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Focal pricing constraints and pass-through of input cost changes

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Focal Pricing Constraints and Pass-Through of Input Cost Changes

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Abstract

Pass-through rates are relevant in a variety of contexts, such as estimating antitrust damages. It is often alleged that focal pricing, the practice of charging only special prices, e.g. ending in 9s, reduces the degree of pass-through in an industry. This claim has serious consequences, including the dismissal of court cases, but it is not grounded in economic theory or evidence. I prove that, in a simple but general framework, expected pass-through is unchanged by the presence of focal pricing constraints. It is therefore not safe to assume that pass-through will be low in industries characterised by focal pricing constraints.

1 Introduction

A firm sells goods to consumers, and incurs costs in the form of input costs and sales taxes. The degree to which changes in these costs are passed-through to changes in the prices paid by consumers is relevant in a variety of contexts, including tax incidence, merger control, and antitrust cases. Governments considering a hike in sales taxes are interested in the degree to which it will be absorbed by firms, and to what degree by consumers. Competition authorities deciding whether to allow a merger consider whether any potential efficiencies[1] from the proposed merger will in fact be passed through to consumers in the form of lower

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[1] Merger efficiencies are cost reductions resulting from the merger.

prices, and therefore may offset detrimental impacts from the merger, including reduced competitive pressure. Courts awarding antitrust damages rely on estimates of pass-through to quantify fair compensation. Because focal pricing has been raised as an issue in several class actions, the discussion in this paper focuses on the case of indirect purchaser class actions. In these cases, it is alleged that the claimants (end-consumers) paid inflated prices for end-products because input costs of production were inflated due to abusive conduct by one or more upstream firms (producers of the input). In order to estimate appropriate damages, it is necessary to estimate the extent to which inflated input costs were passed through to consumers, instead of being absorbed by downstream firms (purchasers of the input and producers of the end-product).[2]

If a £2 increase in input costs leads to a £2 increase in prices, then there is 'complete' pass-through, or an 100% pass-through rate. Depending on the context, pass-through rates can be below, above, or at 100%. They can be affected by numerous industry-specific characteristics, including the nature of competition between firms, the curvature of demand, returns to scale, and any frictions in setting prices. For instance, Bulow and Pfleiderer (1983)) discuss the theoretical impact of different demand curvatures on pass-through rates in the context of tax incidence. This paper aims to answer a question that has received very little academic attention to date: the impact of focal pricing constraints on pass-through rates. To my knowledge, this question has been addressed only by Alexandrov (2013), who sets out a high-level argument why focal pricing will typically not have a substantial impact on pass-through. I formally prove that the expected pass-through rate is unchanged by the presence of focal pricing constraints, under a simple but general framework. I refer to this as the Irrelevance Theorem. The Irrelevance Theorem holds exactly under several standard models of competition between firms, and different demand functions, and as an approximation in other cases. I also discuss the distribution of pass-through, which is generally characterised by more dispersion under focal pricing constraints.

Focal pricing is a widely observed phenomenon, consisting in firms only charging prices with specific characteristics. These are often prices with 9s in the last digits, as noted e.g. by Levy et al. (2011), and Snir et al. (2017). This can be explained if firms face constraints in the form of consumer inattention to certain digits, so that demand is unchanged within non-singleton sets of prices. This phenomenon in turn could be formalised with behavioural models (see e.g. Strulov-Shlain (2023)) or as rational inattention (see e.g. Basu (1997)). Focal pricing constraints may also arise for other reasons. For instance, Knotek (2008) and Knotek (2011)

[2] In some cases there may be additional layers in the industry, e.g. the input may be sold by an upstream firm to an intermediate firm to produce an output which is then sold by a retailer. The logic remains the same, though with a chain of pass-through.

show that 'convenient' prices (multiples of cash denominations) are widespread for frequently purchased goods paid in cash in high-traffic transactions. Moreover, virtually all firms employ focal pricing to some degree, because money denominations constrain most prices to being multiples of pence. We can think of these phenomena in terms of firms facing exogenous constraints on the set of prices they are able to choose from. Alternatively, we might think of a step-wise demand function, where several prices have equivalent effects on quantity demanded. Regardless of the exact nature of these constraints, they make it optimal for firms to charge focal prices, rather than non-focal alternatives.

The focal pricing constraints I study are distinct from two other closely related phenomena. The first are endogenous focal prices, which may emerge for instance as a mechanism to promote tacit collusion in an oligopolistic setting (see e.g. Scherr (1981)). This is a very different context from the exogenous focal pricing studied here, since firms are in fact able to charge non-focal prices, and in some cases might profit from doing so instead of charging a focal price, at least in the short-run. The other related, but distinct, phenomenon is menu costs. Menu costs involve an actual cost to changing prices, which is not present with focal pricing. Moreover, under menu costs, firms are able to price continuously rather than discretely, unlike with focal pricing. While they are distinct phenomena, they are easily conflated because they both cause price rigidity (stickiness), i.e. prices being changed infrequently, and typically not in very small increments. The impact of price rigidities on pass-through has been explored, with a focus on menu costs. For instance, Nakamura and Zerom (2010) investigate different potential explanations for why there is incomplete (sub-100%) pass-through of exchange rate fluctuations. Among other candidates, they include price rigidity (which in their model is explained through menu costs); finding that they introduce a time lag in pass-through, but have a negligible impact on long-run pass-through rates.[3] In this paper, I focus only on the impact of exogenous focal pricing constraints and not on other related phenomena.

This paper is motivated by debunking the misconception that the presence of focal pricing constraints in general reduces pass-through rates. In multiple high-profile antitrust cases, it has been claimed that if an industry adopts focal pricing that in itself implies there will be very little, if any, pass-through of input cost changes. The argument is that if firms round to certain special prices, they are unlikely to adjust their prices in response to small input cost changes. In *re Lithium Ion Batteries antitrust litigation*, the Court struck out the case, amongst other reasons, because the defendants argued that the plaintiffs' expert analysis of pass-through did not take into account focal pricing. The defendants alleged that "focal

[3] The authors find that exposure to local costs plays an important role in dampening the impact of exchange rate fluctuations, and that curvature of demand also explains the observed pass-through patterns.

point pricing is prevalent in the pricing of products within the class definition, and will result in no pass-through when a small cost change—such as the estimated \$2.16 overcharge for a notebook computer battery here—in presence of focal points that are wider apart than the cost difference itself."[4] A similar argument has been made by Qualcomm in antitrust litigation in several jurisdictions.[5]

While this argument has intuitive appeal, it fails to recognise the other side of the coin: in the presence of focal pricing constraints, some prices will be over-adjusted if the input cost change leads to a jump from one focal price to another. Consider the example of laptops, and let their prices be constrained to end in 9. Then it is true that an overcharge on batteries of \$2.16 could result in no impact on the focal price charged, but it could also lead to a \$9 jump (or more). This is confirmed empirically, e.g. Levy et al. (2011) find price changes are less frequent, but bigger in magnitude, with focal pricing. It is also easy to empirically disprove the claim that focal pricing implies very low pass-through rates: for instance, see Conlon and Rao (2020) for examples of high pass-through in industries which adopt focal pricing.

While there is empirical evidence that pass-through can still be high in contexts with focal pricing, little attention has been devoted to theoretical modelling of this issue. This may be due to the fact that many models of competition become highly complex when pricing is discrete. Filling this theoretical gap is important to complement the existing empirical evidence. Under a simple but general framework, I show that focal price constraints have no impact on expected pass-through rates. The claim is not that focal pricing *cannot* impact pass-through rates, in fact it often will; but rather that there is *no general reason* to expect pass-through rates to be lower in the presence of focal pricing constraints. If we knew exactly how an industry worked, in full detail, we might be able to say that for a specific input cost overcharge none of it was passed through to downstream consumers, or conversely that it was passed-through at a higher than 100% rate. However, it is almost impossible for this sort of information to be available in detailed and reliable form at the outset of a court case, even though some of this information may be obtained through a long process of disclosure and economic analysis. Hence, it is not safe to dismiss a court case based on the assumption that the adoption of focal pricing will reduce pass-through relative to a similar context without focal pricing. Instead, as usual, it is best to estimate pass-through rates specifically for the context of interest. Note that the misconception that focal prices lead to little, or no, pass-

[4] United States District Court Northern District of California *In re Lithium Ion Batteries Antitrust Litigation*, Case No.: 13-MD-2420 YGR (2018, March).

[5] For example, see United States District Court Northern District of California San Jose Division *In re: Qualcomm Antitrust Litigation, Defendant Qualcomm Incorporated's Opposition to Plaintiff's Motion for Class Action*, Case No. 5:17-md-02773-LHK-NMC (2018, September).

through, has substantial economic consequences. For instance, the dismissal of class actions covering millions of claimants, and very large total damage claims, has a direct impact on the claimants, who do not receive fair damages, and an indirect impact on firms, who are more likely to engage in abusive behaviour if they are likely to avoid paying for damages.[6]

I also discuss the distribution of pass-through across different purchases. I show that focal pricing generally increases the dispersion in the pass-through rate. This is particularly relevant in class actions for two reasons. Firstly, the class certification stage often requires demonstrating sufficient homogeneity of the class. This involves different requirements in different jurisdictions, but broadly covers the idea that claimants must have been injured in a sufficiently similar manner, and that similar methods may be used to estimate the damages to be awarded to different claimants. Secondly, the quantification of damages may need to be performed separately for sub-classes which were harmed to different extents. Where claimants purchased few, or a single, product(s), it is likely that different pass-through rates applied to different claimants. With multiple purchases at different points in time, the average pass-through rate is a reasonable estimate of the pass-through rate that applies to all claimants. In the former case, it may be best to estimate pass-through rates separately for different groups of products. In the latter case, it can be legitimate to not explicitly consider focal pricing in the economic analysis.

This paper is set out as follows. First, in section 2.1, I set out a general model to assess the impact of focal pricing constraints on pass-through. Then, in section 2.2, I prove the Irrelevance Theorem under a set of assumption. I discuss each of these assumptions in turn, arguing that they are general. To further substantiate that discussion, in section 3, I analyse different models of competition, and different curvatures of the demand function, relating them back to the assumptions required for the Irrelevance Theorem to hold. In section 4, I conclude with a brief discussion of my findings and their relevance to antitrust damages class actions.

[6] Note that, if abusive upstream firms successfully argue that there was no pass-through in consumer class actions, then that may increase the risk of them having to pay damages to downstream firms (who would have absorbed the inflated cost in the absence of pass-through). However, for that to happen, the downstream firms would have to start their own litigation with the upstream firms. Downstream firms may be unwilling to enter into litigation with their input producers, especially if those producers have market power and have already engaged in abusive practices. For instance, they may fear retribution in the form of disruption to their input supply.

2 A general framework to assess the impact of focal pricing constraints on pass-through

2.1 The framework

Consider a firm facing constant marginal costs c and choosing prices to maximise profits. Here, I remain agnostic about the number of firms in the market, the type of competition, and curvature of demand.[7] I explore specific examples in section 3. To understand how focal pricing constraints affect pass-through, I compare pass-through rates under two scenarios: an unconstrained optimisation problem and a constrained optimisation problem.

In the unconstrained optimisation problem, the firm has a strategy $p^u(c)$ mapping any possible cost level to an optimal price.

Now consider the case where there are focal pricing constraints in the market. In this case, the quantity demanded of the firm's product will no longer depend on the charged price, but on the corresponding focal price f. Prices within a certain interval will be associated to a specific focal price: $f = f_i \quad \tau_{i-1} < p \leq \tau_i$. For instance, we can imagine that consumers are inattentive to some digits, so quantity demanded is a function of price rounded up to the next focal price.[8] In this case, $\tau_i = f_i$ and $f = f_i \quad f_{i-1} < p \leq f_i$.[9] In general, it will be optimal for the firm to charge only focal prices.[10] Therefore, while in the unconstrained problem the firm's strategy was $p^u(c)$, now it takes the form $p^c = f_i \quad t_{i-1} < c \leq t_i$.

It is useful to consider for what value of costs a given focal price would be unconstrained optimal: $f_i = p^u(\chi_i)$. Then we can define the gap between costs at which consecutive focal prices are unconstrained optimal as: $\chi_{i+1} - \chi_i$.

Let the firm's input cost increase from c to $c + \Delta$. The pass-through rate of an increase in marginal cost of size Δ in the unconstrained case is:

$$\frac{p^u(c + \Delta) - p^u(c)}{\Delta}$$

[7] Where there are n symmetric firms, the single firm we consider here can be taken to be a representative firm, so that if pass-through rates are not affected by focal pricing for this one firm, they are not affected at an industry level. Where there are multiple asymmetric firms, with potentially different pass-through rates, we can in principle repeat the same analysis for each of them - this paper focuses on symmetric settings, and asymmetric ones are left to future work.

[8] The concept of a latent unconstrained demand function can be thought of as representing the demand function that consumers tend towards as frictions decrease.

[9] Results readily extend to rounding up, or to the nearest, focal price.

[10] In certain oligopolistic settings, it may be possible to have multiple equilibria, some of them involving charging non-focal prices, but we can select equilibria where focal prices are charged to match our empirical observations.

The pass-through rate of the same increase in marginal cost of size Δ in the constrained case is:

$$\frac{p^c(c+\Delta) - p^c(c)}{\Delta}$$

The question at hand is how the unconstrained pass-through rate compares to the pass-through rate under focal pricing constraints. I answer this question formally under the following set of assumptions.

Assumption Set A

1. **Regularity Condition** for focal prices. The distance between each consecutive focal price is consistent: $f_{i+1} - f_i = G$.

2. **Equal Spacing condition** for cost thresholds. The distance between consecutive cost thresholds is constant, and is the same as the distance between costs at which consecutive focal prices are unconstrained optimal. We can therefore write $t_{i+1} - t_i = \chi_{i+1} - \chi_i = \theta G \quad \forall i$. This assumption implies that the unconstrained pass-through rate is constant (see the proof of the Irrelevance Theorem).

3. **Uniformity Condition**. If we observe a firm charging the focal price f_i we can infer that $c \sim U(t_i, t_{i+1})$.

Discussion of Assumption Set A

The Regularity Condition is likely to cover the vast majority of real-life cases which, as discussed above, involve consistently rounding to prices ending in specific digits, generally 9s. Moreover, the discretisation of prices due to currency limitations is also an example of regular focal pricing. Regular spacing of focal prices is a convenient modelling assumption, but it can be dropped with the Irrelevance Theorem still holding, at least approximately. If we wanted to drop the Regularity Condition, we would also relax the Equal Spacing condition for cost thresholds, to require that the (variable) gaps between cost thresholds are approximately the same as the (variable) gaps between costs at which consecutive price thresholds are unconstrained optimal.

The Equal Spacing Condition is key for the Irrelevance Theorem to hold exactly. As discussed in section 3, this assumption holds for monopolistic, perfect, and differentiated Bertrand competition with linear demand. Under perfect competition, the Equal Spacing assumption holds with any demand function. In the monopolistic case the assumption holds exactly for demand functions other than linear demand, including the logarithmic demand function. Under monopolistic competition, the Equal Spacing Condition holds approximately for any curvature of demand (including demand functions for which the unconstrained pass-through rate is not constant over the cost interval).

The Uniformity Condition is necessary for the Irrelevance Theorem to hold – if it does not then the result needn't hold even approximately. This assumption is likely to hold under any realistic context where the Irrelevance Theorem may be usefully deployed, i.e. in situations where we have limited information about the context of interest. This would certainly be the case in the early stages of a class action, when funding has not yet been obtained to carry out in-depth analysis, and disclosure has not been granted. We are also likely to face substantial lack of information at later, intermediate, stages of court cases. In such situations, the Uniformity Condition represents our uncertainty prior to a full analysis. By definition, if we observe a firm charging the focal price f_i we know that $t_i < c < t_{i+1}$. Moreover, in general we do not have sufficiently detailed and reliable information (prior to detailed empirical investigation) whether the marginal cost is close to the upper bound of this interval or not, which is relevant to whether focal pricing constraints will lead to reduced pass-through, or increased pass-through. It is difficult enough to obtain accurate cost estimates, even with court mandated disclosure, which can be challenging to obtain.

Consider the example of an antitrust damages class action. The claimants are end-consumers of a product which included an input which allegedly had an overcharge on it. The defendants are firms who sold the input to downstream firms which in turn produced and sold a product, with this input, to the consumers (for simplicity, I refer to them as the retailers). The claimants, and the defendants, do not possess detailed information about the cost structure and pricing strategy in the relevant retail industry. Moreover, the retail firms generally have no incentive to voluntarily disclose this sensitive information unless a court requires them to do so. Since they are not the ones accused of wrong-doing, obtaining disclosure is particular hard. It is possible in some cases to request disclosure of certain documents, but it can be a lengthy and challenging process, including difficulties such as the redaction of sensitive information which can be crucial to obtaining a full picture. Even after obtaining access to documents, these are typically so numerous and lengthy that it is very resource-intensive to obtain a clear picture of firm strategy from their documents. It might also be possible to interview people who work in the sector, but it can be challenging. This is partly because the abusive firm(s) are likely to command substantial influence in the sector, so that it is challenging to find reputable sources willing to go against them in court.

Even with full disclosure, we are very unlikely to be able to obtain reliable information on the full contingent pricing strategy of a firm, as it may well not exist in written form, or be subject to frequent discussion and alteration. Therefore, we are unlikely to know where the marginal cost was located relative to cost thresholds prior to the alleged overcharge. Hence, prior to detailed analysis (which is warranted by the Irrelevance Theorem) it is reasonable to assume the Uniformity Condition (which allows us to prove the Irrelevance Theorem). It is hard to construct examples where this is not the case in a manner which undermines the Irrelevance Theorem.

The first way in which the Uniformity Condition may not hold is if we have already conducted detailed empirical analysis. In this case, it is no surprise that the theorem is no longer valid: having performed detailed analysis, we will have information on whether in the specific case focal pricing increased, decreased, or did not affect pass-through. The Irrelevance Theorem is about the expected value of pass-though prior to detailed analysis. The point is that, prior to detailed analysis, we cannot assume that focal pricing led to low pass-through, and hence that empirical analysis is required. The second way in which the Uniformity Condition may not hold is if we have reason to believe that focal pricing is not an exogenous constraint (e.g. due to consumer inattention to digits), but an industry practice which is endogenous to firm optimisation (e.g. focal pricing as a tacit collusive strategy). In this case, costs will be located in relation to thresholds in a profit-maximising manner. This is a different phenomenon from the one addressed in this paper. It is potentially of interest too, but challenging to analyse because it is best conceptualised in a dynamic oligopolistic setting, with the ensuing multiplicity of equilibria; for this reason, it is left to future work. The third way in which the Uniformity Condition may not hold is if firms can easily adjust non-price characteristics (such as quality, pack-size, components included in a bundle..). Then we might expect that firms would adjust their marginal cost, by adjusting these other characteristics, so that the focal price being charged is as close to unconstrained optimal as possible. But then the reason to suspect focal pricing may dampen pass-through is moot: holistic pricing, taking into account non-price characteristics, is (almost) unconstrained, so that we would expect the holistic pass-through, capturing characteristic-adjusted prices, to be the same regardless of the presence of focal pricing.

2.2 The Irrelevance Theorem

Irrelevance Theorem *Under Assumption Set A, the expected pass-through rate is the same in the unconstrained setting, and in the setting with focal pricing constraints.*

Proof First consider the unconstrained case. By the definition of χ_i, we can write the pass-through rate of an input cost increase from χ_i to χ_{i+1} as:

$$\frac{f_{i+1} - f_i}{\chi_{i+1} - \chi_i}$$

By the Regularity Condition and Equal Spacing condition, we can write:

$$\frac{f_{i+1} - f_i}{\chi_{i+1} - \chi_i} = \frac{G}{\theta G} = \frac{1}{\theta} \quad \forall i$$

Since we can write this for any value of the focal prices, it follows that the unconstrained pass-through rate is constant. Let the firm's input cost increase from c to $c + \Delta$, then:

$$\frac{p^u(c + \Delta) - p^u(c)}{\Delta} = \frac{1}{\theta}$$

$\theta = 1$ corresponds to complete pass-through of 100%, $\theta > 1$ to incomplete pass-through, and $\theta < 1$ to pass-through above 100%.

In the absence of focal pricing constraints, the charged price increases by:

$$p^u(c + x) - p^u(c) = \frac{\Delta}{\theta}$$

Now consider the constrained problem. By the Equal Spacing assumption, the price charged will jump by $\lfloor \frac{\Delta}{\theta G} \rfloor$ focal prices with certainty where $\lfloor \rfloor$ is the floor operator. If the change in input cost is weakly greater than the gap between cost thresholds, $\Delta \geq \theta G$, then the charged price is sure to increase. By the Regularity Assumption, each of these jumps entails a price change of G. Therefore, the charged price will increase by $\lfloor \frac{\Delta}{\theta G} \rfloor G$ with certainty. Where $\Delta < \theta G$, we do not have certainty about any price jumps.

Additionally, by the Uniformity Assumption and Equal Spacing Assumption, there is a $\frac{\Delta mod \theta G}{\theta G}$ probability of a further jump in focal price (where *mod* is the modulo function). By the Regularity Assumption, this further jump, if it occurs, would lead to an additional increase of G in the charged price.

In expectation, the input price increase therefore leads to the following change in the constrained optimal price charged:

$$E\left[p^c(c + \Delta) - p^c(c)\right] = \lfloor \frac{\Delta}{\theta G} \rfloor G + \frac{\Delta mod \theta G}{\theta G} G = G(\lfloor \frac{\Delta}{\theta G} \rfloor + \frac{\Delta mod \theta G}{\theta G})$$

By definition of the modulo and floor operators:

$$E\left[p^c(c + \Delta) - p^c(c)\right] = G\frac{\Delta}{\theta G} = \frac{\Delta}{\theta} = p^u(c + x) - p^u(c)$$

The expected pass-through rate is $\frac{1}{\theta}$ regardless of the presence of focal pricing constraints. **QED.**

As I discuss in section 3.4, while it is conceivable that in some specific settings the Irrelevance Theorem may not hold, it is hard to find realistic examples of this. Therefore, in general it is not safe to assume that focal pricing reduces pass-through.

2.3 The distribution of pass-through

In general, the presence of focal pricing increases the dispersion of pass-through. Consider an industry where, in the absence of focal pricing constraints, all firms have a pass-through rate of $\frac{1}{\theta}$. Then the price change associated to a cost increase of size Δ[11] is always $\frac{\Delta}{\theta}$, and there is 0 variance in pass-through.

The introduction of focal pricing constraints increases the dispersion in pass-through rates. Let $i\theta G < \Delta < (i+1)\theta G$, where i is a weakly positive integer. Then the impact on the price of a specific product at a specific time is iG with probability $1 - \frac{\Delta mod \theta G}{\theta G}$ and $(i+1)G$ with probability $\frac{\Delta mod \theta G}{\theta G}$. Therefore, in the presence of focal pricing constraints, pass-through may be higher or lower than in the unconstrained problem. The probability with which it is higher (or lower) depends on the size of the cost change relative to the gap between cost thresholds. For instance, if the cost change is much smaller than the gap between focal prices, it will result in no pass-through with higher probability, and extremely high pass-through with lower probability. The variance is now:

$$E\left[(p^c(c+\Delta) - p^c(c))^2\right] - E\left[p^c(c+\Delta) - p^c(c)\right]^2$$
$$= \left(1 - \frac{\Delta mod\theta G}{\theta G}\right)(nG)^2 + \frac{\Delta mod\theta G}{\theta G}\left((n+1)G\right)^2 - \left(\frac{\Delta}{\theta}\right)^2$$

By Jensen's inequality, the variance is weakly positive under focal pricing constraints, and hence higher than in the unconstrained case.

In the context of a class action, where each member of the class purchased a single product, focal pricing constraints may lead to substantial heterogeneity in the pass-through rates which apply to each claimant,[12] so that it may be best to estimate pass-through rates specific to sub-classes. Where each member purchased multiple products, at different times, by the

[11] e.g. because of an overcharge of Δ on an input.

[12] Other input costs change over time, so even in a fully symmetric industry there may be heterogeneity in pass-through due to other costs impacting the degree of pass-through of the overcharge Δ.

law of large numbers, we might expect the average pass-through rate experienced by each claimant to be similar to the expected pass-through rate. In this case, it may be sensible to estimate the pass-through rate as one would in the absence of focal pricing constraints, and consider a single number to be an appropriate estimate for all claimants.

3 Examples of specific models

In this section, I show that the Equal Spacing assumption holds in a variety of standard models, and hence that the Irrelevance Theorem is widely applicable. Given this paper's focus on exogenous constraints on pricing, it is more natural to consider firms as price-setting, rather than quantity-setting. The rest of this discussion is therefore grounded in models of price competition.[13]

In turns, I consider three standard competitive frameworks: monopolistic competition, perfect competition, and differentiated Bertrand competition. In the context of monopolistic competition and perfect competition, I also discuss the impact of different curvatures of demand.[14] Throughout this discussion, I maintain a number of basic assumptions. In particular, I focus on static models, in which all firms face the same constant marginal cost c (so when there is a change in that cost, it is faced by all firms). In section 3.4 I briefly discuss what may happen when relaxing these assumptions.

3.1 Monopolistic competition

3.1.1 Linear demand

I start by considering the simplest possible model, with a single monopolist selling a single good, facing a linear demand function and marginal cost c. The monopolist can only change the price of the good, not any non-price characteristics. As discussed earlier, when firms can alter non-price characteristics, focal pricing constraints are likely to affect pass-through rates, so that I am focusing on the most clear case where focal pricing could matter to pass-through.

The monopolist's optimisation problem is:

$\max\left(\alpha-\beta p\right)\left(p-c\right) \quad \alpha,\beta>0$

[13] It is left to future work to consider how we might incorporate focal pricing constraints in a quantity-setting context, and whether in specific models of quantity competition the Irrelevance Theorem does not hold.

[14] It is left to future work to consider different curvatures of demand with differentiated Bertrand competition.

The optimal price is $p^u = \frac{\alpha}{2\beta} + \frac{c}{2}$ and we obtain the standard result of 50% pass-through under monopolistic competition. In this case, the profit function is symmetric around the optimal price, so that:

$$(\alpha - \beta(p^u + x))((p^u + x) - c) = (\alpha - \beta(p^u - x))((p^u - x) - c)$$

Now consider that the monopolistic faces focal pricing constraints: consumers are inattentive to some digits, so that demand is a function of price rounded up to the next focal price $f = f_i \quad f_{i-1} < p \leq f_i$.[15] Then the monopolist's constrained problem is $\max(\alpha - \beta f_i)(p - c) \quad \alpha, \beta > 0$. The profit maximising price in this constrained problem must be a focal price, because for any other price it would be possible to increase the price up to the next focal price without decreasing demand. Moreover, because of the symmetry of the profit function around the optimal unconstrained price, it is always constrained optimal for the monopolist to charge the focal price nearest to the optimal unconstrained price.

In appendix A, I show that the Equal Spacing Condition holds, with $\theta = 2$. With uncertainty about the full details of the context of interest (the Uniformity Condition), the conditions for the Irrelevance Theorem hold: the expected pass-through rate is 50% regardless of the presence of focal pricing constraints.

3.1.2 Non-linear demand

In practice, demand may well not be linear, and the degree of curvature of demand is an important determinant of the rate of pass-through of cost changes, as discussed by Bulow and Pfleiderer (1983).[16] In the monopolistic case, the Equal Spacing assumption holds exactly under broader demand functions than linear demand, for instance with the logarithmic demand function $p = \alpha - \beta \ln q \quad \alpha, \beta > 0, 0 < q < e^{\alpha/\beta}$.[17] In this case, the unconstrained monopolist chooses $p^u = c + \beta$, so there is a constant mark-up, and 'complete' pass-through of 100%. The monopolist constrained by focal prices (spaced out at regular intervals G) chooses which focal price to charge based on a cut-off rule. As shown in appendix B, these cut-offs are evenly spaced at the same regular intervals G. Since the gap between costs at which consecutive focal prices are unconstrained optimal is also G, the Equal Spacing assumption holds. In the presence of focal pricing constraints, pass-through remains 100%.

[15] Results readily extend to rounding up, or to the nearest, focal price.

[16] There are other important determinants, such as returns to scale, which are left for future work to consider.

[17] It is left to future work to determine whether there is a specific class of demand functions for which this is the case.

With other demand functions, we could have two sources of failure of the Equal Spacing assumption. The first is non-constant pass-through rates. Then the requirement that the gaps between cost thresholds is constant will not hold. This is not a fundamental issue for the Irrelevance Theorem, as long as the (variable) gaps between thresholds are approximately the same as the (variable) gaps between costs at which consecutive price thresholds are unconstrained optimal. If this is the case, then we can adjust the proof of the Irrelevance Theorem to account for non-constant pass-through, and will obtain that focal pricing leaves local pass-through approximately unchanged.

Even where unconstrained pass-through rates are constant, as in the case for the class of demand functions discussed in Bulow and Pfleiderer (1983), for some curvatures of demand, the Irrelevance Theorem will hold approximately, rather than exactly. This will happen if the Equal Spacing condition does not hold locally everywhere, in which case we can see that it will hold as a global approximation. The key intuition here is the following. It is possible, locally, for cost thresholds to be more closely (or widely) spaced out than the costs at which focal prices are unconstrained optimal. Then, locally, focal pricing reduces or increases pass-through because we cannot perform the cancelling out operation that leads us to the Irrelevance Theorem. However, it cannot consistently be the case for cost thresholds to be non-negligibly more closely (or widely) spaced out than the costs at which focal prices are unconstrained optimal. This is because the cost at which a specific focal price is unconstrained optimal must be in the interval between the cost thresholds where this specific focal price is charged in the constrained problem: $t_{i-1} \leq \chi_i \leq t_i$. If we had consistently non-negligibly narrower (or wider) gaps between cost thresholds $t_{i+1} - t_i$ than between the costs at which focal prices are unconstrained optimal $\chi_{i+1} - \chi_i$, then at some point we would find that a cost at which a specific focal price is optimal is in fact outside of the interval when that focal price is charged in the constrained problem:

$$t_{i-1} \geq \chi_i \quad or \quad \chi_i \geq t_i$$

This is a contradiction. Hence, focal pricing may increase (or lower) expected pass-through over specific segments of the cost interval. However, these effects are local, and are typically balanced out by opposite effects on other segments of the cost interval.[18] Therefore, without

[18] I note that it is possible for pass-through to be mildly higher (or lower) in the presence of focal pricing over some of the region, without being cancelled out by the opposite effect, as long as pass-through is almost the same as without focal pricing. For instance, it is possible to construct examples with the constant elasticity demand curve where pass-through with focal pricing constraints is slightly lower than in the unconstrained problem for low value of input costs, and tends towards the same pass-through rate as costs increase. In this case, expected pass-through is only very slightly reduced, so that the Irrelevance Theorem can be said to hold approximately.

very detailed information about the industry, of a variety which is very unlikely to be readily available (even subject to court disclosure), we cannot know a priori whether we are in a cost segment where the presence of focal pricing locally increases or decreases expected pass-through. Moreover, we know that, globally, expected pass-through will be approximately unaffected by the presence of focal pricing. Therefore, the Irrelevance Theorem may be considered to be approximately true regardless of the curvature of demand, in a monopolistic setting.

3.2 Perfect competition

Consider a market characterised by perfect competition (for instance, we might conceptualise this in terms of undifferentiated Bertrand competition). The aggregate demand function Q can take any form, and depends only on the lowest price offered by any firm j in the market $\min_j\{p_j\}$. There are N firms in the market, and the demand function faced by a specific firm n takes the following form:

$$q_n = \begin{cases} \frac{Q}{\sum_j I(p_j = p_n)} & \forall j \quad p_n \leq p_j \\ 0 & \exists j \neq n \quad p_n > p_j \end{cases}$$

All firms face the same marginal cost c. The unique equilibrium involves all firms charging $p = c$, and hence each firm facing demand $q_n = \frac{Q}{N}$, and making zero profits. In this context, there is complete pass-through of input cost changes. Therefore, a Δ overcharge will result in a Δ increase in prices.

We now introduce focal pricing constraints for the firms. For instance, consider a case of consumer inattention to certain digits, so that demand is a function of price p_n rounded up to the nearest focal price $f_n = f_i \quad f_{i-1} < p_n \leq f_i$.[19]

$$q_n = \begin{cases} \frac{Q}{\sum_j I(f_j = f_n)} & \forall j \quad f_n \leq f_j \\ 0 & \exists j \neq n \quad f_n > f_j \end{cases}$$

Once again there is a unique equilibrium, and each firm faces demand $q_n = \frac{Q}{N}$, but now all firms charge $p = f_c$ where f_c is the marginal cost c rounded up to the next focal price.[20] Therefore, all firms now make weakly positive profit. We can easily see that the cost thresholds at which firms adjust prices are identical to the focal prices themselves. Similarly, we know that the costs for which focal prices are unconstrained optimal are also identical to the focal prices themselves. Therefore, the Equal Spacing condition holds under perfect

[19] Results straightforwardly extend for rounding down, or rounding to the nearest focal price.

[20] There is no profitable deviation to a lower focal price because it would entail negative profits, nor any lower non-focal price because it would entail the same firm-specific demand, but at a lower price. There is also no profitable deviation to a higher price because it would entail zero firm-specific demand.

competition, for any demand function. In this context, the expected pass-through rate is still 100% even when we introduce focal pricing constraints.

3.3 Differentiated price competition

Having discussed the two extreme cases of monopoly and perfect competition (which are also informative about collusion and undifferentiated price competition) I now turn to an intermediate case: differentiated price competition.

With continuous pricing, it can be shown under general conditions that there exists a unique equilibrium for differentiated price competition (see Mizuno (2003)). This result no longer holds with discrete pricing, and in general there is a multiplicity of equilibria. With indeterminacy of equilibria it is harder to draw conclusions about the impact of focal pricing, since it is conceptually possible for it to lead to higher or lower pass-through. Here, I focus on a standard setting, and propose a simple equilibrium strategy for firms facing focal pricing constraints. I show that in this case the Equal Spacing assumption holds.

Consider a market with N firms producing a differentiated product, and simultaneously competing on prices in a one-shot game. Demand for firm n's product satisfies standard conditions for differentiated price competition (see Mizuno (2003)). Here, we take it to be:

$$q_n = \begin{cases} Q - p_n + \bar{p}_{-n} & Q > p_n - \bar{p}_{-n} \\ 0 & Q \leq p_n - \bar{p}_{-n} \end{cases}$$

where $\bar{p}_{-n}$ is the average price set by other firms in the market, and Q is some positive constant. Take the symmetric input case, in which all firms face the same marginal cost $c_j = c, \quad \forall j$ and are all exposed to the same changes in input costs (including the one of interest).

In the unconstrained case, the profit function for firm n is $\pi_n = (p_n - c)(Q - p_n + \bar{p}_{-n})$. Best responses are linear and symmetric, and there is a unique symmetric equilibrium where all firms charge $p_n = p = Q + c, \quad \forall n$. Therefore, this industry is characterised by 100% pass-through, i.e. a Δ increase in the input cost faced by all firms will result in a Δ increase in the price charged by all firms.

Now consider the case of exogenously determined focal pricing, e.g. due to consumer inattention to some digits, spaced out with gaps G. Let there be a focal price point at Q, which we refer to as the low price p^L, a medium focal price at $p^M = Q + G$, and a high focal price at $p^H = Q + 2G$ (we focus on this interval, but the logic generalises to the whole interval of possible prices). We can see that the low price is unconstrained optimal when $c = 0$, the medium price is unconstrained optimal when $c = G$, and the high price is unconstrained optimal when $c = 2G$. Therefore, the spacing between costs at which consecutive focal prices are unconstrained optimal is G.

With focal pricing, the demand function can be written as:

$$q_n = \begin{cases} Q - f_n + \bar{f}_{-n} & Q > f_n + \bar{f}_{-n} \\ 0 & Q \leq f_n + \bar{f}_{-n} \end{cases}$$

where f_n is p_n rounded up to the next focal price, and$\bar{f}_{-n} = \frac{\sum_{j \neq n} f_j}{N-1}$.

In this case, the conditions for uniqueness of equilibrium no longer hold. Focusing just on symmetric equilibria, we can divide the support of the marginal cost into segments for each of which there are two possible symmetric equilibria. For $0 \leq c \leq G$ there is an all-low-price equilibrium, and an all-medium-price equilibrium. For $G \leq c \leq 2G$ there is an all-medium-price equilibrium, and an all-high-price equilibrium. Therefore, we can construct an equilibrium where each firm's strategy is to charge medium prices if $0 \leq c \leq G$ and high prices if $G \leq c \leq 2G$ (we could easily extend this to higher costs and prices). Hence the spacing between cost thresholds is G, which is the same as the spacing between he gaps between the costs at which focal prices are unconstrained optimal. Hence, the Equal Spacing assumption holds, and in the presence of uncertainty (the Uniformity assumption) the Irrelevance Theorem holds.

Let us say that prior to the cost increase, the industry was characterised by medium prices, meaning that marginal costs were in the range $0 \leq c \leq G$. Then if costs increase by Δ, there is a $\frac{\Delta}{G}$ chance of costs increasing to the range $G \leq c \leq 2G$, in which case all firms charge high prices. The expected change in prices is therefore $\frac{\Delta}{G}G = \Delta$, i.e. the industry is still characterised by 100% pass-through, as it was in the absence of focal pricing.

3.4 Other models

There are a multiplicity of modelling assumptions that influence pass-through, and its relation to focal pricing. These include the number of firms, whether they compete in a one-shot game or repeatedly, whether there are asymmetries between firms, the degree of product heterogeneity, what characteristics of the product can be chosen by the firm (price, quantity, quality...), etc. Therefore, it is hard to rule out the possibility that there exist models where the Equal Spacing assumption does not apply, and hence focal pricing reduces, or increases pass-through in expectation. However, it is challenging to find examples where this is the case. For instance, consider relaxing the assumption of symmetric costs in the context of undifferentiated price competition. In this case, with focal pricing constraints, if the increase in input costs was sustained only by a subset of firms, then the affected firms would be forced to exit the market. There would be no direct impact on prices,[21] and hence zero direct price pass-through. However, this is exactly what would happen in the unconstrained setting too, so that the pass-through rate is 0% regardless of the presence of focal pricing constraints.

Even in models with multiplicity of equilibria it is hard to see how the Irrelevance Theorem could not hold at least as a global approximation. For instance, consider the differentiated price competition setting discussed above. Consider an alternative equilibrium, where each firm's strategy is to charge low prices if $0 \leq c \leq G$ and high prices if $G \leq c \leq 2G$. Then the expected pass-through is is twice as high as without focal pricing. However, note that we could also construct the opposite example, where firms are in the medium price equilibrium for the whole set of possible costs, and hence there is no pass-through. Similarly to the discussion of different curvatures of demand in section 3.1.2, locally higher (or lower) pass-through rates will in general be 'cancelled out' by other locally lower (or higher) pass-through rates; or at least it cannot be the case that focal pricing systematically increases or decreases pass-through along the cost interval. Therefore, the Irrelevance Theorem still holds as a global approximation when we do not possess detailed knowledge about whether we are in a segment where focal pricing locally increases or decreases pass-through rates. Continuing the differentiated Bertrand example, if we extended our analysis to further along the cost interval, if we were in an all-medium equilibrium at $G \leq c \leq 2G$, that would not be feasible for $2G \leq c \leq 3G$, where there must be either an all-high equilibrium or an equilibrium with the next focal price up: it is not sustainable for there to not be pass-through along more than a local segment of the cost interval.

The conceptual indeterminacy of the impact of focal pricing on pass-through, combined with the fact that the Irrelevance Theorem holds exactly in many standard settings, and approximately in many others, provides a clear rationale for conducting context-specific empirical analysis to estimate pass-through, rather than assuming that the presence of focal pricing will lead to little, or no, pass-through.

4 Discussion

In the context of antitrust damages class actions, the Irrelevance Theorem provides a strong rationale not to accept arguments that cases should be dismissed because there will be low, or zero, pass-through with focal pricing constraints. However, we should take seriously the possibility that focal pricing may increase heterogeneity in the distribution of pass-through. Depending on the specifics of the contexts, this might mean that the majority of the class

[21] It is worth noting that the exit of a subset of firms might lead to a large indirect impact on prices through increased market concentration and higher chance of collusion.

has suffered no harm, while a minority has suffered substantial harm. In other cases, all class members suffered damages, perhaps to similar, or perhaps to different, degrees. It is also possible that no class members suffered any damages at all. In the, unusual, case of all consumer having purchased the same product at the same time, and overcharge being very small relative to the gaps between focal prices, it is likely that there was no pass-through. However, there is also a non-negligible possibility that there was pass-through, and that it was very high.

In some jurisdictions, the increased heterogeneity in the distribution of pass-through introduced by focal pricing may be perceived as a challenge to the homogeneity requirement that class members should all have suffered damages in a similar way. However, it is not clear why heterogeneity in damages arising from focal pricing should be treated any differently from heterogeneity arising from class members having purchased slightly different products at different times, as is standardly the case in class actions. In these cases too, class members may well have suffered from different damages to begin with, and then accumulated differential interest on damages over time. It also seems remarkable that abusive firms can avoid paying damages to consumers simply because those consumers were affected to different degrees by the anti-competitive behaviour. From an economic viewpoint, the way to address this issue is to perform more granular analysis, and obtain estimates of damages specific to members of different sub-classes. Where this kind of detailed expert analysis is considered too expensive by the court, using average estimates for the whole class seems like a more sensible approach than dismissing the case as a whole.

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A A monopolist facing linear demand

Consider the monopolist's problem in the absence of focal pricing constraints. The optimal price is $p^u = \frac{\alpha}{2\beta} + \frac{c}{2}$. Inverting this, we find that the cost associated to a certain optimal price is: $c = \frac{2\beta p^u - \alpha}{\beta}$. Therefore, the cost for which a focal price is unconstrained optimal is $\chi_i = \frac{2\beta f_i - \alpha}{\beta}$. Using the fact that focal prices are regularly spaced out at intervals G (the Regularity Condition), we substitute in $f_i = G * i$: $\chi_i = \frac{2\beta(G*i)-\alpha}{\beta}$.

Therefore, the gap between costs for which two consecutive focal prices are unconstrained optimal is:

$\chi_{i+1} - \chi_i = \frac{2\beta(G*(i+1)) - \alpha - 2\beta(G*i) + \alpha}{\beta}$

$\chi_{i+1} - \chi_i = 2G$

Now consider the monopolist's problem when constrained by focal pricing. To find the cost thresholds at which the monopolist switches from one focal price to another, we consider the following indifference condition: $\pi(f_i, t_i) = \pi(f_{i+1}, t_i)$, which can be written as

$(\alpha - \beta f_i)(f_i - t_i) = (\alpha - \beta f_{i+1})(f_{i+1} - t_i)$

Using the fact that focal prices are regularly spaced out at intervals G, we substitute in $f_i = G * i$:

$(\alpha - \beta (G * i))((G * i) - t_i) = (\alpha - \beta (G * (i+1)))((G * (i+1)) - t_i)$

$\alpha = -\beta (G * i) - \beta ((G * (i+1)) - t_i)$

$t_i = \frac{\alpha + \beta(G*i) + \beta(G*(i+1))}{\beta}$

So the gap between two consecutive thresholds is:

$t_{i+1} - t_i = \frac{\alpha + \beta(G*(i+1)) + \beta(G*(i+2)) - \alpha - \beta(G*i) - \beta(G*(i+1))}{\beta}$

$t_{i+1} - t_i = \frac{\beta(G*(i+2)) - \beta(G*i)}{\beta}$

$t_{i+1} - t_i = 2G$

This is the same gap as the gap between costs for which consecutive focal prices are unconstrained optimal. Because there is a 2G gap between cost thresholds for focal prices that are G apart, pass-through is 50%, as it was in the unconstrained case.

B A monopolist facing a logarithmic demand function

A monopolist faces the following demand function:

$p = \alpha - \beta \ln q \quad \alpha, \beta > 0, 0 < q < e^{\alpha/\beta}$

With constant marginal cost c, the result of unconstrained optimisation is $p^u = c + \beta$, i.e. this market is characterised by a constant mark-up β and 'complete' pass-through of input costs.

Now consider he same monopolist, constrained by focal pricing. Focal price f_i is charged if $t_{i-1} < c \leq t_i$.

The thresholds are found as the points of indifference for the monopolist, i.e. $\pi(f_i, t_i) = \pi(f_{i+1}, t_i)$.

Inverting the demand function, we can write $\pi(f_i, t_i) = (f_i - t_i)\, e^{\frac{\alpha - f_i}{\beta}}$

We find the threshold t_i as $(f_i - t_i)\, e^{\frac{\alpha - f_i}{\beta}} = (f_{i+1} - t_i)\, e^{\frac{\alpha - f_{i+1}}{\beta}}$

Using the fact that focal prices are regularly spaced out at intervals G, we substitute in $f_i = G * i$:

$(G * i - t_i)\, e^{\frac{\alpha - G*i}{\beta}} = (G * (i+1) - t_i)\, e^{\frac{\alpha - G*(i+1)}{\beta}}$

$(G * i - t_i) = (G * (i+1) - t_i)\, e^{\frac{-G}{\beta}}$

$t_i \left(e^{\frac{-G}{\beta}} - 1\right) = G \left((i+1)e^{\frac{-G}{\beta}} - i\right)$

$t_i = G \frac{(i+1)e^{\frac{-G}{\beta}} - i}{e^{\frac{-G}{\beta}} - 1}$

Hence the interval between any two consecutive cost thresholds is:

$t_{i-1} - t_i = G \frac{(i+2)e^{\frac{-G}{\beta}} - i - 1}{e^{\frac{-G}{\beta}} - 1} - G \frac{(i+1)e^{\frac{-G}{\beta}} - i}{e^{\frac{-G}{\beta}} - 1}$

Simplifying, we obtain: $t_{i-1} - t_i = G$

Hence, thresholds are regularly spaced at the same interval as focal prices, and the Irrelevance Theorem holds.