed phenomenon, consisting in firms only charging prices with specific characteristics. In practice, focal prices fall into two main classes. The first are prices with 9s in the last digits. These are widespread, and can be explained through models of consumer behaviour where consumers effectively round down prices when making decisions, e.g. due to left-digit bias (Levy et al. (2011), Snir et al. (2017), Strulov-Shlain (2023), Basu (1997)). For instance, any price between \$100 and \$109.99 may be considered equivalent to \$100 in the consumer's decision-making process. In this context, we can conceptualise the demand function as taking a stepwise form, and it is generally optimal for firms to choose prices from a discrete set of prices, ending in 9s. The second class are 'convenient' prices, which are multiples of certain cash denominations. These are particularly widespread for frequently purchased goods paid in cash in high-traffic transactions (Knotek (2008) and Knotek (2011)). When the non-monetary cost of obtaining exact change is high, it may be optimal for firms to charge only specific prices, for instance either \$10 or \$11 rather than

[1] If a \$2 increase in input costs leads to a \$2 increase in prices, then there is 'complete' pass-through, or an 100% pass-through rate. Depending on the context, pass-through rates can be below, above, or at 100%.

[2] For example, see United States District Court Northern District of California *In re Lithium Ion Batteries Antitrust Litigation*, Case No.: 13-MD-2420 YGR (2018, March); United States District Court Northern District of California San Jose Division *In re: Qualcomm Antitrust Litigation, Defendant Qualcomm Incorporated's Opposition to Plaintiff's Motion for Class Action*, Case No. 5:17-md-02773-LHK-NMC (2018, September)

something in between.[3]

These types of focal pricing constraints are distinct from two other closely related phenomena. Firstly, I use the terminology 'focal pricing constraints'[4] to distinguish my setting from the possibility that oligopolists are choosing to charge certain special prices in an attempt to promote tacit collusion among themselves (see e.g. Scherr (1981)). There is less clear evidence that this is an empirically relevant phenomenon, so I focus on the more practically relevant scenario. The other related, but distinct, phenomenon is menu costs. Menu costs involve an actual cost to changing prices, which is not present with focal pricing. These could be monetary costs of actually changing displayed prices, though these are less relevant in a digital economy, or they may emerge if consumers are boundedly rational (e.g. see Spiegler (2011) for models of reference dependent, loss averse consumers). Both focal pricing and menu costs will lead to price rigidity (stickiness), i.e. prices being changed infrequently, and typically not in very small increments. However, these phenomena are not observationally equivalent. For instance, under menu costs firms will set prices from the whole range of feasible prices, while under focal pricing we will observe only certain 'special' prices.

The impact of menu costs on pass-through has already been studied in the context of money neutrality. Due to the cost of adjusting prices, each period some firms do not adjust prices. However, the firms who do adjust prices choose to do so notwithstanding menu costs because they wish to adjust their prices substantially (Caplin and Spulber (1987), Carvalho and Schwartzman (2015)). Empirically, these contrasting effects may mostly cancel each other out, e.g. Nakamura and Zerom (2010) find that menu costs introduce a time lag in pass-through, but have a negligible impact on long-run pass-through rates.[5] I note that while there are similar economic forces at play at an intuitive level, from a modelling perspective the setting with menu costs is very different from the setting with focal prices, and hence the existing literature on menu costs does not speak to the question of how focal pricing impacts pass-through.

The question of whether, and how, focal pricing affects pass-through is interesting for a variety of policy-relevant reasons. Examples include understanding the welfare effects of

[3] I note that cash payments have been declining in several economies, but remain common in many others. Moreover, even when payments are made digitally, almost all consumer-facing firms employ a related variety of focal pricing: charging only prices that are multiples of the smallest available unit e.g. pennies.

[4] Even though technically firms are able to price continuously, they choose not to do so because of externally imposed constraints.

[5] Nakamura and Zerom (2010) investigate different potential explanations for why there is incomplete (sub-100%) pass-through of exchange rate fluctuations. Among other candidates, they include price rigidity (which in their model is explained through menu costs), finding little impact. Instead, they find that exposure to local costs plays an important role in dampening the impact of exchange rate fluctuations, and that curvature of demand also explains the observed pass-through patterns.

consumers having left-digit-bias or estimating optimal sales tax levels. For instance, Conlon and Rao (2020) show that in the presence of price rigidities (both menu costs and focal prices) the incidence of taxes on spirits varies widely. While the framework I develop may therefore be helpful in a variety of contexts, this paper is motivated by an application to antitrust cases, and that is the focus of the discussion throughout.

Courts awarding antitrust damages rely on estimates of pass-through to quantify fair compensation. For example, in indirect purchaser class actions it is alleged that the claimants (end-consumers) paid inflated prices for end-products due to abusive conduct by upstream firms (producers of an input). In order to estimate appropriate damages, it is necessary to estimate the extent to which inflated input costs were passed through to consumers, instead of being absorbed by downstream firms (purchasers of the input and sellers of the end-products). In multiple high-profile antitrust cases, it has been claimed that the presence of focal pricing in the downstream industry implies there will be very little, if any, pass-through of input cost changes. The argument goes as follows: if downstream firms round to certain special prices, they are unlikely to adjust those prices in response to relatively small input cost changes.

For example, focal pricing was one of the reasons why the court struck out *re Lithium Ion Batteries antitrust litigation*. The defendants alleged that "focal point pricing is prevalent in the pricing of products within the class definition [notebook computers], and will result in no pass-through when a small cost change—such as the estimated \$2.16 overcharge for a notebook computer battery here—in presence of focal points that are wider apart than the cost difference itself."[6] A similar argument has been made in several other high-profile cases, for example by Qualcomm in antitrust litigation in several jurisdictions.[7]

While this argument has intuitive appeal, it fails to recognise the other side of the coin: in the presence of focal pricing constraints, some prices will be over-adjusted if the input cost change leads to a jump from one focal price to another. Consider the example of laptops and let their prices optimally end in 9s. Then it is true that an overcharge on batteries of \$2.16 could result in no impact on the focal price charged, but it could also lead to a \$10 jump (or more).

Empirical studies have found that, with focal pricing, price changes are less frequent, but bigger (see Levy et al. (2011) and Conlon and Rao (2020)). However, to the author's know-

[6] United States District Court Northern District of California *In re Lithium Ion Batteries Antitrust Litigation*, Case No.: 13-MD-2420 YGR (2018, March).

[7] For example, see United States District Court Northern District of California San Jose Division *In re: Qualcomm Antitrust Litigation, Defendant Qualcomm Incorporated's Opposition to Plaintiff's Motion for Class Action*, Case No. 5:17-md-02773-LHK-NMC (2018, September).

ledge only Alexandrov (2013) has set out a conceptual argument why focal pricing need not imply low pass-through. This paper builds on Alexandrov (2013) in a few different directions. Firstly, Alexandrov (2013) only considers the case of a small cost increase, which can lead to at most one jump in focal price, while my theorem holds for any cost change, and any number of potential jumps in focal price. Secondly, Alexandrov (2013) states that expected pass-through will be approximately unaffected in the case of a monopolist facing a log-concave demand curve. Instead, I prove that expected pass-through will be exactly unaffected under my set of assumptions. I show that these assumptions hold for any demand curve under perfect competition, for the linear and logarithmic demand functions under monopolistic competition, and linear demand under differentiated price competition. I also prove that, under monopolistic competition, expected pass-through is approximately unaffected for any well-behaved demand curve. I provide examples of what this approximate result looks like numerically in the case of isoelastic, monomial, and logit demand functions. Thirdly, Alexandrov (2013) makes a high-level statement that expected pass-through will also be approximately unaffected with two or more oligopolists. However, this discussion is informal and does not mention how the author contends with issues, such as multiplicity of equilibria, which arise even in simple oligopoly models with discrete pricing. Instead, this paper formally proves that expected pass-through is exactly unaffected in the widely used differentiated Bertrand model with linear demand, under a simple assumption on equilibrium selection. Under alternative assumptions on equilibrium selection, I prove that expected pass-through is affected locally, but not globally.

The substantial gap in the literature, which this paper intends to fill, has had substantial economic consequences, including the incorrect dismissal of class actions covering millions of claimants and involving hundreds of millions of dollars in damages. These dismissals, grounded in a misconception, have a direct impact on the claimants, who do not receive fair damages, and an indirect impact on firms, who are more likely to engage in abusive behaviour if they are likely to avoid paying for damages.[8]

To convincingly debunk this misconception, I develop a general theoretical framework, under which I prove that focal pricing has no impact on expected pass-through rates. In this way, this paper provides a clear and rigorous rationale for antitrust courts not to assume low

[8] Note that, if abusive upstream firms successfully argue that there was no pass-through in consumer class actions, then that may increase the risk of them having to pay damages to downstream firms (who would have absorbed the inflated cost in the absence of pass-through). However, for that to happen, the downstream firms would have to start their own litigation against the upstream firms. Downstream firms may be unwilling to enter into litigation with their input producers, especially if those producers have market power and have already engaged in abusive practices. For instance, they may fear retribution in the form of disruption to their input supply.

pass-through in the presence of focal pricing, and instead to obtain empirical estimates of pass-through from economic experts. Where feasible, context-specific empirical estimates are best practice to achieve accurate damages quantification. When this is not feasible, e.g. due to resource constraints (including in the early stages of antitrust cases), we can rely on estimated pass-through rates in similar industries, even if they are not characterised by focal pricing themselves. The Irrelevance Theorem shows that these estimates are unbiased, as long as the industries are similar in relevant dimensions such as curvature of demand (even if they are not characterised by similar focal pricing practices).

Note that the claim being made is not that focal pricing *cannot* impact pass-through rates; but rather that there is *no general reason* to expect pass-through rates to be lower in the presence of focal pricing constraints. If we knew exactly how an industry worked, in full detail, we might be able to say that a specific cost overcharge was not passed through to downstream consumers, or conversely that it was passed through at a higher than 100% rate (similarly to what Conlon and Rao (2020) do for tax incidence).[9] However, it is almost impossible for this sort of information to be available in detailed and reliable form at the outset of a court case, even though it may be obtained through a long process of disclosure and economic analysis.

In addition to expected pass-through, I also analyse the distribution of pass-through across different purchases. My model implies that focal pricing generally increases the dispersion in the pass-through rate (consistent with the empirical findings of Conlon and Rao (2020) for spirits). There may be substantial dispersion in pass-through rates if claimants purchased few, or a single, product(s). In this case, it is likely that different pass-through rates apply to different claimants. Instead, in situations where consumers made many purchases at different points in time, the average pass-through rate is a reasonable estimate of the rate that applies to all claimants.

The impact of focal pricing on the dispersion of pass-through is particularly relevant in class actions because it can lead to two challenges. Firstly, the class certification stage often requires demonstrating sufficient homogeneity of the class. This involves different requirements in different jurisdictions but broadly covers the idea that claimants must have been injured in a sufficiently similar manner, and that similar methods may be used to estimate the damages to be awarded to different claimants. The presence of focal pricing may generate some difficulties in meeting the homogeneity requirement. Secondly, class action regimes sometimes require that the quantification of damages be performed separately for sub-classes

[9] Note that the work of Conlon and Rao (2020), while related to this paper, does not answer the question of whether pass-through is lower on average in the presence of focal pricing constraints.

which were harmed to substantially different extents. The presence of focal pricing may give rise to such sub-classes, and hence create the need for more complex quantification.

This paper is set out as follows. First, in section 2.1, I outline a general model to assess the impact of focal pricing constraints on pass-through. Then, in section 2.2, I prove the Irrelevance Theorem under a set of assumptions, which turn out to be quite general. In section 2.3, I discuss other features of the distribution of pass-through under this set of assumptions. In section 3, I analyse different models of competition, and different curvatures of the demand function. I discuss when the assumptions required for the Irrelevance Theorem hold, and when they do not, I show that it remains a good approximation. In section 4, I conclude with a brief discussion of my findings and their relevance to antitrust damages class actions.

2 A general framework to assess the impact of focal pricing constraints on pass-through

2.1 The framework

Consider a firm facing constant marginal costs c and choosing prices to maximise profits. Here, I remain agnostic about the number of firms in the market, the type of competition, and curvature of demand. I explore specific examples in section 3. To understand how focal pricing constraints affect pass-through, I compare pass-through rates under two scenarios: an unconstrained optimisation problem and a constrained optimisation problem.

In the unconstrained optimisation problem, the firm has a strategy $p^u(c)$ mapping any possible cost level to an optimal price.

Now consider the case in which there are focal pricing constraints in the market. In this case, prices within a certain interval (defined by cut-offs τ) will all elicit the same constrained demand q^c as a corresponding focal price:

$$\tau_{i-1} < p \leq \tau_i \quad \Rightarrow \quad q^c\left(p\right) = q^c\left(f_i\right)$$

For instance, with left-digit bias, any price in the interval $\{\$50, \$59.99\}$ may be read by consumers as if it were \$50 and hence yields the same demand as the focal price \$59.99. In this case, we have $\tau_{i-1} = 49.99$, $\tau_i = 59.99$ and $f_i = 59.99$. Now consider an example with convenient prices. Perhaps any price in the interval $\{\$10.01, \$20\}$ is considered equivalent to \$20 by consumers. Here we have $\tau_{i-1} = 10$, $\tau_i = 20$ and $f_i = 20$. In both cases, constrained demand at a certain price is the same as constrained demand at the next focal price up:

$$f_{i-1} < p \leq f_i \quad \Rightarrow \quad q^c(p) = q^c(f_i)$$

In general, it will be optimal for the firm to charge only focal prices because non-focal prices yield the same demand at a lower per-unit profit. To write the new firm optimisation problem, it is useful to relate the constrained demand function to the unconstrained demand function:

$$q^c(f_i) = q^u(r(f_i)) \quad f_{i-1} < r(f_i) \leq f_i$$

All prices strictly greater than f_{i-1} but weakly less than f_i are mapped by consumers onto the same demand, which must equal the value of unconstrained demand evaluated at a price $r(f_i)$ within this range. Under rational expectations:

$$r(f_i) = f_i$$

To see why, consider the example with left-digit bias. Naively, we might expect $r(f_i) = \lfloor f_i \rfloor$[10] since the psychological mechanism involves consumers focusing on left digits. This would mean that any price in the range $\{\$50, \$59.99\}$, including the focal price \$59.99, would not only be read as if it were \$50, but would also yield $q^u(50)$. However, for the firm, any price in this range other than \$59.99 is strictly dominated by \$59.99, since it yields the same demand at a higher price. Hence, if a consumer is charged a price within the range $\{\$50, \$59.99\}$, it will always be \$59.99. Therefore, consumers would be systematically over-buying under $r(f_i) = \lfloor f_i \rfloor$ and in fact under any $r(f_i) < f_i$. Therefore, under left-digit bias and rational expectations, we must have $r(f_i) = f_i$. The same conclusion holds for convenient prices under rational expectations. Therefore, this paper focuses on the case where constrained demand evaluated at a specific price is equal to unconstrained demand evaluated at the closest focal price up:[11]

[10] In this context, the rounding-down operator $\lfloor f_i \rfloor$ should be defined according to the nature of the left-digit bias that affects consumers in that industry (rounding down to nearest units, tens, hundreds...).

[11] The analysis in this paper can be extended to cases where $r(f_i) \neq f_i$.

$$f_{i-1} < p \leq f_i \quad \Rightarrow \quad q^c(p) = q^u(f_i)$$

In this constrained setting, the firm will always charge a focal price. Which focal price it will charge depends on its marginal cost, and on the demand function it faces. Charging a lower (higher) focal price leads to higher (lower) demand, but a lower (higher) per-unit profit. Therefore, while in the unconstrained problem the firm's strategy maps any specific cost c to an optimal price $p^u(c)$, in the constrained problem the firm maps costs within certain intervals to focal prices. These intervals are defined by cost thresholds, or cut-offs, t:

$$t_{i-1} < c \leq t_i \quad \Rightarrow \quad p^c = f_i$$

In the discussion of the Irrelevance Theorem, it will be useful to refer to the cost level at which a given focal price would be unconstrained optimal: $f_i = p^u(\chi_i)$. This allows us to define the gap between costs at which consecutive focal prices are unconstrained optimal: $\chi_{i+1} - \chi_i$.

Before delving into the Irrelevance Theorem, I formalise the question the Theorem aims to answer. Let the firm's input cost increase from c to $c + \Delta$. The pass-through rate of an increase in marginal cost of size Δ in the unconstrained case is:

$$\frac{p^u(c+\Delta) - p^u(c)}{\Delta}$$

The pass-through rate in the constrained case is:

$$\frac{p^c(c+\Delta) - p^c(c)}{\Delta}$$

The question at hand is how the unconstrained pass-through rate compares to the expected pass-through rate under focal pricing constraints. I answer this question formally under the following set of assumptions.

Assumption Set A

1. **Regularity condition** for focal prices. The distance between each consecutive focal price is consistent: $f_{i+1} - f_i = G$.

2. **Equal Spacing condition**.

   (a) The unconstrained pass-through rate is constant and is $\frac{1}{\theta}$. Combined with the Regularity condition, this implies that the distance between costs at which consecutive focal prices are unconstrained optimal is constant and is:[12]

   $$\chi_{i+1} - \chi_i = \theta G \quad \forall i$$

   A value of $\theta = 1$ corresponds to complete pass-through of 100%, $\theta > 1$ to incomplete pass-through, and $\theta < 1$ to pass-through above 100%.

   (b) The distance between consecutive cost thresholds is also constant, and equal to the distance between costs at which consecutive focal prices are unconstrained optimal:

   $$t_{i+1} - t_i = \theta G \quad \forall i$$

3. **Uniformity condition**. If we observe a firm charging the focal price f_i we can infer that $c \sim U\left(t_{i-1}, t_i\right)$.

Discussion of Assumption Set A

The Regularity condition is likely to cover the vast majority of real-life cases which, as discussed above, involve consistently rounding to prices ending in specific digits. Regular spacing of focal prices is a convenient modelling assumption, but it is not central to the Irrelevance Theorem. It is possible to obtain an approximate version of the Theorem while relaxing this assumption and jointly relaxing the Equal Spacing condition to require that the (variable) gaps between cost thresholds are approximately the same as the (variable) gaps between costs at which consecutive price thresholds are unconstrained optimal.

The Equal Spacing condition is required for the Irrelevance Theorem to hold exactly, but it can be relaxed and the Theorem still holds as an approximation.

Part (a) of the assumption is a frequently made simplifying assumption that unconstrained pass-through rates are constant. Under perfect competition, the unconstrained pass-through rate is constant for any demand function. The class of demand functions which entail constant unconstrained pass-through rates under monopolistic competition is set out in Bulow and Pfleiderer (1983), and as discussed in section 3 includes several widely used functional

[12] By the definition of χ_i, we can write the pass-through rate of an input cost increase from χ_i to χ_{i+1} as: $\frac{f_{i+1}-f_i}{\chi_{i+1}-\chi_i} = \frac{G}{\chi_{i+1}-\chi_i} = \frac{1}{\theta}$. Rearranging, we obtain: $\chi_{i+1} - \chi_i = \theta G$.

forms. In the differentiated Bertrand model, unconstrained pass-through is constant under linear demand.

While constant unconstrained pass-through is a frequently made assumption, it is also true that several demand functions give rise to non-constant unconstrained pass-through rates. This includes demand functions like logit, which are widely used in empirical IO. In section 3.1.3, I discuss an example of a monopolist facing logit demand, in which case the Irrelevance Theorem still holds as an approximation.

Part (b) of the assumption additionally requires that, in the constrained version of the problem, the gap between consecutive cost thresholds is constant and the same as the gap between which focal prices are unconstrained optimal. This means that even faced with discrete pricing options, pass-through rates remain constant. In section 3, I show that this is the case for any demand function under perfect competition. Under monopolistic competition, I show that the Equal Spacing assumption holds exactly only for linear and logarithmic demand, but that it is a good approximation for other demand functions. I also show that the condition holds exactly in the differentiated Bertrand model with linear demand.

The Uniformity condition is necessary for the Irrelevance Theorem to hold – if it does not then the result needn't hold even approximately. By definition, if we observe a firm charging the focal price f_i we know that $t_{i-1} < c \leq t_i$. Moreover, in general we do not have sufficiently detailed and reliable information (prior to detailed empirical investigation) about where the marginal cost is located within this interval. We formalise this uncertainty as a uniform distribution over the interval.

The Uniformity condition is plausible in most realistic contexts where the Irrelevance Theorem may be usefully deployed, i.e. in situations where we have limited information about the context of interest. Consider the example of an antitrust damages class action. The claimants are end-consumers of a product which was made with an input which was sold at an inflated price by abusive upstream firms to downstream firms. Neither the claimants nor the defendants possess detailed information about the cost structure and pricing strategy of the downstream firms. Moreover, the downstream firms generally have no incentive to voluntarily disclose this sensitive information unless a court requires them to do so. Since they are not the ones accused of wrong-doing, obtaining disclosure is particularly hard. It is possible in some cases to request disclosure of certain documents. However, obtaining and processing these documents is typically a lengthy, resource-intensive and challenging process, including difficulties such as the redaction of crucial sensitive information. Interviewing people who work in the sector can also be challenging. This is partly because the abusive firm(s) are likely to command substantial influence in the sector, so that it is challenging to find reputable sources willing to testify against them in court. Even with full disclosure, we are very unlikely to be able to obtain reliable information on the full contingent pricing strategy of a firm, as it may well not exist in written form, or be subject to frequent discussion and alteration. Therefore, we are unlikely to know where the counterfactual marginal cost, without the alleged overcharge, was located relative to cost thresholds.

Hence, prior to detailed analysis (which is warranted by the Irrelevance Theorem) it is reasonable to assume the Uniformity condition (which allows us to prove the Irrelevance Theorem). Once we have conducted that detailed empirical analysis then we will have estimated, specifically for that case, both (i) whether counterfactual cost was close to any thresholds, and (ii) whether focal pricing increased, decreased, or did not affect pass-through. The first of these points is relevant to the second, because if counterfactual cost was close to the lower (upper) threshold, pass-through is likely decreased (increased) by focal pricing constraints.

The Uniformity condition may not hold if firms can easily adjust non-price characteristics (such as quality, pack-size, components included in a bundle...). Then we might expect that firms would adjust their marginal cost, by adjusting these other characteristics, so that the focal price being charged is as close to unconstrained optimal as possible. But then the reason to suspect focal pricing may dampen pass-through is moot: holistic pricing, accounting for non-price characteristics, is (almost) unconstrained, so that we would expect the holistic pass-through, capturing characteristic-adjusted prices, to be approximately the same regardless of the presence of focal pricing.

A more serious concern about the uniformity condition may emerge for products that, at the time of purchase, were affected by only a small number of small cost changes since their design. For some products, it may be possible to easily adjust non-price characteristics prior to release, but not after. We might then expect that at release, characteristics were chosen so that the constrained price was not too far from unconstrained optimal. This would likely be the case for a myopic firm, though it is less clear in the case of a firm anticipating future cost changes. If the constrained price was originally not too far from unconstrained optimal, then the cost would likely not be located very close to a cost threshold for switching to another focal price. Therefore, if the firm faced a single cost increase, much smaller than the gap between cost thresholds, it would likely result in no pass-through. If we have good reason to think this is the case, then the Uniformity condition clearly does not hold, nor does the Irrelevance Theorem. In this situation, a court might be justified in dismissing a case on the basis that there likely was no pass-through.

However, this situation is very unlikely to arise in practice, for several reasons. Firstly, it will not necessarily be clear a priori if a cost change was very small relative to the gap between cost thresholds. This is because a cost change being small relative to the gap *between focal prices* does not entail that it is small relative to the *gap between cost thresholds*. The latter is what matters to constrained pass-through, and it depends on the nature of competition and the curvature of demand as well as on the gap between focal prices. This information may not be known prior to some level of analysis.

Additionally, in many real-life cases, at the time of purchase there will have been multiple cost changes prior to, and/or after, the one of interest. When estimating the overcharge, what matters is the difference between the counterfactual price (with no overcharge) and the factual price (with overcharge) at the time of purchase. Even if the original cost was not close to thresholds, cost changes before the one of interest make it hard to know where the updated cost might have been relative to cost thresholds at the time of the overcharge. Moreover, even if the overcharge did not trigger an immediate price change, it may have been passed through at a later date, after one or more other cost changes. Both of these possibilities make it hard to know, before detailed analysis, whether the counterfactual cost was far or close to a threshold.

Moreover, a firm anticipating future cost changes may have chosen product characteristics at release to reduce the discrepancy between its constrained optimal price and its unconstrained optimal price *over time*, rather than at release. In that case, it is perfectly possible that the cost at release was in fact close to a cost threshold. Without substantial analysis, it would be hard to estimate the location of the counterfactual cost at the time of purchase relative to thresholds for switching focal price. Overall, while the distribution of costs may well not *actually* be uniform within cost thresholds, in the absence of further information, we can represent our uncertainty about the distribution itself as a uniform distribution.

2.2 The Irrelevance Theorem

Irrelevance Theorem *Under Assumption Set A, the expected pass-through rate is the same in the unconstrained setting, and in the setting with focal pricing constraints.*

Proof First consider the unconstrained case. By part (a) of the Equal Spacing condition, the pass-through is rate is $\frac{1}{\theta}$. Hence, an increase in the firm's input cost from c to $c + \Delta$

increases the charged price by:

$$p^u(c+x) - p^u(c) = \frac{\Delta}{\theta}$$

Now consider the constrained problem. By the Equal Spacing assumption, the price charged will jump by $\lfloor \frac{\Delta}{\theta G} \rfloor$ focal prices with certainty where $\lfloor \rfloor$ is the floor operator. If the change in input cost is weakly greater than the gap between cost thresholds, $\Delta \geq \theta G$, then the charged price is sure to increase. By the Regularity assumption, each of these jumps entails a price change of G. Therefore, the charged price will increase by $\lfloor \frac{\Delta}{\theta G} \rfloor G$ with certainty. Where $\Delta < \theta G$, we do not have certainty about any price jumps.

Additionally, by the Uniformity assumption and Equal Spacing assumption, there is a $\frac{\Delta mod \theta G}{\theta G}$ probability of a further jump in focal price (where *mod* is the modulo function). By the Regularity assumption, this further jump, if it occurs, would lead to an additional increase of G in the charged price.

In expectation, the input price increase therefore leads to the following change in the constrained optimal price charged:

$$E\left[p^c(c+\Delta) - p^c(c)\right] = \lfloor \frac{\Delta}{\theta G} \rfloor G + \frac{\Delta mod \theta G}{\theta G} G = G\left(\lfloor \frac{\Delta}{\theta G} \rfloor + \frac{\Delta mod \theta G}{\theta G}\right)$$

By definition of the modulo and floor operators:

$$E\left[p^c(c+\Delta) - p^c(c)\right] = G\frac{\Delta}{\theta G} = \frac{\Delta}{\theta} = p^u(c+x) - p^u(c)$$

The expected pass-through rate is $\frac{1}{\theta}$ regardless of the presence of focal pricing constraints. **QED.**

For example, if focal prices are spaced at \$10 intervals, the unconstrained pass-through rate is 100%, the gap between cost thresholds is \$10, and there is a \$12 increase in the price of an input then:

- The constrained price will certainly increase by at least \$10, because at least one cost threshold will be crossed regardless of the starting point relative to the thresholds.
- With probability $\frac{1}{5}$, two thresholds are crossed, so there is a \$20 increase in the constrained price.
- With probability $\frac{4}{5}$, only one threshold is crossed, so there is a \$10 increase in the constrained price.

- Hence, the expected constrained price increase is \$12. The expected constrained pass-through rate is 100%, which is the same as the unconstrained pass-through rate.

2.3 The distribution of pass-through

The expected value of pass-through is not the only metric we might be interested in. We may additionally be concerned with other features of its distribution, particularly its dispersion. In general, the presence of focal pricing increases the dispersion of pass-through. Consider an industry where, in the absence of focal pricing constraints, all firms have a pass-through rate of $\frac{1}{\theta}$. Then the price change associated to a cost increase of size Δ is always $\frac{\Delta}{\theta}$, and there is 0 variance in pass-through.

The introduction of focal pricing constraints increases the dispersion in pass-through rates. Let:

$$n\theta G < \Delta < (n+1)\theta G$$

where n is a weakly positive integer. For instance, if $n = 1$ then the overcharge is greater than the gap between cost thresholds, but less than twice this amount. The price of a product jumps by n focal prices, i.e. increases by nG, with probability $1 - \frac{\Delta mod \theta G}{\theta G}$. The price jumps by $(n+1)$ focal prices, i.e. increases by $(n+1)G$, with probability $\frac{\Delta mod \theta G}{\theta G}$. Therefore, in the presence of focal pricing constraints, pass-through may be higher or lower than in the unconstrained problem. For instance, if the cost change is much smaller than the gap between cost thresholds, it will result in no pass-through with high probability, and extremely high pass-through with low probability. The variance of constrained pass-through is:

$$\begin{aligned} var\left(\frac{p^c(c+\Delta) - p^c(c)}{\Delta}\right) = \frac{var\left(p^c(c+\Delta) - p^c(c)\right)}{\Delta^2} \\ = \left(\frac{G}{\Delta}\right)^2 \frac{\Delta mod \theta G}{\theta G}\left(1 - \frac{\Delta mod \theta G}{\theta G}\right) \end{aligned}$$

The variance is strictly positive unless $\Delta = n\theta G$ for $n \geq 1$, in which case it is zero because the price is guaranteed to jump by exactly n focal prices. Hence, with focal pricing constraints, the variance of pass-through is weakly higher than in the unconstrained case.

In the context of a class action, where each member of the class purchased a single product, focal pricing constraints may lead to substantial heterogeneity in the pass-through rates which apply to each claimant,[13] so that it may be best to estimate pass-through rates specific to sub-classes. In this scenario, and if the cost change is small so that it would have led either to a single jump in focal price, or no price change at all, then it may be useful to distinguish between consumers who are estimated to be harmed (because of a focal price jump) and those who are not (because of no response in price). Where each member purchased multiple products, at different times, we might expect the average pass-through rate experienced by each claimant to be closer to the expected pass-through rate. In this case, it may be appropriate to estimate the pass-through rate as one would in the absence of focal pricing constraints and consider a single number to be an appropriate estimate for all claimants.

Other aspects of the distribution of pass-through may also be of interest to antitrust practitioners. In particular, the likelihood of the realised pass-through rate being zero may be relevant to decisions about whether to invest resources into a case. This likelihood is increased by focal pricing, particularly when the cost change was very small, and when consumers each purchased a single product. However, as discussed in section 4, antitrust policymakers may wish to consider the implications of this type of logic on firm incentives. If firms are systematically less likely to be sued for anticompetitive behaviour when their downstream customers use focal pricing, that may have a negative effect on competition and consumer welfare.

3 Examples of specific models

In this section, I prove that expected pass-through is either exactly or approximately the same with and without focal pricing constraints under a variety of models which are frequently used by economists, particularly in the context of antitrust cases. Note that these are models of the downstream market, rather than the upstream one, as we are interested in the degree to which downstream firms pass through increased input costs. I consider different alternatives for the nature of competition in the downstream market, and the demand curve faced by downstream firms.[14] The main focus of this section is whether the Equal Spacing condition holds under these different models. Where it does, then the Irrelevance Theorem holds, and expected pass-through is unaffected (as long as the Regularity condition and the Uniformity

[13] Regardless of the presence of focal pricing constraints, heterogeneity may be introduced by asymmetries between firms or products. With focal pricing, even in cases where a single product is sold by a monopolist or in a fully symmetric industry, changes in other input costs over time may lead to heterogeneity in the degree of pass-through of the overcharge Δ for consumers who purchased at different times.

[14] Note that the upstream market is almost certainly concentrated if upstream firms have engaged in abusive behaviour resulting in an overcharge. Instead, the downstream market could be very competitive, or it could also be concentrated.

condition are met, which will generally be the case). I also discuss models in which the Equal Spacing condition is not met, and show that expected pass-through remains approximately (rather than exactly) unaffected by focal pricing constraints.

Given this paper's focus on pricing constraints, it is more natural to consider firms as price-setting, rather than quantity-setting. The rest of this discussion is therefore grounded in models of price competition. In turn, I consider three standard competitive frameworks: monopolistic competition, perfect competition, and differentiated price competition. In the context of monopolistic competition and perfect competition, I also discuss the impact of different curvatures of demand. Throughout this discussion, I maintain a number of basic assumptions. In particular, I focus on static models, in which all firms face the same constant marginal cost c (so when there is a change in that cost, it is faced by all firms). In section 3.4 I briefly discuss what may happen when relaxing these assumptions.

3.1 Monopolistic competition

Let us start by considering intuitively whether, under monopolistic competition, pass-through could be much lower with focal pricing than without. Let us remain agnostic about the unconstrained demand function, and just assume it is well behaved, and let there be focal pricing constraints. The monopolist faces cost $c_i = \chi_i$ and charges focal price f_i, which is both unconstrained and constrained optimal. Then, if cost increases to $c_i + \Delta$, assume that pass-through is zero with focal pricing, so that the monopolist continues to charge f_i. Then consider a further cost increase to $c_i + 2\Delta$, and again assume that pass-through is zero with focal pricing, so that the monopolist continues to charge f_i. But then consider that a further cost increase takes us to $c_i + 3\Delta = \chi_{i+1}$, at which cost level it is unconstrained optimal for the monopolist to charge the next focal price up f_{i+1}. If pass-through of this further cost increase was also zero, then we would have a contradiction: the monopolist would not charge the unconstrained optimal price even though it is available in the constrained problem.

If pass-through was zero for the first two cost increases, then it must be that pass-through was not only non-zero, but higher than unconstrained pass-through, for the last cost change. Hence, focal pricing constraints might lead to zero pass-through *locally*, but not globally. In fact, it is logically necessary for constrained pass-through to be higher than unconstrained pass-through for some initial values of costs, to compensate for zero pass-through for some other initial values of costs. A similar logic to that provided for the Uniformity condition suggests that *a priori* we cannot know which of these is the relevant case, and hence that prior to detailed empirical assessment, the best indication of constrained pass-through is the value we would have expected in the absence of focal pricing constraints.

Similar reasoning applies if we consider initial-cost regions where constrained pass-through is not zero, but is lower than unconstrained pass-through. If this is substantially and consistently the case, then we must reach the same contradiction as before. Therefore, if there are initial-cost regions where constrained pass-through is lower (higher) than unconstrained pass-through, then it follows that either (i) this is balanced out by initial-cost regions where constrained pass-through is higher (lower) than unconstrained pass-through, or (ii) constrained pass-through is lower (higher), or equal to, unconstrained pass-through for any initial cost, but the difference is small and tends to zero so that the contradiction caused by accumulation is never reached. Again, we can conclude that expected constrained pass-through is approximately equal to unconstrained pass-through.

Note that this logic does not appeal to the Regularity condition nor to the Equal Spacing condition. Therefore, in the monopolistic case the Irrelevance Theorem holds, at least as a global approximation, under very broad conditions, including with demand functions which yield non-constant pass-through rates.

It is interesting to note that in both cases discussed above, if there are regions where constrained pass-through is lower or higher than unconstrained pass-through, then that entails a non-constant unconstrained pass-through rate. As we will see in some concrete examples below, with a monopolist facing isoelastic and monomial demand, there are cases when unconstrained pass-through rates are constant, but constrained pass-through rates are not. I show that in those cases, and also for a monopolist facing logit demand, the Irrelevance Theorem is a reasonable approximation. The Theorem holds exactly in the case of a monopolist facing linear and logarithmic demand.

3.1.1 Linear demand

I start by considering the simplest possible model, with a single monopolist selling a single good, facing a linear demand function and marginal cost c. The monopolist can only change the price of the good, not any non-price characteristics. As discussed earlier, when firms can alter non-price characteristics, focal pricing constraints are unlikely to affect pass-through rates. Hence, I focus on the case where focal pricing could more plausibly affect pass-through.

The monopolist's unconstrained optimisation problem is:

$$\max\left(\alpha-\beta p\right)\left(p-c\right), \quad \alpha,\beta>0$$

The optimal price is $p^u = \frac{\alpha}{2\beta} + \frac{c}{2}$ and we obtain the standard result of 50% pass-through under monopolistic competition.

Inverting the unconstrained optimal price we find that the cost associated to a certain unconstrained optimal price is:

$$c = \frac{2\beta p^u - \alpha}{\beta}$$

Therefore, the cost for which a focal price is unconstrained optimal is:

$$\chi_i = \frac{2\beta f_i - \alpha}{\beta}$$

Hence, the gap between costs for which two consecutive focal prices are unconstrained optimal is:

$$\chi_{i+1} - \chi_i = \frac{2\beta f_{i+1} - \alpha - 2\beta f_0 + \alpha}{\beta}$$

Using the fact that focal prices are regularly spaced out at intervals G (the Regularity condition), we substitute in $f_{i+1} = f_i + G$. Simplifying, we obtain:

$$\chi_{i+1} - \chi_i = 2G$$

Hence, part (a) of the Equal Spacing condition holds. Now consider that the monopolist faces focal pricing constraints. As argued in section 2.1, only focal prices will be charged, so that the monopolist's constrained problem is:

$$\max\,(\alpha - \beta f_i)\,(f_i - c)\,, \quad \alpha, \beta > 0$$

We find the cost thresholds at which the monopolist switches from one focal price to another from the indifference condition: $\pi(f_i, t_i) = \pi(f_{i+1}, t_i)$. This can be written as

$$(\alpha - \beta f_i)\,(f_i - t_i) = (\alpha - \beta f_{i+1})\,(f_{i+1} - t_i)$$

This can be rearranged to obtain an expression for t_i. We can then find the gap between two consecutive thresholds:

$$t_{i+1} - t_i = \frac{\alpha + \beta f_{i+1} + \beta f_{i+2} - \alpha - \beta f_i - \beta f_{i+1}}{\beta}$$

Simplifying, and using the fact that focal prices are regularly spaced out at intervals G, we obtain:

$$t_{i+1} - t_i = 2G$$

This is the same gap as the gap between costs for which consecutive focal prices are unconstrained optimal, so part (b) of the Equal Spacing condition holds. The assumptions of the Irrelevance Theorem hold exactly, and focal pricing constraints leave expected pass-through unaffected.

3.1.2 Other demand functions giving rise to constant unconstrained pass-through rates

In practice, demand may well not be linear, and the degree of curvature of demand is an important determinant of the rate of pass-through of cost changes, as discussed by Bulow and Pfleiderer (1983).[15] Since part (a) of the Equal Spacing assumption requires constant unconstrained pass-through, we know that the Equal Spacing condition will hold exactly only for demand functions which entail constant unconstrained pass-through under monopolistic competition. These are found by Bulow and Pfleiderer (1983) to be: (i) logarithmic (or negative exponential) demand, (ii) isoelastic (or constant elasticity) demand, and (iii) monomial demand in the form $p = \alpha - \beta q^{\delta}, \delta > 0$, which includes the special case of linear demand when $\delta = 1$. This is a reasonably broad class of widely used functional forms. For instance, the one-product version of the Almost Ideal Demand System is a special case of isoelastic demand, as discussed by Weyl and Fabinger (2013).

Given that the Equal Spacing condition can only hold exactly under monopolistic competition if demand takes one of these forms, let us consider these in turn, to find the class of demand functions for which the Equal Spacing condition holds exactly.

Logarithmic demand

$$p = \alpha - \beta \ln q, \quad \alpha, \beta > 0, 0 < q < e^{\alpha/\beta}$$

In this case, the unconstrained monopolist chooses $p^u = c + \beta$, so there is a constant mark-up, and 'complete' pass-through of 100%. The monopolist constrained by focal prices (spaced out at regular intervals G) chooses which focal price to charge based on a cut-off rule. As shown in appendix A, these cut-offs are evenly spaced at the same regular gap G. Hence, the Equal Spacing assumption holds. The Irrelevance Theorem holds exactly: in the presence of focal pricing constraints, expected pass-through remains 100%.

Isoelastic demand

[15] There are other important determinants, such as returns to scale, which are left for future work to consider.

$$p = \beta q^{-1/\eta}, \quad \eta > 1, \beta > 0$$

This functional form implies constant elasticity of demand $-\eta$. In this case, the unconstrained monopolist optimally chooses:

$$p^u = \frac{\eta}{\eta - 1} c$$

The unconstrained pass-through rate is $\frac{\eta}{\eta-1}$, which is greater than 100%. As shown in appendix B, the Equal Spacing assumption does not hold *exactly* in this setting. We already know from the logic outlined at the start of section 3.1, that the Irrelevance Theorem must still hold at least as a global approximation. In appendix B, I provide numerical examples to illustrate what this means in practice. For instance, for $\eta = 2$, the gap between consecutive cost thresholds is close to $\frac{G}{2}$, and hence the pass-through rate is close to the unconstrained value of 200%. It is noteworthy that, even though the unconstrained pass-through rate is constant, the constrained pass-through rate is non-constant, but close to its unconstrained value.

Monomial demand

$$p = \alpha - \beta q^{\delta}, \delta, \alpha, \beta > 0$$

Facing monomial demand, the monopolist will optimally choose:

$$p^u = \frac{\delta \alpha + c}{\delta + 1}$$

Therefore, the unconstrained pass-through rate is constant at $\frac{1}{\delta+1}$. For any value of $\delta > 0$, this entails incomplete pass-through. In the special linear case, where $\delta = 1$, the pass-through rate is 50%. As shown in appendix C, similarly to the case of isoelastic demand, the Equal Spacing assumption is typically not met exactly (of course it is for $\delta = 1$). Again, from the logic outlined at the start of section 3.1, we know the Irrelevance Theorem must still hold at least as a global approximation. In appendix C, I provide numerical examples to illustrate what this looks like in the case of a monopolist facing monomial demand.

3.1.3 Logit demand

Before turning to alternative models of competition in the downstream market, I consider the case of a monopolist facing logit demand. This is an interesting case both because logit demand is widely used in empirical IO (e.g. see Miravete et al. (2023)) and because it implies non-constant pass-through rates in the unconstrained monopoly problem. The latter means that part (a) of the Equal Spacing condition does not hold. The logit case therefore gives us an interesting example of what might happen when we relax that assumption.

Following the standard logit monopoly set-up, let consumer i's utility from the good be $u_i = a - bp + \epsilon_i$. Here p is the price charged by the monopolist, and ϵ_i is an individual taste shock. These taste shocks are iid type 1 extreme value. Consumers have a reservation utility normalised to 0. It follows that the share of potential consumers who purchase the monopolist's good is:

$$s(p) = \frac{\exp(a - bp)}{1 + \exp(a - bp)}$$

Normalising the size of the potential market to 1, the monopolist maximises the profit function:

$$\pi = (p - c)\, s(p) = (p - c)\, \frac{\exp(a - bp)}{1 + \exp(a - bp)}$$

Taking derivatives and rearranging, this yields:

$$p^u = \frac{1}{b(1 - s(p^u))} + c$$

As the share $s(p^u)$ is a function of the price, further manipulation is required to find the solution in terms of model fundamentals. Using the LambertW function $W(x)$, we can formulate the closed form solution:

$$p^u = \frac{1 + W\left(exp(a - 1 - cb)\right)}{b} + c$$

The pass-through rate can be found by taking the derivative with respect to cost and is $(1 - s(p^u))$. This entails incomplete pass-through at a non-constant rate. Pass-through is close to 1 when costs are high (and hence the market share is low), and close to 0 when costs are low (and hence the market share is high).

We already know that in this setting the expected constrained and unconstrained pass-through rates must be similar in the sense outlined at the start of section 3.1. To illustrate what this looks like in practice, in appendix D, I provide two numerical examples. In both examples, expected pass-through is similar in magnitude, but weakly higher, with focal pricing constraints than without. The difference between constrained and unconstrained expected pass-through tends to zero as costs increase.

3.2 Perfect competition

Now consider a market characterised by perfect competition, or undifferentiated price (or Bertrand) competition. The aggregate demand function Q can take any form and depends only on the lowest price offered by any firm j in the market $p^{min} = \min_j \{p_j\}$. There are N firms in the market, and the demand function faced by a specific firm n takes the following form:

$$q_n^u = \begin{cases} \frac{Q(p^{min})}{\sum_j I(p_j = p^{min})} & p_n = p^{min} \\ 0 & p_n \neq p^{min} \end{cases}$$

All firms face the same marginal cost c. The unique equilibrium involves all firms charging $p^u = c$, and hence each firm facing demand $q_n^u = \frac{Q(c)}{N}$, and making zero profits. In this context, there is complete pass-through of input cost changes. Therefore, a Δ overcharge will result in a Δ increase in prices.

We now introduce focal pricing constraints for the firms. If a firm charges p_n, this will lead to the same constrained demand as charging: $f_n = f_i \quad f_{i-1} < p_n \leq f_i$. The focal price associated to the lowest price charged is f^{min}. Constrained demand takes the form:

$$q_n^c = \begin{cases} \frac{Q(f^{min})}{\sum_j I(f_j = f^{min})} & f_n = f^{min} \\ 0 & f_n \neq f^{min} \end{cases}$$

Once again there is a unique equilibrium. All firms charge $p = f^c$, where f^c is the marginal cost c rounded up to the next focal price. Each firm faces constrained demand $q_n^c = \frac{Q(f^c)}{N}$.[16]

[16]There is no profitable deviation to a lower focal price because it would entail negative profits, nor any

Therefore, all firms now make weakly positive profit. We can easily see that the cost thresholds at which firms adjust prices are identical to the focal prices themselves. Similarly, we know that the costs for which focal prices are unconstrained optimal are also identical to the focal prices themselves. Therefore, the Equal Spacing condition holds under perfect competition, for any demand function. The Irrelevance Theorem holds exactly: the expected pass-through rate is still 100% even when we introduce focal pricing constraints.

3.3 Differentiated price competition

Having discussed the two extreme cases of monopoly and perfect competition (which are also informative about collusion and undifferentiated price competition) I now turn to an intermediate case: differentiated price (or Bertrand) competition. With differentiated price competition, under continuous pricing and other general conditions, there is a unique equilibrium (see Mizuno (2003)). This result no longer holds with discrete pricing, which typically leads to a multiplicity of equilibria. I show that the Equal Spacing assumption holds exactly under a simple equilibrium strategy profile for firms facing focal pricing constraints. I then discuss how the Irrelevance Theorem still holds as a global approximation under alternative equilibrium strategy profiles.

Consider a market with N firms producing a differentiated product and simultaneously competing on prices in a one-shot game. Demand for firm n's product satisfies standard conditions for differentiated price competition (see Mizuno (2003)). Here, we take it to be:

$$q_n^u = \begin{cases} Q - p_n + p_{-n}^- & Q > p_n - p_{-n}^- \\ 0 & Q \leq p_n - p_{-n}^- \end{cases}$$

where p_{-n}^- is the average price set by other firms in the market, and Q is some positive constant. Take the symmetric input case, in which all firms face the same marginal cost $c_j = c, \quad \forall j$, and in which all firms are exposed to the same changes in input costs (including the one of interest).

In the unconstrained case, the profit function for firm n is $\pi_n^u = (p_n - c)(Q - p_n + p_{-n}^-)$. Best responses are linear and symmetric, and there is a unique symmetric equilibrium where all firms charge: $p_n^u = Q + c, \quad \forall n$. Therefore, this industry is characterised by a constant unconstrained pass-through rate of 100%, i.e. a Δ increase in the input cost faced by all firms will result in a Δ increase in the price charged by all firms.

lower non-focal price because it would entail the same firm-specific demand, but at a lower price. There is also no profitable deviation to a higher price because it would entail zero firm-specific demand.

With focal pricing, the demand function can be written as:

$$q_n^c = \begin{cases} Q - f_n + \bar{f}_{-n} & Q > f_n + \bar{f}_{-n} \\ 0 & Q \leq f_n + \bar{f}_{-n} \end{cases}$$

where f_n is p_n rounded up to the next focal price, and $\bar{f}_{-n} = \frac{\sum_{j \neq n} f_j}{N-1}$.

Let there be a focal price point at Q, which we refer to as the low price p^L, a medium focal price at $p^M = Q + G$, and a high focal price at $p^H = Q + 2G$. I focus on this interval, but we can imagine that this pattern of focal prices continues along the whole interval of possible prices. From $p_n^u = Q + c, \quad \forall n$, we know that the low price is unconstrained optimal when $c = 0$, the medium price is unconstrained optimal when $c = G$, and the high price is unconstrained optimal when $c = 2G$. Therefore, the spacing between costs at which consecutive focal prices are unconstrained optimal is G. Hence, part (a) of the Equal Spacing condition holds.

Under the constrained demand function, the conditions for uniqueness of equilibrium no longer hold. Focusing just on symmetric equilibria, we can divide the support of the marginal cost into segments for each of which there are two possible equilibria:

1. For $0 \leq c \leq G$ there is an all-low-price equilibrium, and an all-medium-price equilibrium.

2. For $G \leq c \leq 2G$ there is an all-medium-price equilibrium, and an all-high-price equilibrium.

Consider an equilibrium where each firm's strategy is to charge medium prices if $0 \leq c \leq G$ and high prices if $G \leq c \leq 2G$. Hence the spacing between cost thresholds is G, which is the same as the spacing between the gaps between the costs at which focal prices are unconstrained optimal. Hence, part (b) of the Equal Spacing assumption also holds.

Imagine that, prior to the cost increase, the industry was characterised by medium prices, meaning that marginal costs were in the range $0 \leq c \leq G$. Then if costs increase by Δ, there is a $\frac{\Delta}{G}$ chance of costs increasing to the range $G \leq c \leq 2G$, in which case all firms charge high prices. The expected change in prices is therefore $\frac{\Delta}{G}G = \Delta$, i.e. the industry is still characterised by 100% expected pass-through, as it was in the absence of focal pricing. The Irrelevance Theorem holds exactly.

The same logic holds in the alternative equilibrium where firms charge low prices if $0 \leq c \leq G$ and medium prices if $G \leq c \leq 2G$. Again, the Irrelevance Theorem holds exactly.

Consider another possible equilibrium, where each firm's strategy is to charge low prices if $0 \leq c \leq G$ and high prices if $G \leq c \leq 2G$. Then the expected pass-through with focal pricing is twice as large as unconstrained pass-through. However, this effect is local. If we extended our analysis to further along the cost interval, pass-through could not remain twice as large going into the interval $2G \leq c \leq 3G$, since the equilibrium price for that segment cannot be two focal prices up from the high price. In that segment, the equilibrium could either involve all firms charging high prices (creating an area of locally zero expected pass-through) or the next focal price up (generating an area where, locally, expected pass-through is the same as in the unconstrained case). Moreover, note that we could also construct the opposite example, where firms are in the medium price equilibrium for both $0 \leq c \leq G$ and $G \leq c \leq 2G$, and hence, locally, there is no pass-through. If we extended our analysis to further along the cost interval, an all-medium equilibrium would not be feasible for $2G \leq c \leq 3G$, where there must be either an all-high equilibrium or an equilibrium with the next focal price up: it is not sustainable for there to not be pass-through along more than a local segment of the cost interval.

The intuition is similar to that in the monopoly discussion (see section 3.1), although here we have to be careful to think about what is sustainable as an equilibrium, rather than what is unilaterally optimal. Again, we see that locally higher (or lower) pass-through rates will in general be 'cancelled out' by other locally lower (or higher) pass-through rates; or at least that it cannot be the case that focal pricing systematically and substantially increases or decreases pass-through along the whole cost interval. Therefore, a priori, expected constrained pass-through is still approximately equal to unconstrained pass-through. In this sense, we might say that the Irrelevance Theorem holds as a global approximation even when we depart from the Equal Spacing condition. It is a global approximation in the sense that if we do not know if we are at a segment of initial cost where pass-through is locally increased or decreased by focal pricing constraints, then in expectation there is approximately no effect.

3.4 Other models

Pass-through rates are affected by many modelling assumptions. These include demand curvature, returns to scale, nature of competition between firms, whether firms compete in a one-shot game or dynamically, whether there are asymmetries between firms, the degree of product heterogeneity, etc.[17] Therefore, it is hard to rule out the conceptual possibility that there exist models where expected constrained pass-through is systematically and substan-

[17] Modelling assumptions do not always affect pass-through. For instance, Kate and Niels (2005) discuss examples where pass-through does not depend on the price elasticity of demand nor on market share.

tially different from unconstrained pass-through. However, as noted by Alexandrov (2013), this does not seem plausible in any standardly used model. For instance, consider relaxing the assumption of symmetric costs in the context of undifferentiated price competition. In this case, with focal pricing constraints, if the increase in input costs was sustained only by a subset of firms, and was large enough to cross a cost threshold, then the affected firms would be forced to exit the market. There would be no direct impact on prices,[18] and hence zero direct price pass-through. However, this is exactly what would happen in the unconstrained setting too, so that the pass-through rate is 0% regardless of the presence of focal pricing constraints. As discussed in the differentiated Bertrand example, even in settings with multiplicity of equilibria it is hard to imagine how constrained pass-through could be substantially higher or lower than unconstrained pass-through, other than locally.

This paper has argued that the Irrelevance Theorem holds exactly under several common models of competition, and as a global approximation under others. This means that pass-through may be locally higher, or lower, with focal pricing than without, but globally it is approximately the same. In some cases, expected pass-through is unaffected along the whole cost interval (e.g. monopolist facing linear demand). In others, there may be portions of the cost interval where focal pricing increases, and others where it decreases, expected pass-through (e.g. differentiated price competition under certain equilibrium strategy profiles). In others still, expected constrained pass-through can be systematically slightly higher (e.g. monopolist facing logit demand) or systematically slightly lower (e.g. monopolist facing isoelastic demand) than unconstrained pass-through, with the difference tending to zero in some direction along the cost interval. The conceptual indeterminacy of the impact of focal pricing on pass-through, combined with the fact that the Irrelevance Theorem holds exactly in several standard settings, and approximately in others, provides a clear rationale for conducting context-specific empirical analysis to estimate pass-through, rather than assuming that the presence of focal pricing will lead to little, or no, pass-through.

4 Discussion

In the context of antitrust damages class actions, the Irrelevance Theorem provides a strong rationale not to accept arguments that cases should be dismissed because there will be low, or zero, pass-through with focal pricing constraints. However, we should take seriously

[18] It is worth noting that the exit of a subset of firms might lead to a large indirect impact on prices through increased market concentration and higher chance of collusion. This is true both in the constrained and unconstrained setting.

the possibility that focal pricing may increase heterogeneity in the distribution of pass-through. Depending on the specifics of the context, this might mean that the majority of the class suffered no harm, while a minority suffered substantial harm. In other cases, all class members suffered damages, perhaps to similar, or perhaps to different, degrees. It is also possible that all class members suffered very large damages, or that no class members suffered any damages at all.

Even though antitrust damages class actions should generally not be dismissed on grounds related to the effect of focal pricing on *expected* pass-through, a case could still be plausibly dismissed on grounds related to the impact of focal pricing on the *likely realisation* of pass-through. If there is a high probability that the class suffered no harm, why spend large amounts of resources on the case? However, consider the possible implication of this attitude. Downstream firms may not be willing to initiate antitrust litigation against their input supplier(s) for fear of input supply disruption. Therefore, if focal pricing was seen as a reason to strike out consumer cases, that could entail a situation where upstream abusive firms were able to overcharge downstream firms with little fear of antitrust challenge from anyone. It would be concerning if the presence of focal pricing in downstream markets was considered something of a *carte blanche* for abusive firms upstream. Moreover, even if downstream firms did challenge upstream firms, they would only be able to obtain damages for overcharges that were not passed through to consumers. If any remaining component of damages could not be recovered by consumers,[19] in expectation, abusive firms would still face lower fines, even if they were successfully challenged in court by downstream firms. While this may be viewed as an acceptable down-side of efficiently dismissing cases that are likely to result in no fines, one might be concerned that it would systematically worsen incentives for abusive behaviour upstream.

A separate issue created by the impact of focal pricing on the distribution of pass-through is at the class certification stage. In some jurisdictions, the increased heterogeneity in the distribution of pass-through introduced by focal pricing may be perceived as a challenge to the homogeneity requirement that class members should all have suffered damages in a similar way. However, it is not clear why heterogeneity in damages arising from focal pricing should be treated any differently from heterogeneity arising from class members having purchased slightly different products at different times, as is standardly the case in class actions. In

[19]It may be possible for a consumer class action to rely on pass-through estimates from a successful antitrust challenge by downstream firms. Then the abusive firm would have to pay for the full damages caused. However, this is not always realistic. Antitrust litigation can easily take many years to reach a final judgment, and class actions typically have time limits within which they must be filed. Moreover, in case of a settlement between upstream and downstream firms, there may be no judgment to rely on. Even if there was a judgment, if it was in another jurisdiction it may not be possible to rely on it in court.

these cases too, class members may well have suffered from different damages to begin with, and then accumulated differential interest on damages over time. It also seems remarkable that abusive firms might be able to avoid paying damages to consumers simply because those consumers were affected to different degrees by the anti-competitive behaviour. From an economic viewpoint, the way to address this issue is to perform more granular analysis and obtain estimates of damages specific to members of different sub-classes. Where this kind of detailed expert analysis is considered too expensive by the court, using average estimates for the whole class seems like a more sensible approach than dismissing the case as a whole.

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A A monopolist facing a logarithmic demand function

Consider a monopolist facing a logarithmic demand function, and constrained by focal pricing. Focal price f_i is charged if $t_{i-1} < c \leq t_i$. The thresholds are found as the points of indifference for the monopolist, i.e. $\pi(f_i, t_i) = \pi(f_{i+1}, t_i)$.

The profit function is:

$$\pi(f_i, t_i) = (f_i - t_i)\, e^{\frac{\alpha - f_i}{\beta}}$$

We find the threshold t_i from:

$$(f_i - t_i)\, e^{\frac{\alpha - f_i}{\beta}} = (f_{i+1} - t_i)\, e^{\frac{\alpha - f_{i+1}}{\beta}}$$

Using the fact that focal prices are regularly spaced out at intervals G, we substitute in $f_{i+1} = f_i + G$:

$$f_i e^{\frac{\alpha - f_i}{\beta}} - (f_i + G)\, e^{\frac{\alpha - f_{i+1}}{\beta}} = t_i \left(e^{\frac{\alpha - f_i}{\beta}} - e^{\frac{\alpha - f_{i+1}}{\beta}} \right)$$

Rearranging and simplifying, we obtain:

$$t_i = f_i - G \frac{e^{\frac{-G}{\beta}}}{1 - e^{\frac{-G}{\beta}}}$$

Hence the interval between any two consecutive cost thresholds is:

$$t_{i+1} - t_i = f_{i+1} - G \frac{e^{\frac{-G}{\beta}}}{1 - e^{\frac{-G}{\beta}}} - f_i + G \frac{e^{\frac{-G}{\beta}}}{1 - e^{\frac{-G}{\beta}}}$$

Simplifying, we obtain:

$$t_{i+1} - t_i = f_{i+1} - f_i = G$$

Hence, thresholds are regularly spaced at the same interval as focal prices, and the Equal Spacing condition holds exactly.

B A monopolist facing isoelastic demand

Following the same method as used in appendix A for the logarithmic case, in the isoelastic case I find that:

$$t_i = f_i - G \frac{f_i^{\eta}}{(f_i + G)^{\eta} - f_i^{\eta}}$$

The gap between consecutive cost thresholds is:

$$t_{i+1} - t_i = G\left(1 + \frac{f_i^\eta}{(f_i+G)^\eta - f_i^\eta} - \frac{(f_i+G)^\eta}{(f_i+2G)^\eta - (f_i+G)^\eta}\right)$$

The Equal Spacing condition would require that $t_{i+1} - t_i = G\frac{\eta-1}{\eta}$. In general, this will not hold exactly. Note that the gap between cost thresholds is non-constant.

To map this onto pass-through rates, consider an initial cost between t_i and t_{i+1}. If cost increases by a small amount (less than the gap between focal prices) then the expected constrained pass-through rate is:

$$\frac{G}{t_{i+1} - t_i}$$

Note that this number varies for costs falling between different cost thresholds, because the gaps between thresholds is non-constant, but it is constant for costs falling between the same cost thresholds. Therefore, expected constrained pass-through is non-constant even though unconstrained pass-through is constant.

From the logic outlined at the start of section 3.1, we know that the Irrelevance Theorem must still hold at least as a global approximation. I provide illustrative examples in table 1, table 2 and table 3 which confirm that expected pass-through is approximately the same with and without focal pricing constraints. Constrained expected pass-through is typically slightly lower when initial marginal cost is close to 0, and is the same as unconstrained expected pass-through for high initial marginal cost.

	Cost ranges for which a particular focal price is constrained optimal ($)				
Expected pass-through	$c \in (6.11, 11.48)$	$c \in (26.77, 31.80)$	$c \in (51.88, 56.89)$	$c \in (501.99, 506.99)$	$c \in (5002, 5007)$
Unconstrained	2				
Constrained	1.861496	1.985636	1.995792	1.999951	2
Focal price charged	$19	$59	$109	$1,009	$10,009

Table 1: Numerical example of pass-through in the case of a monopolist facing isoelastic demand, and a small cost change. Illustrated for parameter values: $\eta = 2, G = 10$.

	Cost ranges for which a particular focal price is constrained optimal ($)			
Expected pass-through	$c \in (8.31, 71.64)$	$c \in (523.31, 573.41)$	$c \in (5024.38, 5074.38)$	$c \in (50024.5, 50074.5)$
Unconstrained	2			
Constrained	1.57916	1.99586	1.999951	2
Focal price charged	$109	$1,009	$10,009	$100,009

Table 2: Numerical example of pass-through in the case of a monopolist facing isoelastic demand, and a small cost change. Illustrated for parameter values: $\eta = 2, G = 100$.

	Cost ranges for which a particular focal price is constrained optimal ($)				
Expected pass-through	$c \in (8.99, 18.85)$	$c \in (47.15, 56.36)$	$c \in (92.82, 101.89)$	$c \in (903.52, 912.52)$	$c \in (9003.59, 9012.59)$
Unconstrained	1.111111				
Constrained	1.014425	1.08594	1.102919	1.111011	1.11111
Focal price charged	$19	$59	$109	$1,009	$10,009

Table 3: Numerical example of pass-through in the case of a monopolist facing isoelastic demand, and a small cost change. Illustrated for parameter values: $\eta = 10, G = 10$.

C A monopolist facing monomial demand

Following the same method as used in appendix A for the logarithmic case, in the monomial case I find that:

$$t_i = f_i - G\frac{(\alpha - f_i - G)^{\frac{1}{\delta}}}{(\alpha - f_i)^{\frac{1}{\delta}} - (\alpha - f_i - G)^{\frac{1}{\delta}}}$$

$$t_{i+1} - t_i = G\left(1 + \frac{(\alpha - f_i - G)^{\frac{1}{\delta}}}{(\alpha - f_i)^{\frac{1}{\delta}} - (\alpha - f_i - G)^{\frac{1}{\delta}}} - \frac{(\alpha - f_i - 2G)^{\frac{1}{\delta}}}{(\alpha - f_i - G)^{\frac{1}{\delta}} - (\alpha - f_i - 2G)^{\frac{1}{\delta}}}\right)$$

The Equal Spacing condition requires that $t_{i+1} - t_i = (\delta + 1)\, G$. This will generally not hold exactly.

To map this onto pass-through rates, consider an initial cost between t_i and t_{i+1}. If cost increases by a small amount (less than the gap between focal prices) then the expected constrained pass-through rate is:

$$\frac{G}{t_{i+1} - t_i}$$

Since the gap between cost thresholds varies with f_i, we can see that, while the unconstrained pass-through rate is constant, the constrained pass-through rate is generally non-constant.

When $\delta = 1$, monomial demand reduces to linear demand. In that case, the Equal Spacing condition holds exactly, and so does the Irrelevance Theorem.

As outlined at the start of section 3.1, the Irrelevance Theorem must hold at least as a global approximation for other values of δ too. I provide numerical examples in table 4, table 5 and table 6, which confirm that expected pass-through is approximately the same with and without focal pricing constraints.[20] These examples highlight different patterns for the case of $\delta < 1$ (table 4, table 5) and $\delta > 1$ (table 6). With $\delta < 1$, expected pass-through is typically slightly higher in the constrained case, with the difference decreasing for higher cost values. With $\delta > 1$, expected pass-through is typically slightly lower in the constrained case, with the difference decreasing for lower cost values.

[20] In constructing these examples, it is important to select parameters and prices that are consistent with non-negativity constraints. Cost and quantity are both positive only for unconstrained prices in the range $p^u \in (\frac{\delta}{\delta+1}\alpha, \alpha)$.

	Cost ranges for which a particular focal price is constrained optimal ($)				
Expected pass-through	$c \in (3.68, 12.68)$	$c \in (12.68, 21.68)$	$c \in (21.68, 30.68)$	$c \in (57.66, 66.65)$	$c \in (84.59, 93.40)$
Unconstrained	$\frac{1}{1.8} = 0.\overline{555}$				
Constrained	0.5556672	0.5556929	0.5557287	0.5562267	0.5671516
Focal price charged	$49	$54	$59	$79	$94

Table 4: Numerical example of pass-through in the case of a monopolist facing monomial demand, and a small cost change. Illustrated for parameter values: $\delta = 0.8$, $\alpha = 100$ and $G = 5$. Note that these parameter values are consistent with unconstrained optimal prices in the range $p^u \in (44.\bar{4}, 100)$.

	Cost ranges for which a particular focal price is constrained optimal ($)				
Expected pass-through	$c \in (16.72, 28.72)$	$c \in (124.70, 136.70)$	$c \in (244.66, 256.66)$	$c \in (364.60, 376.59)$	$c \in (578.59, 589.00)$
Unconstrained	$\frac{1}{1.2} = 0.8\overline{333}$				
Constrained	0.8334534	0.833515	0.8336612	0.8340938	0.9605721
Focal price charged	$119	$209	$309	$409	$589

Table 5: Numerical example of pass-through in the case of a monopolist facing monomial demand, and a small cost change. Illustrated for parameter values: $\delta = 0.2$, $\alpha = 600$ and $G = 10$. Note that these parameter values are consistent with unconstrained optimal prices in the range $p^u \in (100, 600)$.

	Cost ranges for which a particular focal price is constrained optimal ($)				
Expected pass-through	$c \in (74.96, 184.98)$	$c \in (184.98, 295.01)$	$c \in (405.04, 515.09)$	$c \in (625.15, 735.26)$	$c \in (845.47, 956.00)$
Unconstrained	$\frac{1}{2.2} = 0.\overline{4545}$				
Constrained	0.4544639	0.4544415	0.4543568	0.4541005	0.4523609
Focal price charged	$659	$709	$809	$909	$1,009

Table 6: Numerical example of pass-through in the case of a monopolist facing monomial demand, and a small cost change. Illustrated for parameter values: $\delta = 1.2$, $\alpha = 1,100$ and $G = 50$. Note that these parameter values are consistent with unconstrained optimal prices in the range $p^u \in (600, 1100)$.

D A monopolist facing logit demand

With focal pricing constraints, the monopolist's profit from charging focal price f_i when faced with logit demand is:

$$\pi(f_i) = (f_i - c)\, s(f_i)$$

Cost thresholds are defined by the monopolist's indifference condition:

$$(f_i - t_i)\, s(f_i) = (f_{i+1} - t_i)\, s(f_{i+1})$$

We can rearrange this to obtain the following expression for cost thresholds:

$$t_i = \frac{f_i s(f_i) - f_{i+1} s(f_{i+1})}{s(f_i) - s(f_{i+1})}$$

The gap between consecutive cost thresholds is:

$$t_{i+1} - t_i = \frac{f_{i+1} s(f_{i+1}) - f_{i+2} s(f_{i+2})}{s(f_{i+1}) - s(f_{i+2})} - \frac{f_i s(f_i) - f_{i+1} s(f_{i+1})}{s(f_i) - s(f_{i+1})}$$

This gap is non-constant and must be evaluated numerically rather than analytically. Consider an initial cost between t_i and t_{i+1}. If cost increases by a small amount (less than the gap between focal prices) then the expected constrained pass-through rate is:

$$\frac{G}{t_{i+1} - t_i}$$

Note that this number varies for costs falling between different cost thresholds, because the gaps between thresholds is non-constant, but it is constant for costs falling between the same cost thresholds. Instead, the unconstrained pass-through rate changes continuously along the cost interval. Therefore, to evaluate its expected value for an initial cost located uniformly within the interval (t_{i+1}, t_i) we must integrate the unconstrained pass-through function:

$$\frac{1}{t_{i+1} - t_i} \int_{t_i}^{t_{i+1}} (1 - s(p^u(c)))\, dc$$

The share can be written directly as a function of cost:

$$s(c) = \frac{W\,(exp(a - 1 - bc))}{1 + W\,(exp(a - 1 - bc))}$$

Substituting this in, we obtain:

$$\frac{1}{t_{i+1}-t_i}\int_{t_i}^{t_{i+1}}\left(\frac{1}{1+W\left(exp(a-1-bc)\right)}\right)dc$$

We can re-write this in the form:

$$\frac{1}{t_{i+1}-t_i}\left[-\frac{1}{b}\ln\left(W\left(exp(a-1-bc)\right)\right)\right]_{t_i}^{t_{i+1}}$$

Note that since the pass-through rate is continuously changing, this approach is only valid for small cost changes (for which the pass-through rate evaluated at the initial cost level is appropriate).

	Cost ranges for which consecutive focal price are constrained optimal ($)				
Expected pass-through	$c \in (63.83, 619.81)$	$c \in (619.81, 630.00)$	$c \in (630.00, 640.00)$	$c \in (640.00, 650.00)$	$c \in (650.00, 660.00)$
Unconstrained	0.01181042	0.717095	0.9991614	0.9999997	1
Constrained	0.01798621	0.9820138	0.9999939	1	1
Focal price charged	$620	$630	$640	$650	$660

Table 7: Numerical example of pass-through in the case of a monopolist facing logit demand, and a small cost change. Illustrated for parameter values: $a = 500, b = 0.8, G = 10$.

	Cost ranges for which consecutive focal price are constrained optimal ($)				
Expected pass-through	$c \in (123.19, 189.11)$	$c \in (189.11, 199.11)$	$c \in (199.11, 204.52)$	$c \in (204.52, 209.55)$	$c \in (209.55, 214.55)$
Unconstrained	0.07099858	0.4251025	0.8696654	0.9865038	0.9988645
Constrained	0.07585818	0.5000000	0.9241418	0.9933071	0.9994472
Focal price charged	$195	$200	$205	$210	$215

Table 8: Numerical example of pass-through in the case of a monopolist facing logit demand, and a small cost change. Illustrated for parameter values: $a = 100, b = 0.5, G = 5$.

In the examples illustrated in table 7 and table 8, constrained expected pass-through is weakly higher than unconstrained expected pass-through. The difference between the two changes non-monotonically along the cost interval: it is small for very low costs, more pronounced for low costs, and goes to zero as costs increase further. When costs are high (i.e. price is high and the share of the potential market is low) both expected pass-through rates are approximately equal to 100%.