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Beyond ranks: inequality in the measurement of mobility

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Beyond Ranks:
Inequality in the Measurement of Mobility*

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Abstract

We argue the change in individuals' Lorenz ordinates—their positions in the Lorenz curve—is informative about economic mobility. Lorenz ordinates combine the ordinal content of ranks with the cardinal content of income differences. In this way, Lorenz mobility is meaningful when there are material differences between incomes across the distribution, explicitly linking inequality and mobility. In a highly unequal society, rank mobility among low-income people produce small income differences, reflected in a relatively flat Lorenz curve; while similar rank movements near the top produce large income differences, reflected in convex Lorenz curves. We show how Lorenz mobility relates to other mobility measures and to standard concepts of income inequality, separating horizontal (or positional) mobility, that holds inequality constant, from vertical mobility, that captures changes in inequality. We also show Lorenz ordinates are axiomatized as individuals' *aspirational* economic status—status increases when an individual's income is closer to those richer than them—and provide an application to intergenerational mobility in the US. Lorenz mobility in the US is lower than rank mobility, highlighting the role of inequality.

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1. Introduction

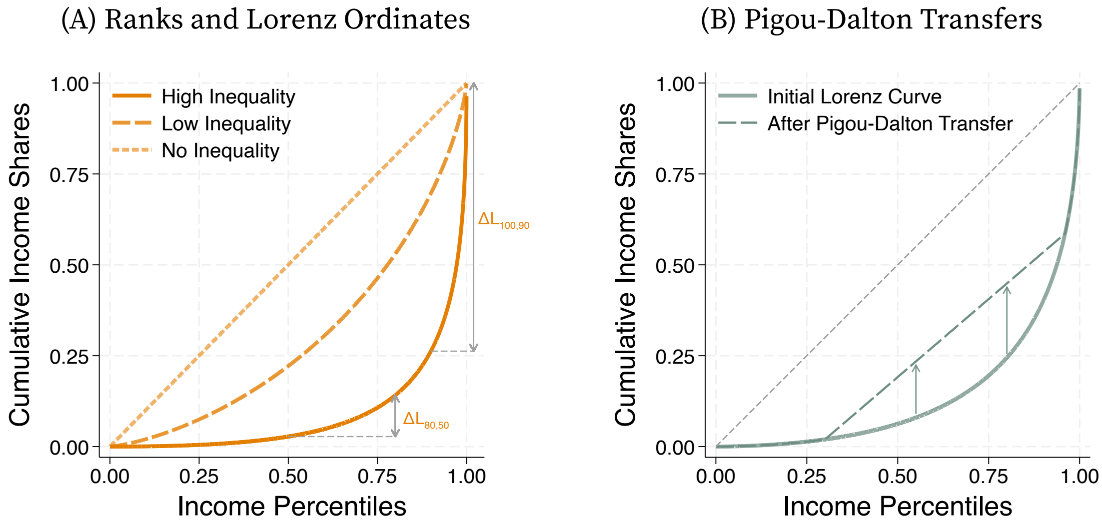
The study of mobility and inequality are intertwined, with many measures of economic mobility focusing on the *relative* position of individuals in the economy. These include intergenerational changes in income ranks (Bartholomew 1973 and Shorrocks 1978), and rank correlations (Schiller 1977, Chetty et al. 2014, or Fagereng, Mogstad, and Rønning 2021).[1] However, purely ordinal measures obscure the connection between mobility and inequality by discarding cardinal information in income. Large changes in ranks may have little material impact when differences in income are small, while small changes in ranks between those with drastically different incomes are meaningful. Without cardinality, relative mobility measures struggle to capture material differences.

The interaction between individuals' positions and cardinal income differences is described by the Lorenz curve—the cumulative income share at each rank. Consider Figure 1A, which shows the Lorenz curve of the log-normal distribution. In more unequal economies, income distributions compress at the bottom, where there is little difference between incomes, and spread at the top, where differences are large. Lorenz ordinates—individuals' positions in the Lorenz curve—capture this relationship: in the presence of inequality, differences in Lorenz ordinates shrink when income differences between ranks are small and spread as these differences increase. We argue this makes the *change in Lorenz ordinates* a desirable measure of economic mobility. They combine ordinal differences between individuals, captured by ranks, with cardinal differences in income levels, reflected in inequality.

Changes in Lorenz ordinates decompose into *horizontal and vertical mobility* making explicit the implications of inequality for mobility. Horizontal mobility comes from changes in position holding inequality constant: moving along the same Lorenz curve.

[1] While most of our discussion applies to different economic outcomes, such as wealth, we focus on income throughout for ease of exposition.

FIGURE 1. Inequality and Mobility



Notes: Panel A shows Lorenz curves for log-normal distributions with different variances, equivalently, after successive proportional Pigou-Dalton transfers. The Lorenz curve of a log-normal distribution with variance σ^2 is $L(r) = \Phi\left(\Phi^{-1}(r) - \sigma\right)$; Φ is the standard normal cumulative density. The no-inequality curve is the limit as $\sigma \longrightarrow 0$. Panel B shows a Lorenz curve before and after Pigou-Dalton transfers.

These changes are meaningful in as much as inequality makes their Lorenz ordinates different (Figure 1A). Vertical mobility comes from changes in inequality that alter the Lorenz curve. Reducing inequality increases mobility (by increasing Lorenz ordinates), as is the case after Pigou-Dalton transfers from richer to poorer individuals (Figure 1B). When individuals keep their position in the distribution (such as after proportional taxation) or when aggregating signed changes in Lorenz ordinates (equivalent to changes in the Gini coefficient) mobility only reflects changes in inequality.[2]

We also show that Lorenz ordinates provide a natural measure of *aspirational* economic status, axiomatically justifying its use for mobility. This characterization arises under two key conditions. First, status is *aspirational* in that it only depends on the income of richer individuals: individuals are *upward-looking*, reflecting the objective of catching up to those richer than them. Second, status satisfies *anonymity*

[2]These results highlight that mobility is not separable from the distribution of income, linking to extensive work interpreting mobility as social welfare (Atkinson and Bourguignon 1982).

with respect to others: it depends on how rich others are, not who they are or how income is distributed among them. The result is that an individual's status is determined not only by their ranks but also by the share of income held by those richer than them, increasing when there is less (income) distance to the top. It is in this sense that transfers from the rich to the poor increase mobility. Receiving transfers increases statusby bringing individuals closer to those richer than them even if relative positions (ranks) are preserved (Figure 1B).

While not driven by equity concerns, our results share properties with theories of inequality aversion (Robson 1992) and fairness (Fehr and Schmidt 1999) where the position of an individual in society is relevant (see also Lazear and Rosen 1981). However, position is not sufficient as an increase in income is valued even if ranks remain constant, underscoring the role of income for the consumption capabilities of individuals (Sen 1973). Our characterization of individual status via Lorenz ordinates is also linked to indexes of *aggregate satisfaction* in the context of relative deprivation (e.g., Runciman 1966; Yitzhaki 1979; Kakwani 1984), despite the two being conceptually different. Satisfaction indexes aggregate income comparisons between individuals, as the axiomatization in Ebert and Moyes (2000) makes clear, while status captures an individual's capabilities and position in society.

We connect mobility using Lorenz curves to existing approaches by separating the axiomatization of aggregate and individual mobility from that of status. Our measure of status can be used to compute aggregate absolute mobility (Fields and Ok 1996, 1999), signed mobility (Bhattacharya and Mazumder 2011; Cowell and Flachaire 2018; Ray and Genicot 2023), and intergenerational correlations (Hart 1983).[3] The justification for these measures, such as the axiomatic derivation in Shorrocks (1993) for Hart's

[3]These are commonly implemented by computing the intergenerational elasticity of income (e.g., Solon 1992; Zimmerman 1992; Dearden, Machin, and Reed 1997; Mazumder 2005; or Bolt, French, Hentall-MacCuish, and O'Dea 2024) or the Spearman correlation of income ranks (e.g., Dahl and DeLeire 2008; Olivetti and Paserman 2015; Ward 2023; or Jácome, Kuziemko, and Naidu 2025).

mobility measure, applies equally to mobility in Lorenz ordinates. Our approach also disentangles overall economic growth from mobility providing a measure comparable across economies with units expressed in income shares.[4]

Finally, we apply our framework to intergenerational status mobility in the US using linked parent-child data. Mobility using Lorenz ordinates is lower than mobility in ranks reflecting the role of inequality in the US. Moreover, mobility is uneven and concentrated among those making large transitions. Despite nearly identical Lorenz curves across generations (hence minimal vertical mobility), changes in position more than offset an increase in inequality between generations and generate status mobility equivalent to 23% of aggregate income.

2. Inequality and Status Mobility

Consider an economy populated by a finite set of dynasties indexed $i = 1, \ldots, N$. There are two observations of income for each dynasty, y^P and y^K, these might be incomes of parents and their children or of the same individual at different dates. We denote by Y^P and Y^K the $(N \times 1)$ vectors of income.

2.1. Status beyond ranks

We are interested in mobility with respect to the *status* of individuals. We see status as capturing the consumption capabilities of individuals (e.g., Sen 1973; Yitzhaki 1979) as well as their position in the economy. Thus, we assume status is a function of an individual's income and the income distribution in the economy, $s : \mathbb{R}_+ \times \mathbb{R}_+^N \longrightarrow \mathbb{R}_+$.

We constrain how status depends on income by imposing 4 conditions expressed in Axioms 1–4 below. We start by normalizing the scale of status, forcing it to be bounded.

[4] Our Lorenz-based approach also complements studies on top income and wealth shares (see, Kopczuk 2015 for a survey). Relatedly, Audoly et al. (2025) use Lorenz ordinates to study wealth mobility as an alternative to ranks and log-wealth that emphasizes top wealth holders.

Axiom 1 (**Status Normalization**). *Status satisfies* $s(y, Y) \in [0, 1]$.

We restrict attention to status functions that are strictly monotone in income and right-continuous. Specifically, status increases with the dynasty's income, holding the rest of the income distribution constant. Monotonicity is key to distinguish purely ordinal representations of status, like income ranks, from cardinal representations that reflect income levels. Right-continuity accounts for ties in the finite economy leading to jumps in status when income changes so as to break those ties.

Axiom 2 (**Continuity and Monotonicity**). *Status* $s\left(y_i, Y\right)$ *is strictly increasing and right-continuous on* y_i *given* $\{y_j\}$ *for* $j \neq i$.

In order to disentangle economic growth from mobility we focus on status functions that capture *relative*, as opposed to *absolute*, mobility. Specifically, we preclude changes in status resulting from overall income growth or changes in units. Therefore, status depends on the shape of Y and the relative position of y in Y, but not on their level. As in Shorrocks (1993), this imposes *scale invariance* within and between generations, making redundant the units in which income is measured.

Axiom 3 (**Growth Independence**). *Status satisfies* $s(y, Y) = s(\kappa y, \kappa Y)$ *for any* $\kappa > 0$.

Finally, we establish how status responds to the shape of the income distribution. Our objective is that status combines the cardinal and ordinal information reflected in income differences. Specifically, we make status *aspirational*: dynasties focus on those richer than them to determine their status. We see this as reflecting the objective of a dynasty to catch up with those richer than them. Status increases when the dynasty gets closer (in income) to those above, not by looking at those below.[5] So, only redistribution *from* those who are richer increases status. We strengthen this by imposing that status be invariant to pure redistribution above or below the dynasty.

[5] As in the TV series Mad Men, copywriters look up to Don Draper, but Draper *does not think about them at all.*

This makes status "*anonymous*:" a dynasty does not care about the identities of others (whose positions can be reshuffled by redistribution) while still caring about their own relative position (unchanged by this redistribution). This property makes the cumulative distribution of income the relevant statistic for determining status.

AXIOM 4 (**Aspirational Status**). *Order dynasties in Y such that* $y_i \leq y_{i+1}$ *for all* $i = 1, \ldots, N$. *Let* $1 < h < j \leq N$ *be two dynasties and make any transfer* δ *between them such that*

$$y_{h-1} < y_h + \delta < y_{j+1} \qquad \text{and} \qquad y_{h-1} < y_j - \delta < y_{j+1} \tag{1}$$

letting $y_{j+1} = y_j$ *if* $j = N$. *The transfer* δ *can be positive or negative. Then,*

(a) *The status of dynasties* $i = 1, \ldots, h-1, j+1, \ldots, N$ *remains unchanged:* $s\left(y_i, Y\right) = s\left(y_i, Y^{\delta}\right)$ *where* Y^{δ} *is the same as* Y *except for the incomes of the* h *and* j *entries; and*

(b) *The status of dynasty* j *is unchanged if their rank is unchanged: if* $y_{j-1} < y_j - \delta < y_{j+1}$, *then* $s\left(y_j, Y\right) = s\left(y_j - \delta, Y^{\delta}\right)$.

Together, these axioms tie status, and hence mobility, to the Lorenz ordinates of dynasties in the income distribution as well as to their income rank. Status reflects cardinal differences in income, because increasing income brings a dynasty closer to those they aspire to reach, even when income ranks remain unchanged.

PROPOSITION 1. *A status function satisfies Axioms 1–4 if and only if*

$$s(y, Y) = \psi\left(r(y, Y), L(y, Y)\right), \tag{2}$$

for $\psi : [0,1] \times [0,1] \longrightarrow [0,1]$ *a continuous function increasing in its first argument and strictly increasing in the second. Functions* r *and* L *give the rank and the Lorenz curve of* Y, *respectively,*

$$r(y, Y) \equiv \sum_{i=1}^{N} \frac{\mathbb{1}_{\{y_i \leq y\}}}{N} \quad \text{and} \quad L(y, Y) \equiv \sum_{\{\tilde{y} \in Y \mid \tilde{y} \leq y\}} \frac{\tilde{y}}{N\mu}; \quad \text{where} \quad \mu \equiv \sum_{i=1}^{N} \frac{y_i}{N}. \tag{3}$$

PROOF. Let Y be such that $y_i \leq y_{i+1}$ for all $i = 1, \ldots, N$ and fix a dynasty i. Consider an alternative $N \times 1$ ordered income vector Y' with the same average income, $\mu' = \mu$, and cumulative income among those richer than i, $\sum_{\{\tilde{y}' > y_i'\}} \tilde{y}' = \sum_{\{\tilde{y} > y_i\}} \tilde{y}$. Assume the vectors coincide in their i^{th} entry, $y_i' = y_i$, so the sum of income below i also coincides.

It is possible to move from Y to Y' by means of transfers among dynasties $1, \ldots, i-1$ and dynasties $i+1, \ldots, N$. Axiom 4 demands $s(y_i, Y) = s(y_i, Y')$. Therefore, status cannot depend on the whole distribution of income but only on y_i, its rank, r, mean income, μ, and the sum of income above y_i, $\sum_{\{\tilde{y} > y_i\}} \tilde{y}$.

A transfer from i to $h < i$, that does not change i's rank does not change status (Axiom 4). So, status cannot depend on y_i directly, except for its effect on the cumulative income sum, $\sum_{\{\tilde{y} \leq y_i\}} \tilde{y}$, which implies income above y_i.

Axiom 3 implies μ is superfluous—re-scaling income results in the same status—so we are left with two arguments (ranks and cumulative income shares) and write $s(y, Y) = \psi\left(r(y, Y), \sum_{\{\tilde{y} \leq y\}} \frac{\tilde{y}}{N\mu}\right)$ for some function ψ. The second argument is the Lorenz ordinate, which, unlike the rank, is strictly increasing in income. Hence, ψ must be strictly increasing in the second argument by Axiom 2. Ranks are only weakly increasing in income, as not all changes in income change relative positions. Thus, ψ depends only weakly on them. Finally, ψ must be continuous to preserve the right-continuity of ranks and Lorenz ordinates per Axiom 1. □

We further isolate how status depends on the shape of the income distribution by restricting how it moves across co-monotone distributions—where generations share the same ranks. Specifically, we require status moves linearly along the ray connecting any two such distributions. This condition sets how status units deal with the cardinality of income, as the position of each dynasty is fixed along these movements.

Axiom 5 (**Co-Monotone Rays**). *Order dynasties in Y such that $y_i \leq y_{i+1}$ for all $i = 1, \ldots, N$. Consider an alternative income vector Y' with the same strict order for all dynasties. An economy along the ray connecting these two vectors has $Y^\alpha = \alpha Y + (1-\alpha) Y'$, for $\alpha \in [0, 1]$. Status satisfies $s\left(y_i^\alpha, Y^\alpha\right) = \alpha s(y_i, Y) + (1-\alpha) s\left(y_i', Y'\right)$.*

Axiom 5 only covers mixing of co-monotone income distributions. For instance, the outcome of transfers that change the rank of a dynasty is not covered, neither are distributions with rank reversals between dynasties. Among co-monotone distributions, we impose no restrictions on the shape of Lorenz curves or the change in status levels between Y and Y'. In particular, Lorenz curves can cross allowing $s(y_i, Y) > s\left(y'_i, Y'\right)$ for some i and $s\left(y_j, Y\right) < s\left(y'_j, Y'\right)$ for some $j \neq i$. Nevertheless, Axiom 5 is enough to obtain that the ψ function in (2) is affine in Lorenz ordinates.

PROPOSITION 2. *A status function satisfies Axioms 1–5 if and only if*

$$s(y_i, Y) = \Omega(r_i) \cdot [\lambda L_i + (1-\lambda)\Xi(r_i)], \tag{4}$$

for weakly increasing functions $\Omega(r) \in (0,1]$ *and* $\Xi \in [0,1]$ *and scalar* $\lambda \in (0,1]$, *where* $r_i = r(y_i, Y)$ *and* $L_i = L(y_i, Y)$ *are the rank and Lorenz ordinate of dynasty* i.

PROOF. Let Y such that $y_i \leq y_{i+1}$ with strict inequality for at least one i and Y' a income with the same strict order. Without loss, we have $\mu = \mu'$, as status and the Lorenz curves are scale invariant (Axiom 3). Under Proposition 1, Axiom 5 requires

$$\begin{aligned} s(y_i^\alpha, Y^\alpha) &= \psi(r(y_i^\alpha, Y^\alpha), L(y_i^\alpha, Y^\alpha)) \\ &= \alpha\psi(r(y_i, Y), L(y_i, Y)) + (1-\alpha)\psi\left(r\left(y'_i, Y'\right), L\left(y'_i, Y'\right)\right). \end{aligned} \tag{5}$$

From Aaberge (2001), the Lorenz is linear under co-monotone mixtures,

$$L(y_i^\alpha, Y^\alpha) = \alpha L(y_i, Y) + (1-\alpha) L\left(y'_i, Y'\right). \tag{6}$$

Hence, ψ satisfies (5) if and only if it is affine on L, with coefficients that depend weakly on ranks, giving (4). The coefficient on L must be greater than zero to ensure strict monotonicity of status on income (Axiom 2). If Y and Y' are such that $y_i = y_j$ and $y'_i = y'_j$ for all i, j, Axiom 3 implies $s(y, Y) = s\left(y', Y'\right) = s(y^\alpha, Y^\alpha)$, which satisfies Axiom 5 immediately. Axiom 1 restricts ranges to ensure $s \in [0,1]$. □

Thus, status depends on the Lorenz ordinates of dynasties, their position in the Lorenz curve, and not just in their ranks, their position in the income distribution. This endows status with cardinal information, sets the units and range of status, and makes explicit the link between inequality and status (as we discuss momentarily). The (increasing) weights on the Lorenz ordinates can capture social preferences that depend on the position of individuals, reinforcing the role of inequality.[6] Finally, while our representation of status goes beyond ranks, they are a limiting case when $\lambda \longrightarrow 0$, $\Omega(r) = 1$, and $\Xi(r) = r$. We return to this case when discussing mobility where we also discuss the particularly illustrative and novel case of *Lorenz status*, when $\lambda = 1$ and $\Omega(r) = 1$ so that status coincides with Lorenz ordinates.

Proposition 2 sets a mapping from income distributions to individual status, providing clear and interpretable units for our status measure (and later for mobility) in terms of *income shares*. Crucially, Lorenz ordinates (and hence status) are the same under incomes $\{Y^P, Y^K\}$ as in an economy where average income is constant across generations, $\{\mu^K/\mu^P Y^P, Y^K\}$. This separates overall income growth from mobility in economic status. Consequently, an increase in status by 0.1 for dynasty i corresponds to the dynasty increasing their own income by 10% of aggregate (fixed at the income level of either generation).

Inequality and status. The conditions we have imposed imply that status depends on the *Lorenz ordinates* of dynasties. This clarifies the implications of inequality for mobility: movements towards equality increase status. This is more easily seen by exploring the effects of Pigou-Dalton transfers, which modify the shape of the income distribution locally by making it less unequal.[7] These transfers towards equality necessarily increase status in the economy in the sense of the usual stochastic order, as they lead to a Lorenz curve that is pointwise above the original as in Figure 1B, producing an order over inequality and status (see Atkinson 1970). Moreover, when Pigou-Dalton transfers preserve ranks, the status of each dynasty increases.

For instance, consider proportional Pigou-Dalton transfers which smoothly reduce

[6]This is consistent with rank based weights in Yaari's 1988 evaluation measure, the parametric weighting in Donaldson and Weymark's 1980 S-Gini index, as well as generalized concentration indexes (Kakwani, Wagstaff, and Van Doorslaer 1997, Wagstaff 2002).

[7]Consider two dynasties $h, j \in \{1, \dots, N\}$ with $y_h < y_j$. A transfer $\delta > 0$ from j to h is a Pigou-Dalton transfer if j is not poorer than h after the transfer, $y_h + \delta \leq y_j - \delta$.

inequality across the distribution, delivering a new vector $Y^{\alpha} = \alpha Y + (1 - \alpha)\mu$ for some $\alpha \in [0, 1]$. These transfers towards equality preserve ranks and thus increase the status of all dynasties (as seen in Figure 1A). The new status satisfies

$$s\left(y_i^{\alpha}, Y^{\alpha}\right) = \alpha\psi\left(r_i, L_i\right) + (1 - \alpha)\psi\left(r_i, r_i\right) , \tag{7}$$

where $r_i = r\left(y_i, Y\right) \geq L\left(y_i, Y\right) = L_i$ are the rank and Lorenz ordinate of y_i. Thus, status is maximized under equality with $s\left(\mu, [\mu]_{i=1}^{N}\right) = 1$.

2.2. Status mobility

We now turn to characterizing how to measure status mobility taking the measurement of economic status as given. We impose mobility is symmetric, thus it measures the absolute change in status, similar to the measure in Fields and Ok (1996) for income levels. We discuss signed mobility in Section 3.1.

PROPOSITION 3 (**Absolute Status Mobility**). *A mobility function* $m : \mathbb{R}_{+} \times \mathbb{R}_{+} \longrightarrow \mathbb{R}_{+}$ *is*

(a) *Continuous in all arguments;*

(b) *Symmetric in that* $m\left(s, s'\right) = m\left(s', s\right)$*;*

(c) *Step-additive in that* $m\left(s, s''\right) = m\left(s, s'\right) + m\left(s', s''\right)$ *for all* $s \leq s' \leq s''$*; and*

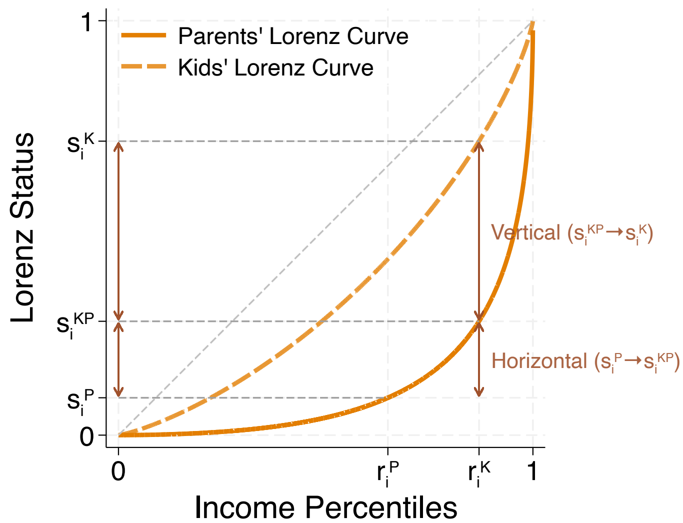
(d) *Translation-invariant in that* $m\left(s + \zeta, s' + \zeta\right) = m\left(s, s'\right)$ *for all* s, s' *and* $\zeta > 0$*;*

if and only

$$m\left(s^{P}, s^{K}\right) \propto \left|s^{K} - s^{P}\right| . \tag{8}$$

PROOF. Consider status s. By translation invariance $m(s, s + \zeta) = m(0, \zeta)$ for $\zeta \geq 0$. So, mobility does not depend on s, only on the gap ζ. Define mobility by a gap ζ as $g(\zeta) \equiv m(0, \zeta)$. g is immediately continuous because m is continuous. Moreover, mobility is additive on the gap, that is, $g\left(\zeta + \zeta'\right) = g(\zeta) + g\left(\zeta'\right)$. To see this let $\zeta \geq \zeta' \geq 0$,

FIGURE 2. Horizontal and Vertical Mobility



Notes: The Figure illustrates horizontal and vertical mobility for a dynasty moving from rank r_i^P in the parents' Lorenz curve (dark solid line) to rank r_i^K in the kids' curve (light dashed line). Horizontal mobility corresponds to the movement from initial status s_i^P to the status of individuals with rank r_i^K in the parents' Lorenz curve, s_i^{KP}. Vertical mobility corresponds to the movement from s_i^{KP} to the final status s_i^K.

step-additivity of m gives

$$g\left(\zeta + \zeta'\right) = m\left(0, \zeta + \zeta'\right) = m(0, \zeta) + m\left(\zeta, \zeta + \zeta'\right) = g(\zeta) + g\left(\zeta'\right). \tag{9}$$

Therefore, $g(\zeta) = \kappa\zeta$ for some $\kappa \geq 0$, because continuous additive functions are linear.

Consider s and s' with $s \leq s'$. We have $m\left(s, s'\right) = g\left(s' - s\right) = \kappa(s' - s)$. Symmetry extends the characterization to all status pairs s and s' with $m\left(s, s'\right) = \kappa\left|s' - s\right|$.

□

Horizontal and vertical mobility. We further distinguish between changes in status coming from changes in rank or position under the same income distribution, *horizontal mobility*, and changes in status coming from changes in inequality while holding rank constant, *vertical mobility*. This distinction is evident in the case of *Lorenz mobility*, when status coincides with Lorenz ordinates, as we illustrate in Figure 2.

Formally, consider an economy with income vectors $\{Y^P, Y^K\}$, and write mobility as

$$m\left(s_i^P, s_i^K\right) = \left|s_i^K - s_i^P\right| = \left|\overbrace{s_i^K - s_i^{KP}}^{\text{Vertical Mobility}} + \underbrace{s_i^{KP} - s_i^P}_{\text{Horizontal Mobility}}\right|, \tag{10}$$

where $s_i^{KP} \equiv \widetilde{L}^P\left(r_i^K\right)$ is the status of generation K under P's income distribution while retaining the rank they have under K's distribution, and $\widetilde{L}^G(r)$ is the Lorenz curve of generation $G \in \{P, K\}$ defined over ranks—the income share of individuals in G with rank lower than or equal to r. *Horizontal mobility*, the difference between s_i^{KP} and s_i^P, captures status changes coming from rank changes across generations, holding inequality constant. This corresponds to movements along generation P's Lorenz curve. *Vertical mobility*, the difference between s_i^K and s_i^{KP}, captures changes in the distribution of status across generations, holding the dynasty's rank constant. This corresponds to a (vertical) move from generation P's Lorenz curve to K's curve.

Mobility between co-monotone generations comes only from vertical mobility: solely reflecting changes in inequality. A decrease in inequality between generations implies positive vertical mobility: status is higher at each rank in the less unequal generation. Mobility between generations with the same distribution of income (as captured by the Lorenz curve) come only from horizontal mobility.

2.3. Aggregating mobility

Finally we aggregate mobility via a function $\mathcal{M} : \mathbb{R}^N \longrightarrow \mathbb{R}$ from a vector M of dynastic mobilities, with a typical element m_i. We adopt the axiomatic characterization of general *quasi-arithmetic* aggregators in Kolmogorov-Nagumo-de Finetti's theorem (see Hardy, Littlewood, and Pólya 1952, Thm. 215).

THEOREM 1 (**Kolmogorov-Nagumo-de Finetti**). *A family of aggregators $\mathcal{M}_N : \mathbb{R}^N \longrightarrow \mathbb{R}$, indexed by N, is*

(a) *Continuous and strictly increasing in all arguments;*

(b) *Symmetric in that* $\mathcal{M}_N(PM) = \mathcal{M}_N(M)$ *for any permutation matrix* P*;*

(c) *Reflexive in that* $\mathcal{M}_N(m\mathbb{1}_N) = m$ *for any* m*; and*

(d) *Associative in that* $\mathcal{M}_N(M) = \mathcal{M}_N([\bar{m}, \ldots, \bar{m}, m_{K+1}, \ldots, m_N])$*, where* $\bar{m} = \mathcal{M}_K([m_1, \ldots, m_K])$*;*

if and only if there is a continuous function Γ *such that*

$$\mathcal{M}_N(M) = \Gamma^{-1}\left(\frac{1}{N}\sum_{i=1}^{N}\Gamma(m_i)\right). \tag{11}$$

The conditions imposed by the Kolmogorov-Nagumo-de Finetti theorem are standard and are shared by additive aggregators common in the mobility literature (e.g., Bartholomew 1973; Fields and Ok 1996, 1999; Cowell and Flachaire 2018; Ray and Genicot 2023). The first condition ensures higher mobility for any dynasty is reflected in higher aggregate mobility. The second condition ensures *anonymity*, so the indexes of dynasties are superfluous. The third condition ensures consistency in that if all dynasties have the same mobility $(m_i = m)$ then aggregate mobility must coincide with this movement ($\mathcal{M}_N = m$). The fourth and final condition is the most restrictive, but it is necessary for the additivity of the aggregator (and the reason behind the *quasi-arithmetic* moniker). It implies common properties in the study of inequality and mobility such as replication invariance, so $\mathcal{M}_{nN}\left(\widehat{M}\right) = \mathcal{M}_N(M)$ for $\widehat{M} = \mathbb{1}_n \otimes M$ obtained by replicating the population $n \in \mathbb{N}$ times.

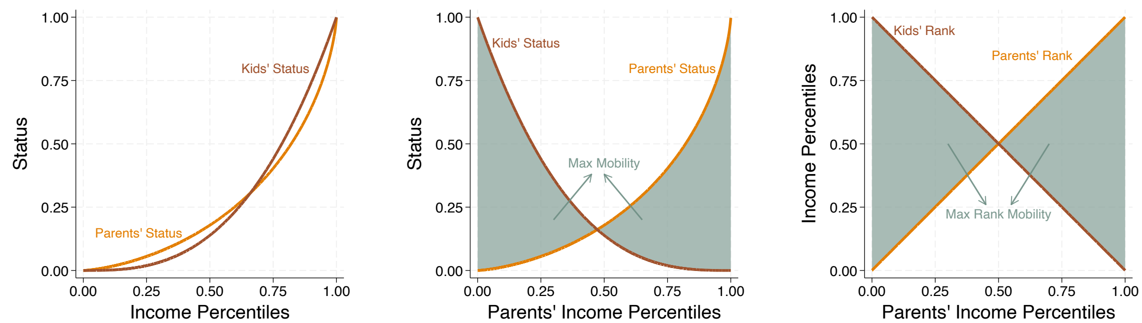
We further pin down the aggregator by requiring $\mathcal{M}$ be homogeneous of degree 1. So, if mobility is re-scaled, aggregate mobility is re-scaled in the same way. The result is standard and we state it without proof (see Hardy, Littlewood, and Pólya 1952, Thm. 84).

Lemma 1 (**Homogeneous Aggregation**). *Let* $\mathcal{M}_N : \mathbb{R}^N \longrightarrow \mathbb{R}$ *satisfy Theorem 1.* $\mathcal{M}_N$ *is homogeneous of degree 1, so that* $\mathcal{M}_N(\kappa M) = \kappa\mathcal{M}_N(M)$ *for all* $\kappa > 0$*, if and only if* $\Gamma(m) = m^\gamma$ *for some* $\gamma \neq 0$*, with the* $\gamma = 0$ *case corresponding to the geometric average.*

The curvature of the aggregator, γ, determines the emphasis on broad-based versus

FIGURE 3. Maximal Mobility

(A) Initial Status Curves (B) Counter-Monotone Ranks (C) Maximal Rank Mobility



Notes: Panel A shows Lorenz curves for parents and kids. Panel B shows the maximal mobility given these curves obtained by the counter-monotone assignment of parents to kids. Panel C shows the maximal rank mobility. Mobility $\mathcal{M}$ with $\gamma = 1$ corresponds to the shaded areas.

concentrated mobility in just the same way as in measures of inequality aversion (Atkinson 1970) or choice under uncertainty (as $\mathcal{M}$ also gives the family of certainty equivalents over $\{m_i\}$ under constant relative risk aversion, Ackerberg, Hirano, and Shahriar 2017). When $\gamma = 1$ we recover the arithmetic mean, which neither favors nor penalizes dispersion in mobility. For positive γ, lower values favor broad-based mobility while higher values favor dispersion, effectively placing a higher weight on dynasties with the highest mobility. Negative values of γ place more weight on the dynasties with the lowest mobility. Although γ can tilt aggregate mobility towards the highest or lowest mobility, our representation of aggregate mobility precludes weighting who moves as there are no weights assigned to dynasties based on their identity or initial conditions.

Bounds on Mobility. We conclude this section by providing a tight upper bound for aggregate mobility. Given two income distributions (Figure 3A), the highest mobility is achieved when dynasties are ordered in counter-monotone fashion[8] as in Figure 3B

[8]The dynasty with the i^{th} highest income in the parent's generation has the $(N+1-i)^{\text{th}}$ highest income in the children's generation.

because pair-wise mobility is represented by a Monge matrix—a sub-modular discrete function. This result holds regardless of the shape of the status functions and covers horizontal mobility, when the two income distributions are identical, and rank mobility, as in Figure 3C, see Burkard, Dell'Amico, and Martello (2012, Prop. 5.7).

We obtain a global upper bound by maximizing counter-monotone mobility over the space of Lorenz curves. This is achieved when Lorenz curves are extremal in the sense of Baillo, Carcamo, and Mora-Corral (2022). The upper-bound on *Lorenz mobility* when $\gamma = 1$ is obtained as,[9]

$$0 \leq \mathcal{M}(M) \leq 2 - \sqrt{2}\,. \tag{12}$$

Maximal *rank mobility* is 1/2 and corresponds to maximal exchange mobility (Bartholomew 1973). See Online Appendix A for details.

3. Extensions

3.1. Signed status mobility

In Proposition 3, we required symmetry of the individual status mobilities leading to a measure of absolute status mobility. However, in some settings, society may value the *direction* of mobility. The following characterization of mobility replaces symmetry in favor of *signed-symmetry*

PROPOSITION 4 (**Signed Status Mobility**). *A mobility function* $m : \mathbb{R}_+ \times \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ *satisfies properties (a), (c), and (d) of Proposition 3 and is signed-symmetric, in that* $m\left(s, s'\right) = -m\left(s', s\right)$ *with* $m\left(s, s'\right) > 0$ *for all* $s < s'$*, if and only if*

$$\widetilde{m}\left(s^P, s^K\right) \propto s^K - s^P\,. \tag{13}$$

[9] Interestingly, this bound is achievable by horizontal mobility. So, it does not require changes in inequality, it instead reflects how inequality shapes changes in status across positions.

Proof. Consider status s and $s^{'}$ with $s < s^{'}$. We have $m\left(s, s^{'}\right) = g\left(s^{'} - s\right) = \kappa(s^{'} - s) > 0$ from the proof of Proposition 3. Signed-symmetry extends the characterization to all status pairs s and $s^{'}$ with $m\left(s, s^{'}\right) = \kappa\left(s^{'} - s\right)$ and $\kappa > 0$. □

Since neither the characterization of status nor of the mobility aggregator depend on the symmetry of status mobility, our results yield a plug-in alternative using the signed mobility measure in (13). When mobility is aggregated uniformly, $\gamma = 1$, this yields a particularly intuitive closed form expression for *net* mobility.

Lemma 2 (**Net Mobility**). *Under Lemma 1 with* $\gamma = 1$*, and Propositions 2 and 4, aggregate mobility has the form:*

$$\widetilde{\mathcal{M}}_N(M) \propto \widetilde{G}\left(Y^P\right) - \widetilde{G}\left(Y^K\right), \tag{14}$$

where $\widetilde{G}(Y) \equiv 1 - \frac{2}{N}\sum_{i=1}^{N}\Omega\left(r_i\right)L_i$ *is Weymark's (1981) Generalized-Gini coefficient. When* $\Omega(r) = 1$ *for all* r *we obtain the usual Gini coefficient* $G(Y) = 1 - \frac{2}{N}\sum_{i=1}^{N} L_i$.

This result clarifies conditions under which changes in inequality, summarized by the Gini, capture mobility across the population. When inequality decreases, so $G\left(Y^K\right) < G\left(Y^P\right)$, net mobility is positive. A decrease in inequality increases the status of dynasties on net. For instance, implementing proportional Pigou-Dalton transfers lowers the Gini coefficient and unequivocally increases mobility as it increases the status of all dynasties (see equation 7 and Figure 1A).

However, comparisons are not straightforward in general. Even under arbitrary Pigou-Dalton transfers, dynasties are subject to both vertical and horizontal mobility as in (10) so that their status can go down even as the distribution of status in the economy improves. Nevertheless, Lemma 2 establishes net mobility is entirely captured by changes in the Gini coefficient or vertical mobility (horizontal mobility is zero-sum).This completes the link between mobility and inequality even when the Lorenz curves of Y^K and Y^P intersect. See Atkinson (1970) and Aaberge and Mogstad (2011) for alternative discussions of ordering intersecting Lorenz curves.

3.2. Intergenerational status correlation

An alternative approach to measure mobility estimates the correlation of individual characteristics across generations. Typically, researchers use intergenerational elasticities of income and the Spearman correlation of income ranks, often obtained from *Galtonian* regressions. The coefficient from log-income regressions can be derived axiomatically as a Hart's 1983 mobility measure, as shown in Shorrocks (1993), or as the reduced form of Becker and Tomes (1979) models.

Our approach provides a justification for measuring intergenerational correlations of status, using Lorenz ordinates, instead of income or income ranks. The result is an extension of our analysis to the measurement of the intergenerational correlation or elasticity of status.[10] For example, computing $1 - \text{corr}\left(s_i^P, s_i^K\right)$ or estimating

$$\log\left(s_i^K\right) = \beta_0 + \beta_1 \log\left(s_i^P\right) + u_i\,, \tag{15}$$

where the mobility measure is $1 - \hat{\beta}_1$.

4. Lorenz Mobility in the US

We now apply our framework to the intergenerational mobility of Americans. We follow Davis and Mazumder (2024) and use data from the National Longitudinal Surveys of Older Men and Young Men and Mature Women and Young Women (NLS66) and the National Longitudinal Survey of Youth 1979 (NLSY79). We exploit a feature of the initial sample design, where multiple respondents were surveyed within the same household, to link parents with children. As Davis and Mazumder demonstrate, this allows researchers to use detailed longitudinal income information on both generations.

[10] In the elasticity case $\log\left(s\left(y_i, Y\right)\right)$ is well-defined as long as $Y > 0$; the same condition required by $\log\left(y_i\right)$. A regression approach can also be motivated as the best linear prediction of status.

We pool the different birth cohorts (approximately those born in the early 1950s and the early 1960s) and weight our analysis using the child's first-round survey weights. The remainder of our sample selection follows Davis and Mazumder (2024) and we refer the interested reader there.[11] Our measure of income for each generation corresponds to total family income averaged over three years.

The distributions of children moves rightward relative to their parents, reflecting overall income growth of 7.8%, and have less mass concentrated at low incomes (Figure 4A). The changes in the shape of the income distributions across generations are reflected in the Lorenz curves (Figure 4B) which are markedly similar, despite crossing at the top of the income distribution.

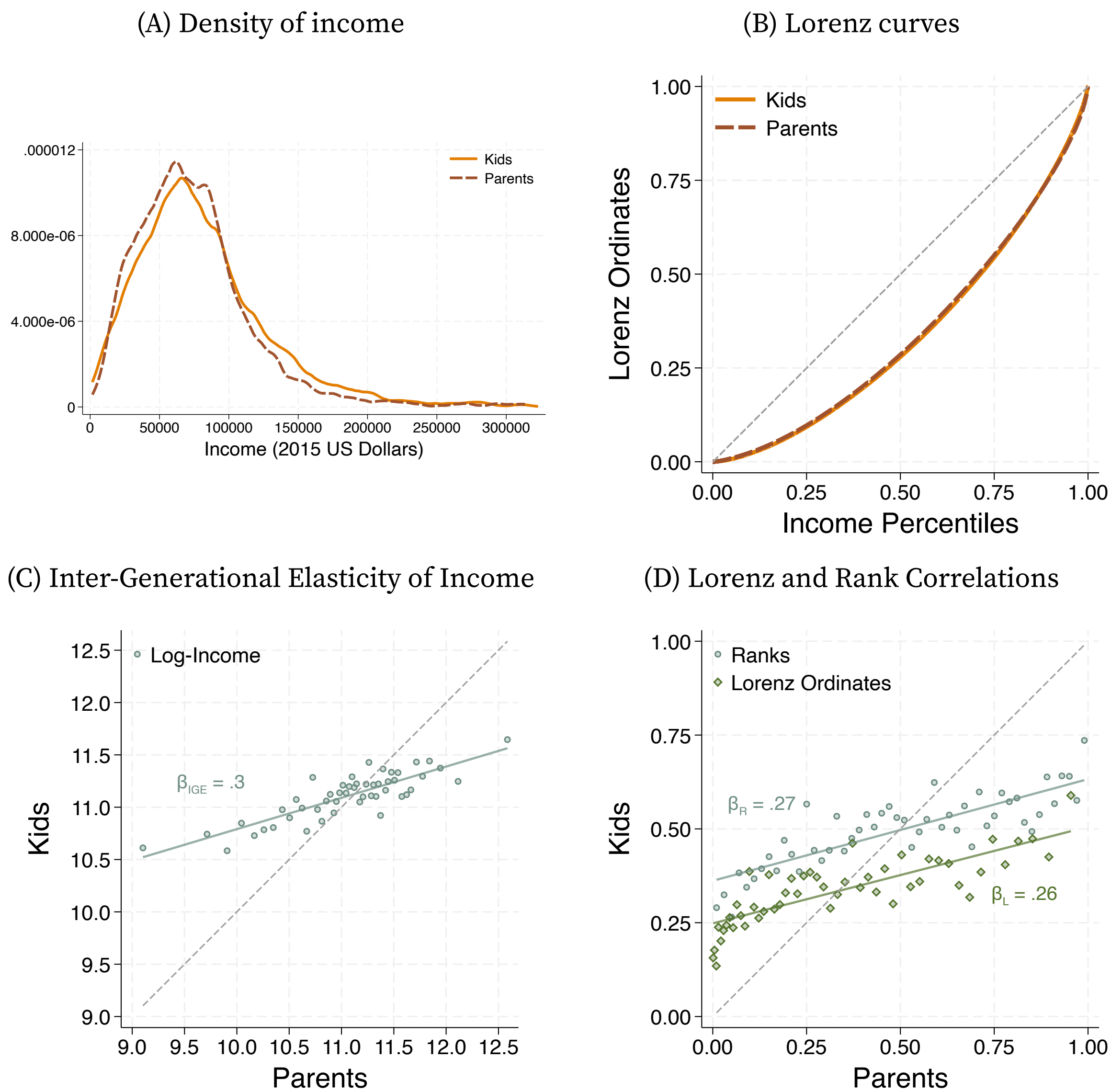
Intergenerational status mobility. We first focus on Lorenz and rank mobility and then compare to existing mobility measures in Table 1. We find Lorenz mobility is lower than rank mobility, reflecting the role of income inequality in shaping differences across generations. As we see from our main mobility measure ($\mathcal{M}$), Lorenz status across generations moves on average by 0.23, while aggregate rank mobility is higher at 0.26.[12] Lorenz mobility corresponds mostly to horizontal mobility, reflecting changes in the position of dynasties, as there are only small differences between the Lorenz curves of both generations (Figure 4B). This indicates that the higher level of rank mobility is partly driven by "spurious" changes in ranks corresponding to small moves in relative income. The lack of vertical mobility is further evidenced by low net (or Gini) mobility ($\widetilde{\mathcal{M}}$). In fact, vertical mobility is negative as the children's generation is more unequal than the parents'.

Mobility itself is also unequal. We see this by comparing our main measure of

[11]We are indebted to the authors for providing a clear and transparent replication package which allows us to reconstruct their analysis data.

[12]Furthermore, measured Lorenz mobility is 39% of its maximal value, while rank mobility is 53% of its maximal value.

FIGURE 4. Income distribution across generations



Notes: Panels A and B show the kernel density of income in each generation and the corresponding Lorenz curves. Panel C shows a binned scatter of log income across generations along with its trend line. Panel D shows scatters of Lorenz ordinates in circles and income ranks in diamonds.

aggregate mobility $\mathcal{M}$ without curvature, $\gamma = 1$, to measures with lower and higher curvature. When we set $\gamma = 1/2$, aggregate mobility favors a more even distribution of mobility, reducing $\mathcal{M}$ if mobility is concentrated among few dynasties. Indeed, Lorenz mobility is reduced to just under 18 percent of aggregate income. Conversely,

TABLE 1. Intergenerational mobility measures

Income Growth	Status Mobility: $\mathcal{M} = \left(\frac{1}{N}\sum (m_i)^{\gamma}\right)^{\frac{1}{\gamma}}$ Rank	Lorenz				Intergenerational Correlations: $1 - \rho\left(x_i^P, x_i^K\right)$			
	$\gamma = 1$	$\gamma = 1$	$\gamma = 1/2$	$\gamma = 2$	$\widetilde{\mathcal{M}}$	Lorenz	Log-Lorenz	Rank	Log-income
0.08	0.264	0.232	0.177	0.319	−0.032	0.742	0.728	0.729	0.701

Notes: The table reports income growth along with measures of intergenerational mobility. Income growth is in the first column. Status mobility measures in the next 4 columns correspond to Lemma 1 and Proposition 1. The sixth column corresponds to net mobility (Lemma 2). Intergenerational correlations are estimates from regressions of each variable between generations.

$\mathcal{M}$ increases substantially when setting $\gamma = 2$, as it places more weight on dynasties with higher mobility, rising to 32 percent of aggregate income.

Finally, we look at standard measures of intergenerational correlations for log-income, ranks, and status levels. We report them in the last four columns of Table 1 and the corresponding scatter plots in Figures 4C and 4D. In practice these measures of intergenerational correlation produce similar evaluations of economic mobility. This is in part because Lorenz ordinates across generations have similar concordance as ranks and income.[13] This is despite substantial income inequality in each generation, whose impact on the dispersion of mobility referenced above is missed by these statistics.

Alternative contexts. In Online Appendix B we reproduce our analysis using Norwegian administrative data on the universe of Norwegian tax residents. This allows us to verify that the patterns of Lorenz mobility we cover are not driven by particular US institutional features, including the high degree of income inequality, or the coverage of our survey data. The Norwegian data allows us to study the individual incomes of mothers and daughters as well as fathers and sons. Our results for mothers

[13] Concordance produces a common ordering over mobility measures (McGee 2025).

and daughters echo our findings for the US: measured mobility is almost entirely horizontal, as inequality stayed roughly constant across generations despite significant income growth of 18% between generations. In contrast, we find an important role for decreased inequality for fathers and sons driving vertical mobility of roughly the same magnitude as the increase in inequality in the US sample.

5. Conclusions

We connect the definition of economic status and the Lorenz curve bringing standard concepts from the study of inequality to the study of mobility. This incorporates both ordinal and cardinal information. In sufficiently equal societies, Lorenz ordinates are close to rank-based measures of status. In unequal societies, differences in Lorenz ordinates are small where the income distribution is compressed and large where income is dispersed, capturing material differences between individuals. The link between inequality and mobility is complete in a special case of net status mobility, which is equivalent to changes in the generalized Gini coefficient.

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Online Appendix
Not For Publication

Beyond Ranks:

Inequality in the Measurement of Mobility[1]

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Appendix A. Bounds on Lorenz Mobility

We consider the maximum mobility, $\mathcal{M}$, when $\gamma = 1$ given two income distributions. This corresponds to the value of an assignment from generation P to generation K. Order dynasties in Y^G such that $y_i^G \leq y_{i+1}^G$ for all $i = 1, \ldots, N$ and $G \in \{P, K\}$, the index i determines the order of income and not the identity of the dynasty. Denote by s_i^G the corresponding status of the dynasty with income position i in generation G, we know that $s_i^G \geq 0$ is increasing in i. The assignment (or transport) that maximizes mobility solves

$$\max_T \frac{1}{N} \sum_{i=1}^{N} \left| s_{T(i)}^K - s_i^P \right| \tag{A.1}$$

The optimal assignment is counter-monotone because pair-wise mobility is represented by a Monge matrix M (a sub-modular discrete function) with typical entry $m_{ij} = \left| s_j^K - s_i^P \right|$, so that $T^{\star}(i) = N + 1 - i$. This result is a direct application of the rearrangement of linear sums in Burkard, Dell'Amico, and Martello (2012, Prop 5.7). The key step requires establishing that M satisfies the Monge property, that for $1 \leq i < k \leq n$ and $1 \leq j < \ell \leq n$ the entries of M satisfy

$$m_{ij} + m_{k\ell} \leq m_{i\ell} + m_{kj} \; ; \tag{A.2}$$

$$\left| s_j^K - s_i^P \right| + \left| s_\ell^K - s_k^P \right| \leq \left| s_\ell^K - s_i^P \right| + \left| s_j^K - s_k^P \right| . \tag{A.3}$$

To prove this, consider $i < k$ and define

$$f(x) \equiv \left| x - s_i^P \right| - \left| x - s_k^P \right| = \begin{cases} s_i^P - s_k^P, & x \leq s_i^P , \\ 2x - (s_i^P + s_k^P), & s_i^P \leq x \leq s_k^P , \\ s_k^P - s_i^P, & x \geq s_k^P . \end{cases} \tag{A.4}$$

hence f is non-decreasing in x. We know $s_j^K \leq s_\ell^K$ because $j < \ell$, so $f\left(s_j^K\right) \leq f\left(s_\ell^K\right)$:

$$\left| s_j^K - s_i^P \right| - \left| s_j^K - s_k^P \right| \leq \left| s_\ell^K - s_i^P \right| - \left| s_\ell^K - s_k^P \right| . \tag{A.5}$$

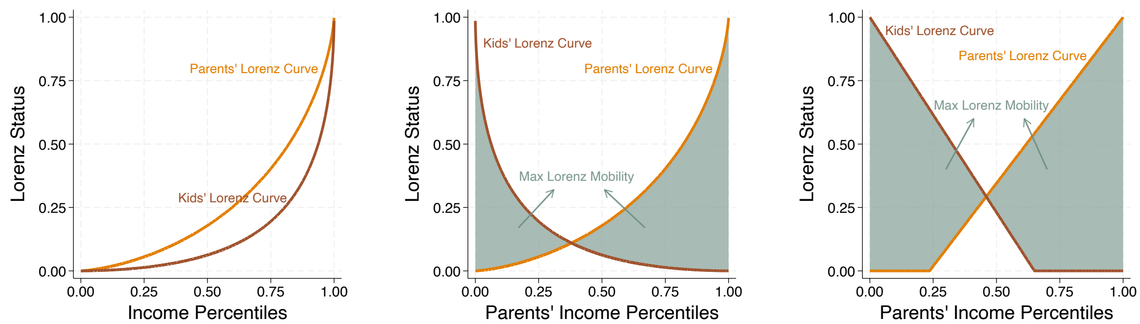
Rearranging yields the inequality in (A.3), following from the sub-modularity of m.

To prove that the counter-monotone assignment is optimal, consider any candidate assignment T that is not strictly decreasing, so that is $i < k$ with $T(i) < T(k)$. Swap the assignment of i and k, leading to a new assignment with $\widehat{T}(i) = T(k)$, $\widehat{T}(k) = T(i)$, and $\widehat{T}(\omega) = T(j)$ for $\omega \neq i, k$. Then, set $j = T(i)$ and $\ell = T(k)$ and apply (A.3) to get that

$$m_{i,T(i)} + m_{k,T(k)} \leq m_{i,\widehat{T}(i)} + m_{k,\widehat{T}(k)} . \tag{A.6}$$

FIGURE A.1. Maximal Lorenz Mobility

(A) Initial Lorenz Curves (B) Counter-Monotone Ranks (C) Extreme Lorenz Curves



Notes: Panel A shows Lorenz curves for parents and kids. Panel B shows the maximal mobility given these curves obtained by the counter-monotone assignment of parents to kids. Panel C shows the maximal mobility across Lorenz curves obtained by extreme curves of the form $\tilde{L}_c(r) = {}^{r-c}/_{1-c}$ for $r > c$ and 0 otherwise. Mobility $\mathcal{M}$ with $\gamma = 1$ corresponds to the shaded areas.

So, the new assignment weakly improves mobility. Save from ties in status, only the counter-monotone assignment is strictly decreasing and maximizes mobility.

Horizontal mobility is also maximized by the counter-monotone assignment as it consist of the special case when the two income distributions coincide and $s_i^K = s_i^P$.

The upper envelope of mobility is obtained by maximising over Lorenz curves. For analytic convenience we compute this in the limiting case as $N \longrightarrow \infty$.[2] Baillo, Carcamo, and Mora-Corral (2022) show that the extreme points in Lorenz space are piecewise linear functions. As the objective function is linear in each curve and we maximise over a convex set, it is sufficient to search for the kink in the two-segment Lorenz curves maximising mobility.[3] That is Lorenz curves defined by kink $c^G \in [0,1]$ so that $\tilde{L}_{c^G}(r) = 0$ for $r \in [0,c]$, and $\tilde{L}_{c^G}(r) = \frac{r-c^G}{1-c^G}$ for $r \in (c,1]$.

Mobility is therefore bounded by the maximum mobility under the counter-monotone assignment, integrating $\tilde{m}(r) = \left| \tilde{L}_{c^K}(1-r) - \tilde{L}_{c^P}(r) \right|$,

$$\max_{c^P, c^K \in [0,1]} \int_0^1 \left| \tilde{L}_{c^K}(1-r) - \tilde{L}_{c^P}(r) \right| \, dr \, . \tag{A.7}$$

There are two cases determined by the breaks in (A.7) at $r = c^P$ and $r = 1 - c^K$:

(a) $0 \leq c^P < 1 - c^K \leq 1$: There are four intervals to consider. One of the curves being

[2] The counter-monotone assignment can be now expressed in terms of ranks, so that $T^\star(r) = 1 - r$. The argument is the same as above but builds directly in the sub-modularity of the difference in status, now defined as functions of rank, r.

[3] Formally, Baillo, Carcamo, and Mora-Corral show that this is the extremal curve for any given value of the Gini coefficient. Searching over kink-points then amounts to searching over Gini coefficients.

zero in two intervals the other two are determined by their crossing at $r = \frac{1-c^K}{2-c^P-c^K}$.

$$\tilde{m}(r) = \begin{cases} \frac{1-c^K-r}{1-c^K} & r \in \left[0, c^P\right] ; \\ \frac{1-c^K-r}{1-c^K} - \frac{r-c^P}{1-c^P} & r \in \left(c^P, \frac{1-c^K}{2-c^P-c^K}\right] ; \\ \frac{r-c^P}{1-c^P} - \frac{1-c^K-r}{1-c^K} & r \in \left(\frac{1-c^K}{2-c^P-c^K}, 1-c^K\right) ; \\ \frac{r-c^P}{1-c^P} & r \in \left[1-c^K, 1\right] . \end{cases} \tag{A.8}$$

The integral is then

$$\int_0^1 \tilde{m}(r)\, dr = \frac{1+c^P}{2} - \frac{\left(c^P\right)^2}{2\left(1-c^K\right)} + \frac{\left(1-c^P-c^K\right)^2\left(\left(1-c^P\right)^2 + \left(1-c^K\right)^2\right)}{2\left(1-c^K\right)\left(1-c^P\right)\left(2-c^P-c^K\right)} - \frac{\left(1-c^P-c^K\right)^2}{2\left(1-c^P\right)} . \tag{A.9}$$

(b) $0 \leq 1-c^K \leq c^P \leq 1$: There is a gap between $1-c^K$ and c^P where both functions are zero. So the integral is

$$\int_0^1 \tilde{m}(r)\, dr = \int_0^{1-c^K} 1 - \frac{r}{1-c^K}\, dr + \int_{c^P}^1 \frac{r-c^P}{1-c^P}\, dr = \frac{1-c^K}{2} + \frac{1-c^P}{2} . \tag{A.10}$$

When $1-c^K = c^P$ the lines touch but do not intersect and the integral is equal to 1/2. We can now establish the maximal mobility by selecting the values of c^P and c^P. Crucially, mobility does not depend on the values of c^P and c^P independently, but on the gap between these values and 1: $g \equiv 1-c^P-c^K$. The first case above corresponds to $g > 0$ and the second case to $g \leq 0$. Then, mobility is

$$\int_0^1 \tilde{m}(r)\, dr = \begin{cases} \frac{1+2g-g^2}{2(1+g)} & g > 0 ; \\ \frac{1+g}{2} & g \leq 0 . \end{cases} \tag{A.11}$$

The second case is maximized when $g = 0$ and mobility is 1/2. The critical values of the first case are the roots of $1-2g-g^2$. The only admissible root is $g^\star = \sqrt{2}-1$ which delivers a value of $2-\sqrt{2} \approx 0.59$.

Therefore, the maximal mobility is attained by Lorenz curves of the form $\tilde{L}_{cG}$ under counter-monotone assignments between generations such that $c^P + c^k = 2-\sqrt{2}$. This tight upper bound implies that

$$0 \leq \mathcal{M}(M) \leq 2 - \sqrt{2} . \tag{A.12}$$

Moreover, this is also the upper bound for horizontal mobility, equivalent to constraining $c^P = c^K = c$ and setting $c = 1 - \frac{1}{\sqrt{2}}$.

Appendix B. Lorenz Mobility in Norway

We now apply our framework to the mobility of Norwegians across generations. We use detailed longitudinal data on individual income collected by the Norwegian tax authority between 1993 and 2017 as well as demographic information available in their population files. We focus on individual pre-tax market income from wages and capital. See Blundell, Graber, and Mogstad 2015 for details on the Norwegian income tax records.

There are several key advantages to using the Norwegian administrative data compared with survey and administrative data available in other countries. The data covers the entire population, including the richest Norwegians, allowing us to construct accurate income ranks and Lorenz ordinates. Second, income is third-party reported, eliminating concerns about measurement error from self-reporting and censoring. Third, we link incomes across generations without relying on imputation (e.g., Collins and Wanamaker 2022; Ward 2023; or Jácome, Kuziemko, and Naidu 2025) or bounding exercises (e.g., Chetty et al. 2017; Berman 2022; or Manduca et al. 2024).

Sample selection. We study Norwegians born between 1961 and 1970 and record their average income between the ages of 31 and 35. We drop individuals who earn less than $1,000 on average. We link these Norwegians to their fathers and mothers and record the parents' average income between the ages of 51 and 55. We construct two samples with this data. First, a sample of mothers and daughters defined by the female Norwegians born 1961–1970 with recorded information on their mothers. This sample contains a total of 128,562 paired observations. Second, an equivalent sample for sons and their fathers with a total of 101,595 observations.

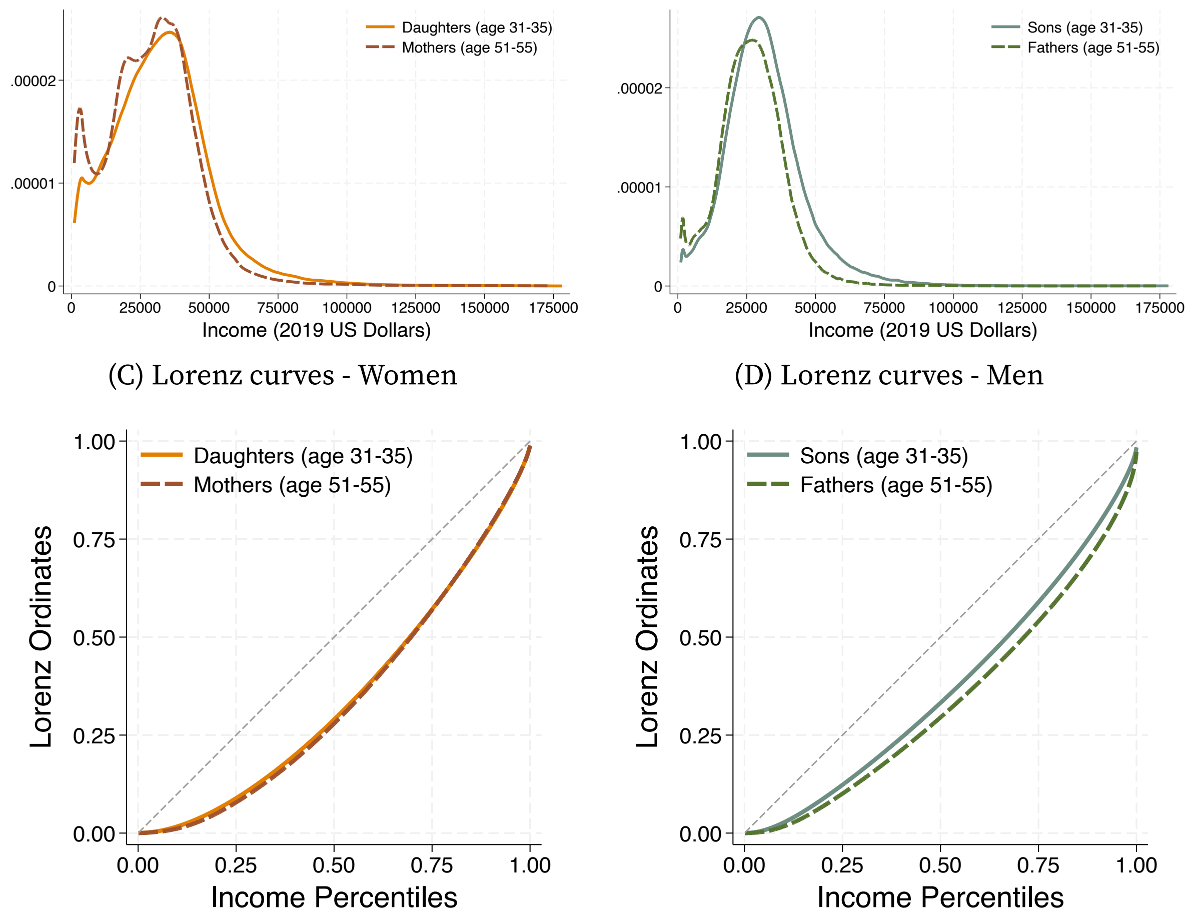
The distributions of daughters and sons move rightwards relative to their mothers and fathers, reflecting overall income growth, and have less mass concentrated at low incomes, as seen in Figures B.1A and B.1B. Daughters' income is 18% higher than their mothers', with the bottom 10 percentiles of the distribution growing more than 70%. Sons' income growth is smaller, slightly below 11%, and less widespread, with the top 15 percentiles of sons actually mapping to lower income levels than their fathers'.

Changes in income distributions across generations are reflected in the Lorenz curves. As seen in Figure B.1C, there are no significant differences between the Lorenz curves of daughters and mothers, despite crossing at the top of the income distribution. In contrast, the Lorenz curve of sons shifts towards equality, Figure B.1D, reflecting the higher income levels of the bottom 10 percentiles and lower income levels of the top 15.

Intergenerational status mobility. We find higher mobility in economic status for women than men across most mobility measures presented in Table B.1. As we see from our main mobility measure ($\mathcal{M}$), women's status across generations moves on average by 29% of their aggregate income compared to men's 26%. Status mobility among women corresponds mostly to horizontal mobility, reflecting changes in the position of dynasties, as there are only small differences between the Lorenz curves of mothers and daughters (Figure B.1C). The lack of vertical mobility is further evidenced

FIGURE B.1. Income distribution across generations

(A) Density of income - Women
(B) Density of income - Men
(C) Lorenz curves - Women
(D) Lorenz curves - Men



Notes: Panels A and B show the kernel density of income in each generation for women and men respectively. Panels C and D show the corresponding Lorenz curves.

by the lack of net (or Gini) mobility ($\widetilde{\mathcal{M}}$).

Mobility of economic status is broad-based for women and men alike. We can see this by comparing our main measure of aggregate mobility $\mathcal{M}$ without curvature, $\gamma = 1$, to measures with lower and higher curvature. When we set $\gamma = 1/2$, aggregate mobility favors a more even distribution of mobility, reducing $\mathcal{M}$ if mobility is concentrated among few dynasties. However, mobility remains high, with women's status moving by 23% of total income and men's moving by 22%. Conversely, $\mathcal{M}$ increases when setting $\gamma = 2$, as it weights more dynasties with higher mobility. These results imply that mobility is broad-based, but with some dynasties experiencing larger status changes.

Finally, we look at intergenerational correlations for status, ranks, and log-income

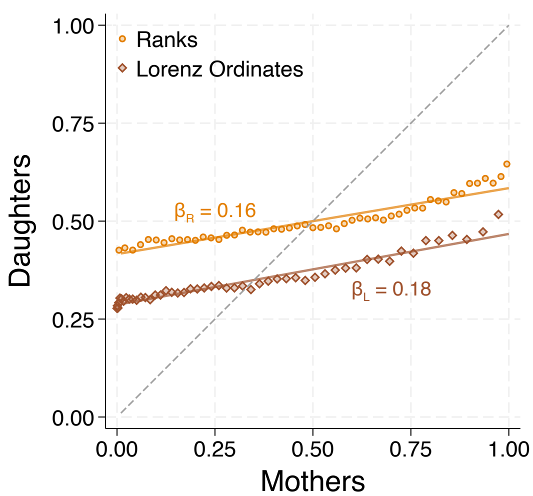
TABLE B.1. Intergenerational mobility measures

	Income Growth	Lorenz Mobility $\mathcal{M} = \left(\frac{1}{N}\sum(m_i)^{\gamma}\right)^{\frac{1}{\gamma}}$ $\gamma = 1$	$\gamma = 1/2$	$\gamma = 2$	$\widetilde{\mathcal{M}}$	Intergenerational Correlations $1 - \rho\left(x_i^P, x_i^K\right)$ Lorenz	Log-Lorenz	Rank	Log-income
Women	0.18	0.285	0.234	0.366	0.006	0.819	0.891	0.832	0.892
Men	0.11	0.261	0.215	0.334	0.033	0.746	0.876	0.777	0.879

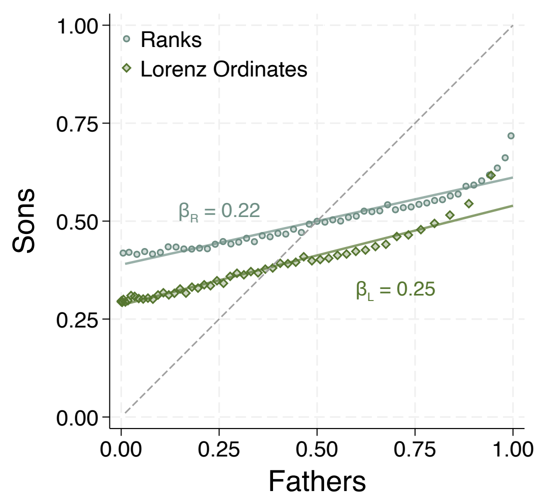
Notes: The table reports income growth along with measures of intergenerational mobility for the samples of Norwegian women (top row) and men (bottom row). Income growth is in the first column. The Lorenz mobility measures in the next 3 columns correspond to Lemma 1 and Proposition 3. The measure in the fifth column corresponds to net mobility (Lemma 2). Intergenerational correlation measures are obtained from regressions of each variable between generations.

FIGURE B.2. Intergenerational correlations: Lorenz status and ranks

(A) Women



(B) Men



Notes: The figures show binned scatters of Lorenz status in circles and income ranks in diamonds across generations along with their corresponding trend lines for women in panel A and men in panel B.

levels in the last four columns of Table B.1 and the corresponding scatter plots for the cases of status and ranks in Figure B.2. As expected, these different measures provide a consistent picture of intergenerational mobility (see McGee 2025). In particular, the correlation of economic status and income ranks are quantitatively similar. This is because the relatively low income inequality in our application results in Lorenz curves that are approximately linear between the 25th and 75th percentiles of income.