Institute for Fiscal Studies

Francesca Arduini

Working paper

25/17

Estimating intra-household sharing from time-use data

Economic and Social Research Council

Estimating Intra-Household Sharing from Time-Use Data

Francesca Arduini*

30th April 2025

Abstract

Estimating intra-household sharing is crucial to understanding overall inequality. Standard measures of consumption inequality only take into account inequality between, and not within, households, because expenditure surveys are generally available only at the household level. I develop a new approach to estimating intra-household sharing, which is both grounded in a general collective household model, and simple to implement with widely available data. I propose using individual-level variation from time-use data to identify the way households share resources between members. For UK working couples, my methodology reveals substantial intra-household inequality, and the poverty rate is 20.59% higher for women than men.

Keywords: Collective model, Sharing rule, Household bargaining, Bargaining power, Consumption inequality, Time-use

*University College London; Institute for Fiscal Studies; University of Oxford. Email: francesca.arduini.21@ucl.ac.uk. Telephone: +44 7597 504 342.

This paper benefited from discussions with Sam Altmann, Ingvild Almås, Orazio Attanasio, Barbara Biasi, Richard Blundell, Martin Browning, Alex Bryson, Rossella Calvi, Paula Calvo, Michela Carlana, Marco Castelluccio, Laurens Cherchye, Pierre-André Chiappori, Edoardo Ciscato, Ian Crawford, Angela Crema, Lorraine Dearden, Bram De Rock, Olivier Donni, Sylvie Lambert, Valérie Lechene, Arthur Lewbel, Ilse Lindenlaub, Corinne Low, Costas Meghir, Cormac O'Dea, Gizem Kozar, Yoko Okuyama, Krishna Pendakur, Laura Pilossoph, Fabien Postel-Vinay, Ana Reynoso, Elena Stancanelli, Michela Tincani, Frederic Vermeulen and Selma Walther. I also thank seminar participants at Yale, UCL and IFS, and Conference participants at the 2024 Annual Meeting of the American Economic Association (ASSA), the 2024 New Advances in Family Economics Workshop, the 2024 Stone Centre at UCL PhD Conference on Income and Wealth Inequality, the 2023 Doctorissimes Paris School of Economics (PSE) Conference, and the 2023 annual meeting of the Society of Economics of the Household (SEHO). This work was supported by the Economic and Social Research Council [grant number ES/P000592/1].

1 Introduction

It is empirically challenging to measure individual-level consumption inequality because expenditure data is typically collected at the household level. The simplest solution to this problem, which is still widely used, for instance by the World Bank, is to assume that there is no intra-household inequality, so that household-level expenditure, divided by the number of household members, directly provides estimates of individual-level consumption (the per capita approach). A growing body of evidence suggests that the equal sharing assumption is unrealistic and that it is crucial to account for intra-household, as well as inter-household, inequality (e.g. Lechene et al. (2022)). Not doing so leads to inaccurate, generally downward-biased, estimates of the aggregate level of consumption inequality, such as the poverty rate. It also fails to appropriately capture key dimensions of inequality, such as gender and age, and their intersection (e.g. see Calvi (2020)). Therefore, standard measures may lead to incorrect conclusions on trends of inequality over time, and about the impact of specific policies. Moving towards more accurate individual-level estimates of inequality is important to improve the targeting of public funds to prioritise more effective policies, and policies aimed at groups that are particularly in need.

I develop a new approach to estimating intra-household sharing, which is both grounded in a general collective household model, and simple to implement with widely available data. My approach incorporates both private and public goods, and both material goods and detailed time-use. Estimation proceeds by OLS from a small number of estimating equations. Instead of requiring the whole expenditure system at the individual level, I need only individual-level expenditure data on a single private good (the 'assignable good'), which is much more commonly available. The responsiveness of assignable good expenditure to household budget is informative about how that budget is shared between members. Together with an identifying assumption restricting preference heterogeneity, this yields point-identification of the sharing rule i.e. household-specific estimates of sharing, as a function of household characteristics. Applying sharing estimates to household expenditure surveys enables estimation of individual-level consumption and inequality.

In the collective household model, the household agrees on time-use and material consumption in the same bargaining problem, and hence the same sharing rule applies both to time-use and material goods. Therefore, sharing can in principle be estimated just as well with a material or a time-use assignable good. This paper is the first to identify sharing from time-use data, and uses a novel assignable good: private leisure. This is time spent on leisure activities without other household members co-present, for instance reading a book alone or having a coffee with a friend. This can be measured accurately from time-use data combining activ-

ity and co-presence information. I provide conceptual arguments and empirical tests that suggest that this new source of identifying variation may satisfy the required assumptions more credibly than alternatives, contributing to more accurate estimates of individual-level consumption inequality.

I adapt the identification result of Dunbar et al. (2013) to a setting with time-use, price variation and public goods. The first two extensions are required in order to use my novel assignable good. The third extension is required for theoretical consistency with my proposed approach to applying estimated resource shares to estimating individual-level consumption, where the distinction between private and public goods is important to avoid overestimating inequality. To make these three extensions tractable, my implementation uses Cobb-Douglas expenditure functions, rather than Almost Ideal Demand System Engel curves, as estimating equations.[1] I estimate sharing from time-use data alone, using the UK Time-Use Survey (UKTUS). Women generally command a lower proportion of resources than men,[2] 45% on average, but this varies substantially between households. Characteristics affect resource shares in a manner consistent with bargaining theory, e.g. women with higher wages, or matched with less educated men, command a higher resource share.

I then apply the sharing rule to a separate but comparable expenditure dataset, the Living Costs and Food Survey (LCF), allowing me to estimate individual-level material consumption. I define this as the monetary market value of an individual's consumption, i.e. the sum of (i) the household's expenditure on public goods, and (ii) the individual's estimated share of the household's expenditure on private goods. This is a useful metric to calculate objective and policy-relevant measures of individual-level consumption inequality. I find that, on average, men's material consumption is 8.53% higher than women's, with a wider gap for poorer households, so that the poverty rate is 20.59% higher for women than for men. These estimates suggest that policymakers should target policies to reduce female poverty in the UK, as further discussed in section 5.2. When I incorporate time-use as well as material consumption, I estimate a wider average gender gap in 'full' consumption of 10.1%. This supports the view that in order to fully estimate gender inequality we must take into account time-use as well as material expenditure.

These gender gaps are very substantial, especially when considering the context. The UK is one of the most gender equal countries in the world[3] and previous literature finds that

[1] It is possible to use my approach with other functional forms, including Almost Ideal Demand System Engel curves, as discussed in online appendix A.

[2] In this paper I use 'resource share' and 'share of resources' as a short-hand for 'conditional resource share' or 'share of household private expenditure'.

[3] United Nations Development Programme. (2022). Human development report 2021-22. http://report.hdr.undp.org

women command higher resource shares in working couples than in households with children or where the woman does not work (e.g. Bargain, Donni and Hentati (2022)). Therefore, we can think of this as an approximate lower bound for the kind of intra-household inequality we are likely to encounter in other countries and with broader household compositions. This reinforces previous findings in the literature that equal sharing does not hold, and that it is important to estimate intra-household inequality instead of using a per capita approach.

Finally, I note that my results also shed light onto the apparently puzzling fact that women with higher wages tend to have less leisure, even though they have more bargaining power in the household. These facts are squared by noticing that *expenditure* on women's leisure is higher for women with higher wages, even if their *quantity* of leisure is somewhat lower; and also that expenditure on material consumption increases in a way that compensates for lower leisure. Importantly, we can account for all this with a collective household model, with a single sharing rule which applies both to material goods and time-use, because changes in wages have an income effect (both through the household budget and through bargaining) as well as a price effect (on the price of time).

1.1 Contributions to the literature

This paper builds on a large and growing literature on estimating intra-household sharing. A small number of papers estimate sharing directly from data on individual-level expenditure e.g. Cherchye, De Rock and Vermeulen (2012). However, this type of data is rarely available in practice and most of the literature focuses on estimating individual-level consumption from household-level expenditure. Several papers have shown that this is theoretically possible under different versions of the collective household model, and with different restrictions (e.g. Chiappori (1992), Browning, Bourguignon et al. (1994), Blundell, Chiappori and Meghir (2005), Blundell, Chiappori, Magnac and Meghir (2007) and Chiappori and Ekeland (2009)).

A variety of papers have estimated intra-household sharing in different countries, both developing and developed, based on a range of approaches, with different underlying versions of the model, identifying assumptions, and estimation approaches. For example, Cherchye, De Rock, Lewbel and Vermeulen (2015) draw on revealed preference techniques to estimate bounds on sharing. Other papers, including Browning, Chiappori and Lewbel (2013) (henceforth BCL), Lewbel and Pendakur (2008), Bargain and Donni (2012) and Brown et al. (2021) point-identify the sharing rule from estimates of Engel curves of multiple goods combined with identifying assumptions restricting preference heterogeneity. Dunbar et al. (2013) (henceforth DLP) shows that, in a setting with non-public material consumption and no price variation, it is possible to identify the sharing rule from individual-level data for

a single private good, known as the 'assignable good'. A similar approach is followed by Bargain, Donni and Hentati (2022) and Lechene et al. (2022) (henceforth LPW). Lise and Seitz (2011) also estimates sharing from data on a single assignable good, but based on a different identifying assumption, restricting the bargaining process so that men and women with equal earnings potential share resources equally. Most papers in the literature find substantial inequality in intra-household sharing, with women and children typically receiving fewer resources than men.

While several approaches in this literature involve complex estimation, recent development have moved towards simpler methods. In particular, LPW shows that, building on the identification result of DLP, it is possible to estimate resource shares by linear regression. I adapt the identification approach of DLP, and the linear estimation framework of LPW, to a setting with time-use, price variation and public goods. In this sense, this paper attempts to combine the tractability of DLP and LPW and the generality of a model with time-use and public goods such as in Lise and Seitz (2011).

Many papers in this literature, including BCL, DLP and LPW, treat all goods as non-public and model household economies of scale through the shareable goods framework. This framework is very tractable, but it does not allow us to impose the restriction which is at the core of public consumption, i.e. the requirement that all household members consume the same quantity of a public good (see Browning, Chiappori and Lewbel (2013) and Chiappori, Meghir and Okuyama (2024)). For this reason, household optimisation under shareable goods and public goods generally yields different solutions. Since households spend a very large proportion of their budgets on public goods, e.g. housing, it is important to accurately capture economies of scale driven by public consumption. This aspect is particularly important when it comes to applying the estimated sharing rule to estimate individual-level consumption and inequality. The sharing rule should apply only to private expenditures since public goods are, by definition, non-rivalrous and consumed by all members in the same quantity. If we incorrectly treat public expenditure as if it were private (as done e.g. by LPW), or exclude it, we substantially over-estimate intra-household inequality. For theoretical consistency, I include public consumption in my model and derive estimating equations for the sharing rule that are consistent with the presence of public goods. In principle this inclusion can make a substantial difference at the stage of resource share estimation. However, in my application, my use of separable preferences and of a further assumption, restricting preference heterogeneity on the private-public good split, entail that the public good extension only really bites in section 5, where I estimate individual-level consumption by applying the estimated sharing rule only to private expenditure, and assigning full public expenditure to each household member. As discussed in section 5.2, the inclusion of public goods matters

substantially, even when it only bites in this stage of the analysis.

Time-use, similarly to public consumption, generates challenges for identification. For this reason, many papers in this literature focus on material consumption. However, as argued by Becker (1965), time-use is a crucial component of household decision-making and well-being. My extension to time-use allows me to estimate 'full' individual-level inequality, inclusive of time-use as well as material consumption. This paper is the first to provide such estimates. Additionally, recognising the endogeneity of time-use is important to correctly defining the household budget as the full budget (how much the household could earn if all members worked all the time, in addition to any unearned income), rather than realised earnings, often used in other papers in this literature, and which are endogenous. Finally, incorporating time-use opens up an alternative source of identifying variation to the standardly used expenditure data on clothing.

The literature to date has been constrained in its choice of individual-level variation to identify the sharing rule from. This is because papers to date use household expenditure surveys both to (i) estimate the sharing rule, and (ii) apply the estimated household-specific shares to household expenditure, to obtain individual-level expenditure. Household expenditure data contains very little individual-level variation, and for this reason most papers in the literature have used clothing as the assignable good, as it is often available split into men's, women's and children's clothing. However, as discussed in more detail in section 3.1, clothing has limitations, including a substantial proportion of zeros in the data, which reduces accuracy and theoretically may bias estimates towards equal sharing. As alternatives to clothing, a small number of papers has attempted using individual-level food expenditure or the residual from usual hours worked (non-market-work) as the assignable good. The former is rarely available in practice. The latter can often be obtained from recall data on usual hours worked in expenditure surveys. As discussed in more detail in section 3.1, non-market-work can be thought of as a proxy of leisure, but is conceptually not a private good because it includes domestic work and joint leisure (time spent on leisure activities with other household members co-present),[4] conceptually biasing estimated towards equal sharing.

I propose splitting the estimation of individual-level consumption into two phases. The first, estimating the sharing rule, can be conducted with time-use data. In the second phase, the estimated sharing rule can then be applied to a separate but comparable household expenditure survey, to obtain individual-level consumption. This is advantageous because time-use data (i) is widely collected and recorded at the individual level, and (ii) typically

[4]The distinction between private and joint leisure is an extension of the distinction between individual and spousal leisure in Fong and Zhang (2001).

includes sufficiently detailed activity and co-presence information to enable careful definition of the assignable good in a way that is both credibly private and has a large enough budget share, with few or no zeros. I propose using private leisure as the assignable good, and providing conceptual reasons why this may lead to more accurate estimates in some contexts, as appears to be borne out in my application to UK data (see section 4.4).

The main limitation of private leisure is that its price is only known for those who work, for whom it coincides with the observed wage. For non-participants, the price of time is higher than their (unobserved) wage and we would have to estimate that in order to use time-use data to identify sharing. That extension is left for future work: in my application to UK data, I focus on working couples. Focusing on this sub-sample also allows for more credible testing of assumptions using singles data, as discussed in section 3.1.4. In section 3.1.4, I also explain why this sample restriction is not problematic in the UK context.

My paper is the first to use private leisure as the assignable good. This is partly because several papers in this literature, e.g. DLP and LPW, cannot use private leisure as the assignable good because their identification result precludes price variation, and the price of time varies at the individual level. I adapt the identification approach in DLP to a context with price variation. Other papers in the literature have shown identification is possible with price variation, but they either do not allow for point identification of the levels of the sharing rule (e.g. Blundell, Chiappori, Magnac and Meghir (2007), which additionally does not allow for public goods) or require more complex estimation than this paper (e.g. Lise and Seitz (2011)).

My paper is closely related to two papers which also focus on the UK context: Bargain, Donni and Hentati (2022) and Lise and Seitz (2011).[5] In section 4.4, I compare my results to those from these papers: my findings can be reconciled with those from Lise and Seitz (2011), but are qualitatively different from those of Bargain, Donni and Hentati (2022). A systematic comparison of different approaches is left to future work, but in section 4.4 and section 5 I illustrate how my estimates vary in response to varying the assignable good, the identifying assumption, and how public consumption is incorporated.

[5]My paper builds on Lise and Seitz (2011) in a few dimensions. Firstly, I add detailed time-use to the model, use a more credibly private assignable good (private leisure instead of non-market work) and estimate the sharing rule from high-quality time-use diary data, instead of expenditure data with recall question on hours worked. Secondly, I drop the 'symmetry assumption' in Lise and Seitz (2011), that men and women with the same hourly wage share household full income equally, and instead use an identifying assumption restricting preference heterogeneity. I also show that it is possible to estimate sharing from time-use data alone. Finally, I adopt the simpler linear estimation approach pioneered by LPW. Relative to this paper, Lise and Seitz (2011) incorporates the complexity of an income taxation system based on household earnings, which was an important feature of the UK economy in the time-period they studied, but is no longer in use in the UK.

As well as contributing to the literature on estimating intra-household sharing, I contribute to the literature on measuring individual-level consumption inequality. I define individual-level consumption similarly to Lise and Seitz (2011), but build on existing approaches in two directions. Firstly, I propose a methodology that carefully takes into account (i) durable goods as well as consumables, (ii) goods with prices that vary substantially at a regional level, and (iii) partly public goods, as well as fully private and public goods. This allows me to incorporate large expenditures which are often excluded, such as housing and cars. Additionally, this is the first paper to set out an approach to including time-use as well as material consumption, to consider 'full' consumption.

1.2 Structure of this paper

This paper is organised as follows. Section 2 sets out the structural model of the household. In section 3 I propose a new methodology to estimate intra-household sharing. Section 4 discusses the estimation of the sharing rule from UK time-use data, the findings and their interpretation, empirical performance of the model and comparison to other approaches. In section 5, I set out a methodology to estimate individual-level consumption. I implement it by applying my estimated sharing rule to a separate but comparable dataset on UK household material expenditure, and discuss policy implications. I conclude with some reflections for future work in section 6.

2 The model

In this section I set out a general static collective model of the household with both private and public goods and both material goods and time-use. To date, the collective model of the household is both more general, and provides a better empirical fit, than alternatives in the literature (e.g. see Browning, Chiappori and Weiss (2014)). This structural model underpins the methodology proposed in this paper.

2.1 Framework

An individual i belongs to a household h. Each individual has a person type t (e.g. man, woman, child).[6] Households are in the same category g (e.g. singles, heterosexual couples...) if they have the same composition, i.e. the same number of household members of each

[6]Depending on the application of interest, types can be defined more granularly to model more heterogeneity. Children may be modelled as decision-makers or as public goods, depending on their age.

type: $N^t_{h,g} = N^t_g, \forall h \in g$. To keep the notation leaner, I index individual variables only by i and h (even though these have an associated t and g), and household variables only by h (not g). I index type-specific parameters that vary between household categories with both t and g, and household category-specific parameters only with a g. Where summing over multiple individuals, I use the letter s instead of i, and the notation st for the type of person s. A household may have multiple members of the same type. Individuals have a vector of characteristics $\pi_{i,h}$ (e.g. age, educational attainment) and their households have a vector of characteristics ζ_h (e.g. the gender ratio in the region).

The household purchases two types of goods on the market: private material goods and public material goods.[7] $c_{i,h}$ is the vector of market purchased private goods consumed by individual i. c is the sum of these vectors over all household members. A specific good, e.g. food, is indexed by $j \epsilon \Omega^c$. The private good j has price p^j, and the vector of prices is p. X is the vector of market purchased public goods. Since these are public goods, all household members consume the full amount purchased by the household. A specific good, e.g. housing, is indexed by $j \epsilon \Omega^X$.[8] The public good j has price r^j, and the vector of prices is r.[9] I do not index p^j and r^j by h because in many applications they will be constant across the sample, but where they do vary between households everything carries through with the small addition of h subscripts to the material good prices.

Time is continuous,[10] and each individual has time-endowment normalised to 1, which can be spent in different activities. For exposition, I distinguish between four types of time-use: private leisure, joint leisure, market work and domestic work. Private leisure $\ell_{i,h}$ includes leisure activities enjoyed by an individual without the co-presence of other household mem-

[7]This paper focuses only on the private-public framework (which can incorporate partially public goods by appropriately defining goods e.g. splitting car fuel into car fuel used for holidays and car fuel used for work trips) but it readily extends to a model with both shareable and public goods. In this case, actual consumption of each member is obtained by multiplying the vector of market purchases for that member by an economies of scale matrix A. The A matrix depends only on household size. Analysis remains the same, with the addition of this matrix (potentially leading to more complex forms for demands, especially where cross-good economies of scale are allowed). An important strength of the shareable goods framework is that it does not impose the degree of economies of scale of different goods. This can be put to use in papers, such as BCL, which are able to estimate economies of scale parameters in addition to the sharing rule. However, the identification approach in this paper, similarly to DLP and LPW, would not allow recovering these additional parameters.

[8]I do not restrict the types of goods. They can be normal or inferior, goods or bads...

[9]The more commonly used notation in this literature is q and p for the quantity and price of private goods, and Q and P for those of public goods. I deviate from it for two reasons. Firstly, to highlight the fact that c and X are only material private and public goods. This allows me to use Q to refer to the set of all public goods. Secondly, for the very practical reason that it is easy to confuse lower and upper case instances of the same letter, particularly for the letter p. For this reason, I use R, rather than P for the price vector associated to Q.

[10]By examining UK time-use data used for the application in this paper, this modelling assumption appears realistic.

bers, e.g. reading a book alone or having coffee with a friend. It is therefore a private good, which enters the utility function directly. Joint leisure $jt_{i,h}$ and domestic work $d_{i,h}$ may instead be thought of as inputs to public goods $D = f^D(\boldsymbol{d})$ and $JT = f^{JT}(\boldsymbol{jt})$ enjoyed by the household (or by specific sub-sets of members in the case of joint leisure involving only some household members). These production functions can accommodate heterogeneous productivity by type.[11] Market work $m_{i,h}$ does not enter the utility function.[12] In order to use private leisure as the assignable good, the key is to distinguish between it and other types of time-use (a more detailed categorisation is consistent with the model, but is not necessary). In particular, it is important to distinguish between private and public leisure[13] as well as between leisure and non-leisure activities such as domestic work.

We write Q_h for the vector of public goods including both material and time-use public goods (X_h, JT_h, D_h) and R_h for the associated price vector. R_h is indexed by h because it includes the wages $w_{i,h}$ of each of its members. Each individual commands an exogenous wage $w_{i,h}$ for a unit of market work.[14] We can think of an individual's unobserved skills as determining their hourly pay. Individuals then choose, within the set of jobs available to them given their pre-determined skills, whether to work in a longer-hour, higher-overall-pay job, or a shorter-hour, lower-overall-pay job. For instance, someone with high numerical literacy will likely command a high hourly wage, and might choose between the longer hours and higher overall salary of investment banking, and the shorter hours and lower (though still substantial) overall pay of industry forecasting. While contracted hours cannot be chosen freely in many jobs, in this model m_i are actual hours worked, which can be chosen freely and are more naturally modelled as a continuous choice variable.

[11] Productivity at home, and preference parameters, can in principle depend on characteristics such as age and productivity in labour markets. Hence, the model can account for very general patterns of behaviour, such as those documented in the time-use literature (e.g. see Bastian and Lochner (2020, August)). To simplify exposition, I treat preferences and productivity as constants which vary at the person type level. The theoretical results of this paper extend to the context with heterogeneity.

[12] This model can be extended to accounting for individuals taking some pleasure in their work (and/or domestic work) by modifying the time budget constraint so that an hour spent working reduces leisure time by less than an hour, capturing the fact that part of the time spent working is enjoyed. See Browning, Chiappori and Weiss (2014) for a discussion.

[13] Browning, Donni and Gørtz (2020) finds that these are far from perfect substitutes.

[14] A static collective model is incompatible with hourly pay being endogenous (see e.g. Browning, Chiappori and Weiss (2014) section 4.4.2). Since hourly pay generally affects bargaining power in the household, the household problem becomes inefficient if it is a choice variable. In a static framework, we can reconcile (i) modelling wages as exogenous in the household problem, and (ii) wages in actuality depending on time spent working, if individuals are myopic about the impact of their present time-use on future periods. If instead we wish to allow bargaining weights to be endogenous (through time-use affecting wages, or another channel), we must depart from the static collective model either by (i) employing a dynamic collective model with limited commitment, or (ii) remaining in a static framework, but choosing an inefficient household model instead of the static collective model (which is efficient by assumption). I note that this is a general issue for this literature, and is not specific to using private leisure as the assignable good.

Given the time endowment has been normalised to 1, we model each individual as having a labour income endowment equal to $w_{i,h}$. In addition, each member can be endowed with non-labour income $y_{i,h}^{NL}$ (or alternatively the household as a whole can have non-labour income y_h^{NL}). An individual's overall endowment is $y_{i,h} = y_{i,h}^{NL} + w_{i,h}$. The household's endowment (or full income) is $y_h = \sum_{i \epsilon h} y_{i,h}$. I refer to this as the household budget.

Each individual i of type t living in a household category g has utility function $u_{t,g}$. This allows preference heterogeneity across types and household compositions. For instance, a woman living alone can have different preferences from a woman living with a partner, and also different from a man living with a partner. Person types and household categories can be chosen to be arbitrarily granular, allowing additional heterogeneity in preferences. However, for implementation, it will be practical to restrict the number of person types and household categories. Types should be chosen to capture the key likely dimensions of heterogeneity in the context of interest, but without reducing the sample size for each category too drastically.

Estimation of the sharing rule must be conducted separately for households of different categories. This is because the form of the bargaining solution of each household depends on the number and types of members, so that the sharing rule for different household categories must be estimated separately.

2.2 The household's optimisation problem

In a household, the constituent individuals bargain over how to divide resources. Depending on the bargaining process, and on the outside options of the individuals, the different individuals will have different bargaining power and the resulting division of resources will be different.

The collective model of the household does not restrict bargaining to any specific solution, and only requires that this process be efficient.[15] Relative bargaining power will in general depend both on (i) market variables such as prices (including wages), and (ii) distribution factors, which enter the household's optimisation problem only indirectly through the distribution of bargaining power, e.g. age and education of members (elements of the vectors of individual and household characteristics $\pi_{i,h}, i \in h$ and ζ_h). I refer to the vector of variables that affect bargaining power as z_h. Note that a specific member's bargaining power will depend not only on their own characteristics, but also on the characteristics of all other household members, hence the household-level subscript.

A key result from the existing literature (see Browning, Chiappori and Weiss (2014)) is that the problem solved by any collective household, regardless of the underlying bargaining

[15] As opposed to models with specific bargaining solutions e.g. McElroy and Horney (1981).

process and outside options, can be represented as an optimisation problem where the maximand is the weighted sum of the members' utility functions. Each member's utility function is weighed by their Pareto weight $\mu_{i,h}(z_h)$ normalised so that $\sum_{i \epsilon h} \mu_{i,h}(z_h) = 1$. The higher an individual's Pareto weight, the more weight the collective household gives their utility in determining its choices. The household's optimisation problem is therefore to maximise $\sum_{i \epsilon h} (\mu_{i,h}(z_h)\, u_{t,g}(c_{i,h}, X_h, l_{i,h}, JT_h, D_h))$.

2.3 A problem in two stages

We can re-cast this as a two-stage problem.[16] This representation is very helpful for identifying individual-level resources. In the first stage, the household chooses expenditure on public goods $R_h Q_h = \sum_{j \epsilon \Omega^X} r^j X_h^j + \sum_{i \epsilon h} w_{i,h}(d_{i,h} + jt_{i,h})$, and how to divide the remaining household budget into individual budgets $\rho_{i,h} = (y_h - R_h Q_h)\, \eta_{i,h}$ for members. The sharing rule[17] determines the share of household budget net of public good expenditure assigned to each member (the individual's resource share $\eta_{i,h}$), with the shares normalised to sum to one $\sum_{i \in h} \eta_{i,h} = 1$. In the second stage, members decide how to allocate their individual budgets $\rho_{i,h}$ to private expenditure.

First stage: public goods and individual budgets

$\max_{\rho,X,d,jt} \sum_{i \epsilon h} (\mu_{i,h}(z_h)\, v_{t,g}(\rho_{i,h}, X_h, D_h, JT_h))$ s.t. the following constraints:

- Budget constraint: $\sum_{i \epsilon h} w_{i,h}(d_{i,h} + jt_{i,h}) + \sum_{j \epsilon \Omega^X} r^j X_h^j + \sum_{i \in h} \rho_{i,h} = y_h$
- Time feasibility constraint: $d_{i,h} + jt_{i,h} \leq 1$
- Non-negativity constraints: $\rho_{i,h}, d_{i,h}, X_h, jt_{i,h} \geq 0$
- Domestic and joint leisure production functions: $D = f^D(\boldsymbol{d})$, $JT = f^{JT}(\boldsymbol{jt})$

Second stage: individual optimisation over private good consumption

$\max_{c_{i,h}, l_{i,h}, m_{i,h}} u_{t,g}(c_{i,h}, l_{i,h}, X_h, D_h, JT_h)$ s.t. the following constraints:

- Budget constraint: $\sum_{j \epsilon \Omega^c} p^j c_{i,h}^j + w_{i,h} l_{i,h} = \rho_{i,h}$
- Time feasibility constraint: $l_{i,h} + m_{i,h} = 1 - (d_{i,h} + jt_{i,h})$

[16] Separability is often assumed when employing the two-stage representation of a collective model with public goods (e.g. Lise and Seitz (2011)), but it is not a necessary assumption. Without separability, second-stage demands generally depend on public good consumption.

[17] In a model with private and public goods, what is called the sharing rule here is sometimes referred to as the conditional sharing rule. This is because it only affects the share of private expenditure of members, conditional on the household's choice of public good expenditure.

- Non-negativity constraints: $c_{i,h}, l_{i,h}, m_{i,h} \geq 0$

In general, second-stage demand depends on the prices of all private goods, including leisure (but not directly on the prices of public goods). Unless preferences are separable in private and public goods, it depends on public good consumption as determined in the first stage. Finally, it depends on the individual's second-stage budget, which is their resource share multiplied by the household budget net of public good expenditure (determined in the first stage). The assignable good c^a is a private good for which we observe individual-level demand (or expenditure) data. Individual i's (of type t) second-stage demand for the assignable good c^a takes the form: $c^a_{t,g}\left(p, w_{i,h}, Q_h, \left(y_h - R_h Q_h\right)\eta_{i,h}\right)$.

Note that this two-stage representation allows us to think of foregone wages from public time-use as foregone first-stage household expenditure on material public goods and on individual second stage budgets. At the same time, the two-stage representation implies that foregone wages from private leisure are foregone individual second-stage expenditure on private material goods. This is *not* an assumption that is imposed on a model, but rather a feature of the static collective model with time-use. The result is the same regardless of whether the model is solved in its original form or its two-stage form.

3 Methodology

Crucially for identification, individual i's second-stage demand for any private good, including the assignable good, depends only on observables, i's own resource share, and i's own preference parameters. Hence, if we observe individual-level expenditure for one assignable good for each household member, and make an identifying assumption restricting preference heterogeneity, we can in general identify resource shares. The advantage of this approach is that identification requires individual-level demand for one good only, rather than the whole unobservable individual-level expenditure system. The structural model simplifies the mapping from household-level expenditures to individual-level expenditures, so that we only need to estimate a very limited number of parameters (the resource share parameters).

In this paper I propose a new method to estimate the sharing rule under the model set out in section 2, with (i) Cobb-Douglas preferences, (ii) leisure as the assignable good, (iii) an identifying assumption restricting preference heterogeneity, and (iv) an additional restriction on preference heterogeneity allowing estimation of the sharing rule from time-use data alone. In section 4 I employ this very tractable approach to estimate resource shares from UK time-use data. As explained in section 3.1.4, I focus my application only on heterosexual working couples without cohabiting children. The model seems to fit well

and the assumptions made do not appear to be strong-arming the data in this context. The methodology set out in this section can straightforwardly be applied to different assignable goods: in section 4.4, I compare my baseline results to findings based on using clothing and non-market-work as the assignable good. Similarly, the methodology can be adapted to different identifying assumptions. In section 4.4 I discuss how the results for clothing change substantially depending on which identifying assumption is used for clothing. While Cobb-Douglas preferences greatly simplify the problem, it is possible to extend the approach in this paper to different parametric specifications. Online appendix A sketches out a general identification result and the required restrictions on functional form of preferences. In Online appendix A, I also outline a detailed example with the Almost Ideal Demand System.

3.1 Assignable good: private leisure

Regardless of our choice of assignable good and exact methodology, the core approach underlying papers including DLP, LPW and this paper is the following. The variation in assignable good expenditure with household budget is taken to be informative about how household budget is split between individual members' second stage budgets. This relies on the strong assumption that men and women have the same preference parameter for the assignable good, or else that we already know the preference parameters of men and women, e.g. from singles data. These types of identifying assumptions can be criticised for any assignable good, and any disputes on their credibility must ultimately be resolved empirically. This paper provides some analysis in this regard, but there is substantial scope for future work to comment on the credibility of the assumptions used in this, and other, papers in the literature.

If the identifying assumption is considered to be credible enough (at least better than the assumption that men and women share resources equally) then the rest of the exercise is rather straightforward. The variation in assignable good expenditure allows for the identification of the sharing rule, i.e. how the budget is allocated to members. This is all that is required to compute estimates of individual-level *overall* consumption. There is no assumption that expenditure on the assignable good is informative about the split of expenditure on other *specific* goods, and it is maintained that men and women may have heterogeneous and unknown preference parameters on all goods apart from the assignable good. Therefore, when using private leisure as the assignable good, this should not be interpreted as assuming that expenditure on any specific material good is split between members in the same manner as leisure. Rather, the assumption that I make in this paper, and which is supported by UK singles' data, is that men and women have a similar preference parameter for private leisure

specifically. This means that, if they have the same wages and the same individual budget, then they will have the same hours of private leisure. Hence we can use data on expenditure on private leisure, together with household budget and wages, to infer how the budget was split between men and women. This does not mean, say, that men and women also spend a similar proportion of their budget on clothing - in fact that is strongly rejected by my data.

3.1.1 Advantages of using private leisure

Clothing has been commonly used to identify sharing because it is often available split into women's, men's and children's clothing in household expenditure surveys. However, it also has limitations, as discussed for instance by LPW. Perhaps most concerning, clothing is a durable good, with a high proportion of 0s in expenditure surveys. In my UK data, 73.20% of households have zero recorded expenditure on at least one of male and female clothing. This is an issue both because it reduces the heterogeneity we can leverage for identification, and because most of the literature treats all observations as interior solutions, and when a large proportion of them are in fact corner solutions this can lead to inaccuracy and also bias towards equality because people with lower resource shares are more likely to hit the corner.

LPW show that they obtain similar estimates from clothing and individual-level food, using data from Bangladesh, and argue this supports the use of clothing as an assignable good because individual-level food expenditure is conceptually a more reliable assignable good. However, individual-level food expenditure cannot generally be used as the assignable good because that type of data is rarely available due to how resource-intensive it is to collect.

Other papers, including Lise and Seitz (2011), have used non-market work as the assignable good. This is obtained as a residual by subtracting usual hours worked (which is often asked as a recall question in expenditure surveys) from the time endowment. The idea is that this is a proxy of leisure which is often available in expenditure data. As acknowledged by authors who have used this assignable good, it is conceptually a good with substantial public components because it includes both domestic work and leisure time spent jointly with other household members. This is likely to bias estimates towards equal sharing.

I propose using individual time-use, instead of household expenditure, data to identify sharing. This is helpful because it allows us to strip out domestic work from non-market-work, based on activity information. Additionally, time-use data typically includes co-presence information, allowing us to also strip out leisure time spent jointly with other household members. Having removed the public components of non-market-work, we are left with a more credibly private assignable good: private leisure. Another advantage of private leisure

is there are no zeros in the data, unlike clothing, and in fact it commands a very large budget share.

For these reasons, we may expect private leisure to yield more accurate estimates of intra-household sharing than commonly available alternative assignable goods, at least in some contexts. In section 4.4, I show that the conceptual arguments outlined above appear to be borne out empirically in my UK data: for instance, the distribution of estimated resource shares obtained using non-market-work is shifted towards equal sharing relative to that obtained from private leisure.

3.1.2 Theoretical consistency of using time-use data to estimate sharing

One may worry that household decision-making over time-use in reality is quite separate from decisions about material consumption, and that we cannot conceptualise a single resource share as applying to both types of goods. This would require a departure from the static efficient collective model which underpins this paper and other papers in this literature. If in reality there was such a divergence in sharing between material and time-use goods, then it may be problematic to use private leisure as the assignable good to estimate a sharing rule to apply to material goods, and vice versa it may be problematic to use a material assignable good to estimate a sharing rule to apply to time-use. Reassuringly, Calvi et al. (2022) find that estimates of women's resource shares (from material assignable goods) are closely related to those women's satisfaction with their availability of leisure time, suggesting that the collective model is right to conceptualise the same sharing rule being applied both to time-use and material goods. Moreover, my resource share estimates, obtained from private leisure as the assignable good, are highly correlated to the wage ratio,[18] a commonly used proxy of sharing.[19]

Note that the model underpinning this paper is compatible with a woman's leisure consumption and her material consumption moving in opposite directions when her resource share increases. For example her material consumption may increase while her leisure decreases if her increased bargaining power is driven by an increased wage (the price effect may outweigh the income effect).

It is also important to note that private leisure is never 'imposed' by other household members in the collective model with participation. Intuitively, while it is true that I cannot take joint leisure with my partner if they are not taking joint leisure with me at that time, I am able to choose whether to undertake private leisure, domestic work, or market work instead. This

[18] Defined as the ratio of female wages to the sum of female and male wages.
[19] The correlation coefficient is 0.76.

is similar to what the model implies for public goods: while it is true that I cannot consume more heating if we do not collectively decide to spend more on heating, this does not imply that I must spend any additional money on my private consumption of food, e.g. I could purchase more clothing or spend more time having private leisure. In the same way, if we collectively choose not to spend more of our budget on public time-use, I could spend more budget, and more time, on private leisure, or I could spend more on material goods (and spend more time working). In order for this to be the case, it is important that hours worked can be chosen.

3.1.3 Wage endogeneity

In the static collective household model wages are exogenous. However, we may worry that in reality wages are endogenous. If time-use affects wages, and wages affect bargaining, that cannot be accounted for within the static collective framework, as it would lead to inefficient bargaining. The fact that previous papers have found the collective model to be a good empirical fit (see Browning, Chiappori and Weiss (2014)) suggests that wage endogeneity is not a substantial issue. For instance, it may be that individuals do not take into account the transitive implication of (i) their wage depending on their time-use, and (ii) their bargaining power depending on their wage, when making decisions. Note that if there was a substantial endogeneity issue, while this issue is more salient when using private leisure as the assignable good, it would be problematic regardless of the choice of assignable good, as the estimating equations derived from the static collective model would not be valid.

3.1.4 Restricting the sample to working couples

For people who participate in market work, the price of time is the observed wage. For those who do not, the price of time is an unobserved quantity, which exceeds the (also unobserved) wage, as explained in online appendix E. Therefore, in general to estimate sharing for households with non-participation we will require either a different approach,[20] or additional data, assumptions and estimation steps, to recover this unobserved price. This is not necessary if we choose an assignable good which is separable from private leisure, but it is always relevant when using private leisure itself as the assignable good.

Partly for this reason, incorporating non-participation is left to future work. In this paper, I focus on working couples without cohabiting children. This sample restriction is not unusual in the literature. In particular, other papers which explicitly incorporate time-use generally

[20] e.g. Blundell, Chiappori, Magnac and Meghir (2007) estimate the sharing rule, up to a constant, using a revealed preference approach.

exclude children or else model expenditure on children as a public good (Almås et al. (2020)). This sub-sample of the population is still of great interest, and in particular can shed light on gender inequalities and some of their drivers.

A further reason why I focus on working couples is that we can more credibly test the identifying assumptions for this sub-sample. As discussed in section 4.4.2, tests from data on working singles appear to be highly informative for this sub-sample, but, as explained in section 4.3.1, may be less convincing for other household compositions.

It is important to note that, in the UK context, selecting only couples where both members work is unlikely to introduce any bias in the estimates. The reason is that non-participation in couples without cohabiting children is similar between genders in the UK and is generally due to reasonably exogenous drivers such as long-term illness or temporary unemployment (see online appendix E). In other countries, with much lower female participation in labour markets, it will be important to incorporate non-participation.

3.2 Parametric specification: Cobb-Douglas

Utilities and domestic production functions are Cobb-Douglas, so that we can write:[21]

$$u_{t,g} = \sum_{j \epsilon \Omega^c} \left(\alpha_{t,g}^{cj} \ln(c_{i,h}^j) \right) + \sum_{j \epsilon \Omega^X} \left(\alpha_{t,g}^{Xj} \ln(X_h^j) \right) + \alpha_{t,g}^l \ln(l_{i,h}) + \sum_{s \epsilon h} \left(\alpha_{t,g}^{Tst,g} \ln jt_{s,h} \right) + \sum_{s \epsilon h} \left(\alpha_{t,g}^{Dst,g} \ln d_{s,h} \right)$$

In the context of resource share estimation, Cobb-Douglas preferences have several advantages. Importantly, they lead to parsimonious estimating equations, even with a rich characteristics vector z_h affecting bargaining power. This enables estimation from realistic, widely available data, which generally involves small sample sizes. Small sample sizes are common in this literature, partly due to limitations of existing data, and partly by construction, because estimation must proceed separately for households of different categories, so that even with large data the sample is divided into smaller sub-samples for estimation.

Additionally, Cobb-Douglas preferences, unlike e.g. the Almost Ideal Demand System, have a direct utility representation, which presents several advantages. Firstly, it allows for very clear interpretation of identifying assumptions in terms of restrictions on preferences. Moreover, the tractable direct utility representation of Cobb-Douglas enables estimation of the sharing rule from time-use data alone. Furthermore, under Cobb-Douglas preferences, resource shares are equal to Pareto weights scaled by a composite preference parameter. In my application to UK data, I find a significant effect of wages on the sharing rule, which

[21] See online appendix F for a more detailed discussion, including normalising assumptions.

with Cobb-Douglas preferences implies that Pareto weights vary with wages. This is inconsistent with the unitary model and can be interpreted as evidence for the collective model. This finding is in keeping with several other findings in the household economics literature (see Cherchye, De Rock and Vermeulen (2012)). Additionally, the simple, explicit utility representation of Cobb-Douglas preferences allows one to sense-check estimates. In online appendix F.6, I set out a back-of-the-envelope method to check that the estimated sharing rule parameters are quantitatively, as well as qualitatively, consistent with bargaining theory.

The separability properties of Cobb-Douglas are also helpful in reducing the amount of data required for estimation. For instance, assuming that private leisure is separable from various types of public time-use means I do not need to estimate time spent on specific other types of time-use. That exercise can be complex due to the sometimes fine line between joint leisure and domestic work, and would require careful consideration of household production functions.

As explained in online appendix A, while it is possible to identify the sharing rule using more flexible functional forms than Cobb-Douglas, this often requires either focusing on type-specific constant shares or else making some additional linear approximations. For instance, with covariates z, an additional small linear approximation is required for Almost Ideal Demand System Engel curves to be estimated by OLS (similarly to LPW). This limits the degree to which alternative preferences would better capture any non-linearities. Moreover, more flexible parametric forms often lead to additional practical issues for estimation because, in order to allow resource shares to depend on several characteristics z_h, estimation requires a large number of regressors, many of which are highly correlated with each other (see online appendix A.5). For these reasons, implementation will often benefit from using very parsimonious functional form assumptions, such as Cobb-Douglas.

The price of the tractability and ease of application achieved with Cobb-Douglas is a strong parametric assumption about preferences. In particular, Cobb-Douglas preferences impose homotheticity, which may seem to strong an assumption for certain goods. It is important to note that I do not require the whole utility function to be Cobb-Douglas: more specifically, I require preferences that are Cobb-Douglas over the assignable good and some aggregate good. Therefore, I am imposing homotheticity on the assignable good, but not on other specific goods. As discussed in section 4.3.3, I test homotheticity of private leisure using data on working singles, and cannot reject it (while, for instance, clothing appears to be a necessity good in my singles data). In section 4.3.3, I outline additional tests that suggest that the functional form assumption is not driving my results in my application to UK data. The Cobb-Douglas assumption is less likely to be realistic for other candidate assignable goods such as food and clothing, which are strongly non-homothetic.

3.3 Assumptions restricting preference heterogeneity

3.3.1 Identifying assumption: private leisure C-D SAP

Identification of the sharing rule from individual-level demand data for a single assignable good requires a strong identifying assumption. It is important to note that the chosen assumption has an important impact on estimates, and should be chosen carefully. Assumptions could be made on the bargaining process e.g. Lise and Seitz (2011) assume that women and men with the same potential earnings divide full income equally. Other papers have focused on restricting preference heterogeneity. For example, Lewbel and Pendakur (2008) and Bargain and Donni (2012) rely on the 'SAT' assumption (similarity across household types), that preferences for the assignable good are stable across household compositions, allowing for identification of the preference parameter from singles data. However, we may be concerned by findings that, at least in some contexts, preference stability across household composition is rejected empirically (see Hubner (2020)). DLP and LPW use the alternative 'SAP' assumption (similarity across people) that women and men in the same household composition have the same preference parameter for the assignable good. Whether this is a good assumption depends on whether we have reason to suspect substantial gender differences for the assignable good.

My data does not allow me to directly test this identifying assumption for private leisure, but I am able to check that it is consistent with singles data, as explained in section 4.3.1, and hence use the SAP identifying assumption. In other contexts, SAP may not be a realistic assumption: as discussed in section 4.4, singles data suggest that women have much stronger preferences than men for clothing. Therefore, if using clothing as the assignable good, it may be better to opt for SAT, or a somewhat weaker version of that assumption, which I term 'SRAT' (similarity across household types). I use this novel SRAT assumption in my robustness check with clothing as the assignable good, finding that the results are much closer to my baseline results from private leisure than when using clothing and the SAP assumption. I also show that, under my baseline sharing estimates, structural estimates of the ratio between men and women's preference parameters for clothing are consistent with the ratio observed in singles' clothing expenditure patterns.

I note that the interpretation of the identifying assumptions differs depending on the functional form of preferences and the assignable good of choice. Because I use a SAP-type identifying assumption in conjunction with Cobb-Douglas preferences and private leisure as the assignable good, the specific identifying assumption I am making may be referred to as 'private leisure C-D SAP'. Everyone, regardless of their type (in a given household category) is assumed to have the same Cobb-Douglas preference parameter for private leis-

ure: $\alpha^l_{t,g} = \alpha^l_g$. Preference heterogeneity across types is maintained for all other preference parameters, and across household compositions.

In the context of heterosexual couples, this assumption means that men and women spend the same share of their individual second-stage budget on leisure, but will split the remaining budget differently between other private goods. This means that I can infer how the budget is split between them by observing their expenditure on private leisure. It *does not* imply that I conclude that expenditure on each other good is split in the same manner as expenditure on private leisure.

Where SAP is not considered to be realistic, we can use alternatives from the literature such as SAT. In the context of private leisure as the assignable good, and Cobb-Douglas preferences, SAT implies that: $\alpha^l_{t,g} = \alpha^l_t$. The preference parameter for private leisure is stable across household composition for people of a specific type. This does not require preferences for other goods to also be stable across household compositions. Alternatively, I propose a novel, milder version of SAT: SRAT. Instead of requiring the level of preference parameters for the assignable good to be stable across household composition, I require only that the ratio between men and women's preference parameter for the assignable good is stable across household compositions. Private leisure C-D SRAT requires that: $\frac{\alpha^l_{t,g}}{\alpha^l_{st,g}} = \frac{\alpha^l_t}{\alpha^l_{st}}$. Hence, it is implied by (but does not in turn imply) SAT. Where we cannot reject equality between men and women's preference parameters for the assignable good in singles data, as is the case for private leisure, SAP and SRAT can be considered equivalent. Further work is required to carefully test the validity of different identifying assumptions applied to different assignable goods and different samples.

3.3.2 Additional restriction for ease of estimation: public good C-D SAP

The second-stage demand for assignable goods is conditional on public good expenditure (even when preferences are separable) because the resource share enters multiplicatively with the household budget *net of public good expenditure*. This presents two challenges. Firstly, it entails the substantial data requirement of having high-quality data both on time-use and material expenditure to construct overall expenditure on public goods, including domestic time and joint leisure as well as material goods. Ideally, this data would be in the same dataset. Alternatively, there would be a separate but comparable dataset including similar household characteristics - then the missing data could be estimated based on the separate dataset. The second challenge is correctly constructing a net of public expenditure budget, which in reality has substantial inter-temporal components, within a static framework. Squaring a static full income with observed public expenditure and implied budget for

private consumption, requires careful consideration of savings, investments and large durables (especially housing). In section 5 I explain how I tackle these challenges for the purpose of estimating individual-level material consumption (after having recovered the sharing rule). A similar approach could be taken here too, but there is a risk that it might give rise to inaccuracies which could affect the accuracy of sharing rule estimates.

To circumvent the challenges presented by estimating household budget *net of public good expenditure*, one can impose an additional preference restriction that total expenditure on public goods does not vary with the distribution of bargaining power in the household (public good SAP). This assumption *does not* amount to a unitary model in that the distribution of bargaining power still affects the composition of public good (and private good) expenditure; it simply does not affect the *overall* split between private and public expenditure. For example, female empowerment could lead to more spending on housing and less on television subscriptions.

Intuitively, public good SAP may seem too strong assumption because some papers have found gender differences on investment in children, although the evidence is mixed (e.g. see Gitter and Barham (2008)). Regardless of this debate, there is no clear reason why public good SAP would be problematic for the working couples in my UK application. In fact, this assumption appears to be consistent with the data in the context of this paper's application, as discussed in section 4.3.2. Care should be taken extending this assumption to other contexts, especially in households with children if children are modelled as public goods, as gender differences are more likely to emerge in the desired split between private and public expenditure.

With Cobb-Douglas preferences, we can achieve the desired restriction with public good C-D SAP, as explained in more detail in online appendix F.3. Public good C-D SAP imposes that the sum of all public good preference parameters is the same for all individuals of the same household category g:[22]

$$\left(\sum_{j\in\Omega^X}\alpha^j_{t,g}+\sum_{st\epsilon h}\alpha^{Tst,g}_{t,g}+\sum_{st\epsilon h}\alpha^{Dst,g}_{t,g}\right)=a^Q_g$$

This assumption allows heterogeneity in how different types of people would choose to divide the budget between specific private goods, and also between specific public goods, while requiring that they would choose the same overall split between private and public goods. As a result of the public good SAP assumption, it is possible to estimate the sharing rule from time-use data alone, if using private leisure as the assignable good. Alternatively, if using a material assignable good, it is possible to account for time-use as well as material goods, and

[22] Equivalently, the sum of all private good parameters is also homogeneous within category: $\left(\sum_{j\in\Omega^c}\alpha^j_{t,g}+\alpha^l_{t,g}\right)=a^{c,l}_g$.

still estimate the sharing rule from expenditure data alone. Note that this assumption is not required if we instead choose to estimate household budget net of public good expenditure. I also note that, for the purposes of estimating the sharing rule, this assumption allows us to proceed as if all goods were private, though in a manner that is consistent with a model with public goods. The extension to public goods bites primarily when it comes to using the estimated sharing rule to estimate individual-level consumption. At that stage it makes a substantive difference whether we consider some goods as public, as in this paper, or not, as in LPW (see section 5.2).

3.4 Linear approximation of resource shares

Resource shares will generally depend on household budget, prices (including wages of all household members) and Pareto weights of household members: $\eta_{i,h}\left(y_h, p, r, w_h, \mu_h\right)$. Recall that Pareto weights μ_h are an unknown function of a vector of variables z_h that determine relative bargaining power in the household, and that the collective model does not pin down a specific bargaining solution. Since z_h in general includes the other variables which affect $\eta_{i,h}$, we can write $\eta_{i,h}(z_h)$. To estimate resource shares by linear regression, we linearly approximate them as:

$\eta_{i,h} = \eta^0_{t,g} + \sum_z \eta^z_{t,g}\left(z_h - \bar{z}\right)$

- $\eta^0_{t,g}$ is the average resource share of a type (living in a specific household category). This is the resource share evaluated at the average characteristics in the sample. By definition, $\sum_{t \epsilon h} \eta^0_{t,g} = 1$. In the context of heterosexual couples, the average resource share of men and women sum to one.

- $\eta^z_{t,g}$ captures the impact on sharing of a household's characteristic z_h deviating from the sample average $(z_h - \bar{z})$. For instance, a higher-than-average wage for the woman might increase the woman's resource share, so that she would have a higher-than-average-for-women resource share. Since resource shares must sum to one within the household, this implies her partner must have a correspondingly lower-than-average-form-men resource share: $\sum_{t \epsilon h} \eta^z_{t,g} = 0$. We can interpret $\eta^z_{t,g}$ as the marginal impact of characteristic z_h on the resource share.

A linear approximation does not guarantee estimates of resource shares which fall within the unit interval. However, by construction, only estimates in this range are theory-consistent. This provides a useful test of model fit. Reassuringly, in my application to UK data, my baseline resource share estimates from time-use data are all within the unit interval.

3.5 Estimation

Under the framework set out above, I structurally derive a linear and parsimonious system of estimating equations. This comprises an equation for each type of person in the form: $w_{i,h}l_{i,h} = \alpha^l_{t,g} y_h \left(\eta^0_{t,g} + \sum_z \eta^z_{t,g} \left(z_h - \bar{z}\right)\right)$

3.5.1 A structural approach to the error term

To proceed to estimation, we must consider the source of any error terms. In this case, there are three likely sources of error in our estimating equations. The first is approximation error from linearly approximating the resource share based on the characteristics vector z_h. The second is household optimisation error at the first stage of the household problem. The third is individual optimisation error at the second stage of the household problem. As explained in online appendix F.4 we can write $w_{i,h}l_{i,h} = \alpha^l_{t,g} y_h \left(\eta^0_{t,g} + \sum_z \eta^z_{t,g} \left(z_h - \bar{z}\right)\right) + \epsilon_{i,h}$ where $\epsilon_{i,h}$ is mean-zero. If z_h includes all key characteristics that affect bargaining (or at least all those that are correlated with regressors) then estimates should be unbiased. The errors are negatively correlated within household (with a correlation weaker in magnitude than -1), so that a SURE estimation approach is recommended.

3.5.2 Estimation procedure

For estimation, I focus on heterosexual working couples without cohabiting children[23] for the reasons discussed above. The estimating equations are:

1. Equation for women: $w_{i,h}l_{i,h} = \beta^0_{f,g} y_h + \sum_z \beta^z_{f,g} y_h \left(z_h - \bar{z}\right)$
2. Equation for men: $w_{i,h}l_{i,h} = \beta^0_{m,g} y_h + \sum_z \beta^z_{m,g} y_h \left(z_h - \bar{z}\right)$

where $\beta^0_t = \alpha^l_{t,g}\eta^0_t$ and $\beta^z_t = \alpha^l_{t,g}\eta^z_t$
Estimation follows these steps:

1. Run linear SURE regressions of leisure expenditure, one for men and one for women, with the restriction that $\beta^z_f + \beta^z_m = 0$. This is because $\beta^z_t = \alpha^l_t \eta^z_t$ and from private leisure C-D SAP and the definition of the linear approximation parameters, $\sum_{t \epsilon h} \beta^z_t = \alpha^l_t \sum_{t \epsilon h} \eta^z_t = 0$. Note that there are other restrictions imposed by the model which I do not impose during estimation, and that can be used to test the fit of the model as discussed in section 4.3.

[23] Where there are multiple household categories, the approach set out below should be carried out separately for each household category.

2. Estimate each type's average resource share as $\hat{\eta_t^0} = \frac{\hat{\beta_t^0}}{\hat{\beta_m^0}+\hat{\beta_f^0}}$. To see why, first note that $\frac{\beta_t^0}{\sum_{st\epsilon h}\beta_{st}^0} = \frac{\alpha_t^l \eta_t^0}{\sum_{st\epsilon h}\alpha_{st}^l \eta_{st}^0}$ and by the leisure SAP assumption the α parameters cancel out so that $\frac{\beta_t^0}{\sum_{st\epsilon h}\beta_{st}^0} = \frac{\eta_t^0}{\sum_{st\epsilon h}\eta_{st}^0} = \eta_t^0$ since resource shares sum to one by definition.

3. Estimate the marginal impact of different characteristics as follows. First, estimate $\hat{\alpha^l} = \frac{\hat{\beta_t^0}}{\hat{\eta_t^0}}$ since $\beta_t^0 = \alpha^l \eta_t^0$. Then, estimate $\hat{\eta_t^z} = \frac{\hat{\beta_t^z}}{\hat{\alpha^l}}$ since $\beta_t^z = \alpha^l \eta_t^z$.

4. The estimated parameters yield the estimated sharing rule $\hat{\eta_{i,h}}$

Armed with the sharing rule, one can estimate individual-specific resource shares in the dataset used for estimation (the time-use dataset in this case): $\hat{\eta_{i,h}} = \hat{\eta_t^0} + \sum_z \hat{\eta_t^z}(z_h - \bar{z})$. Moreover, one can take the sharing rule across to other comparable datasets (e.g. expenditure data) and estimate household-specific resource shares there by applying the sharing rule.[24] This can be helpful to proceed to further applications of sharing rule estimation, as exemplified in section 5.

3.5.3 Intuition for identification

The identification problem we face is that the man's expenditure on private leisure may be more or less responsive than the woman's to changes in the household budget for one of two reasons, or a combination of them. The first possibility is that the man receives a larger proportion of the household budget (net of public good expenditure), i.e. he has a higher resource share than the woman. The second possibility is that the man's preference for private leisure is stronger relative to the woman's, so that he spends a larger proportion of his individual budget on private leisure. In order to disentangle these two channels, I shut down the preference channel with the identifying assumption, allowing me to identify the sharing rule. Under CD-SAP, private leisure expenditure for individuals of any type (within a given household category) $l_i w_i = \alpha^l y_h \eta_{i,h}$ responds in the same way to the same increase in individual budget $y_h \eta_{i,h}$, so differences in responsiveness to changes in y_h identify differences in sharing.

Empirically, identification of the levels of resource shares, and the marginal effects of different characteristics on them, is driven by different sources of variation in the data. For example, ceteris paribus, a higher wage for the man will reduce his leisure demand through a price effect, but increase it through a twofold income effect: (i) increased overall household budget,

[24] To do so, I calculate how household characteristics in the expenditure dataset deviate from the averages in the time-use data, and substitute these deviations $(z_h - \bar{z})$ in the estimated sharing rule $\hat{\eta_g}$.

and (ii) increased own-resource share.[25] Therefore, the leisure patterns of two similar couples that differ in the man's wage help identify both the effect of male wage on sharing, and, by increasing the overall household budget, the levels of resource shares. Similarly, couples which are identical in characteristics apart from the age gap, and have different leisure patterns, help identify the effect of intra-couple age gaps on sharing.

3.6 Data requirements

In order to identify the sharing rule we require:

- A cross-section household dataset with both household-level information and individual-level data for all members.
- Individual-level data on demand / expenditure on an assignable good for all members. Sometimes assignable good demand or expenditure data will be directly available in the dataset. In other cases, it may require constructing. For instance, when using private leisure as the assignable good, it may be necessary to classify time-use data based on activity and co-presence information.
- Data on full household income. Full income will generally have to be constructed from information on wages, together with any non-labour income of members (in this model, the household budget is not equal to realised earnings nor to expenditure on material goods).

In order to identify household-specific shares (instead of type-specific constants) we additionally require:

- Data on key characteristics which are likely to affect bargaining e.g. sex, age, educational attainment and wages of all members. Wages may need to be estimated from earnings and hours worked.

Depending on the choice of assignable good and parametric specification, we may additionally require:

- Price data for any goods for which (i) their price enters the estimating equations, and (ii) their price varies substantially in the sample. This includes wage data if private leisure is the assignable good. No price data is required if assignable good preferences are separable from preferences of all goods with price heterogeneity.

[25] Browning and Gørtz (2012) find that the 'unitary effect', that leisure demand falls as its price increases, dominates the 'collective effect' of the individual's bargaining power increasing in own-wage.

- Expenditure data on specific public goods, if there are any which enter the estimating equations (none if the assignable good is separable from public goods).
- Total public expenditure (on both material goods and time-use). As discussed in section 3.3.2, this requirement can be challenging to meet and can be avoided by making an additional assumption restricting preference heterogeneity.

4 Estimating resource shares from UK time-use data

I apply the novel methodology set out in section 3 to UK time-use data. In section 4.1, I describe the data (the UK Time-Use Survey). In section 4.2, I set out my findings and how to interpret them. Women command substantially lower resources than men, with substantial variation between households. The directions of marginal effects and their magnitudes are consistent with bargaining theory. In section 4.3, I discuss several tests of the model and assumptions, which appear realistic in this context. In section 4.4, I illustrate how my findings vary with different assignable goods, and compare my findings to those from two other papers on household sharing in the UK.

4.1 UKTUS

The UK Time-Use Survey (UKTUS 2000, 2014),[26] is a high-quality time-use survey that has been used in the economics literature (e.g. Kalenkoski et al. (2005)) but never before with the goal of estimating intra-household inequality. It is a national household-based study composed of: (i) a household questionnaire, (ii) an individual questionnaire, and (iii) individual time-diaries. A single household representative answered the household questionnaire, including questions on household characteristics such as composition, dwelling type, and location. The other components were answered by the individual in question. This is likely to substantially increase the quality of the data relative to datasets where a single member answers on behalf of all individuals. The individual questionnaire asks about individual characteristics including age, educational attainment and earnings. Each member completed a weekday and weekend time diary identifying primary and secondary activities for each 10-minute interval over the two days. The time-use data includes very detailed activities, location, and co-presence of others (distinguishing between household and non-household members). This enables me to define private leisure very precisely, as time spent

[26] Office for National Statistics. (2019b). United Kingdom Time Use Survey [data series] 2nd Release. https://doi.org/http://doi.org/10.5255/UKDA-Series-2000054

doing leisure activities without other household members co-present. For a list of activities, see online appendix B. The time-diaries are constructed carefully to minimise measurement error, for instance with the possibility of writing a simple sign to signify the same activity for multiple time intervals. The quality of the data is very high, with approximately 95% of observations having more than 5 distinct activities recorded in a day, and less than 90 minutes of unrecorded time. The data is nationally representative.

As discussed in section 3, I focus on heterosexual working couples without cohabiting children.[27] After cleaning the data, the final pooled (2000 and 2014) sample comprises 711 households (1,422 individuals). While the sample size is not particularly large, it is of a good size relative to this literature.[28]

Summary statistics for some of the key variables are reported in table 1. Hourly pay is obtained by dividing labour income by actual hours worked, rather than contractual hours.[29] Women on average command lower hourly wages than men. Couples generally form with an older, higher-earning man and a younger, lower-earning woman. Household budget is full income: the sum of the labour endowment of members (hourly wage multiplied by 24 hours).[30] Women on average are more qualified.

		UKTUS sample					LCF sample				
		mean	sd	p25	p50	p75	mean	sd	p25	p50	p75
Hourly pay	female	9.87	5.58	6.6	8.61	11.6	10.5	5.39	7.51	9.37	12.2
(2020 GBP)	male	11.6	8.74	7.51	9.78	13.1	12.2	7.61	7.79	10.4	14.3
Age	female	41.9	13	30	43	53	42.1	13.6	29	43	55
(years)	male	44	13.1	31	46	55	44	13.6	30	44	57
Qualification	female	.864	.824	0	1	2	1.04	.871	0	1	2
(levels 0,1,2)[31]	male	.816	.813	0	1	2	.956	.857	0	1	2

Table 1: Summary statistics

[27] The sample includes both couples who never had children and couples who have children who no longer cohabit with them. I am not able to run the analysis separately for these two types of household because the data does not contain information on non-cohabiting children over the age of 15 (none of the households in my sample have children under 15 who do not cohabit with them). Therefore, the age effect in the resource share (older couples assign lower resource shares to women) could be partly driven by older couples being more likely to have non-cohabiting children.

[28] e.g. Cherchye, De Rock and Vermeulen (2012) estimate a closely related model on a sample of 212 Dutch households. Small sample sizes are to some extent inevitable in applications where different household compositions are analysed separately.

[29] This, together with self-employed labour, explains the lower end of hourly wages (which are sometimes lower than the official minimum hourly wage).

[30] Accurate non-labour income data is not available in UKTUS.

[31] Highest qualification obtained, simplified into three categories. Category 2 is equivalent to an undergraduate degree or higher. Category 1 is equivalent to end-of-school diplomas e.g. A levels, IBDP, or equivalent technical qualifications. Category 0 is anything less than that, including lower technical qualifications and school diplomas obtained before the end of school e.g. GCSEs.

4.2 Sharing rule estimates

Using the UKTUS pooled sample, I estimate the resource sharing rule for UK working couples. I find that households do not share resources equally: the mean resource share for women is 0.45 and 0.55 for men (the median is 0.44 and 0.56). As shown in table 4, and explained in more detail in online appendix G, I comfortably reject equal sharing. Household-specific resource shares vary substantially (as can be seen in figure 1). This heterogeneity is driven both by market variables (e.g. wages) and distribution factors (e.g. the gap between the age of the man and the woman). These findings confirm the importance of accounting for intra-household unequal sharing, and for household-specific variation in sharing as a function of characteristics. Moreover, these findings reinforce the prevalence of systematic inequality in sharing by gender, with the distribution of female resource shares to the left of that for men, as illustrated in figure 1.

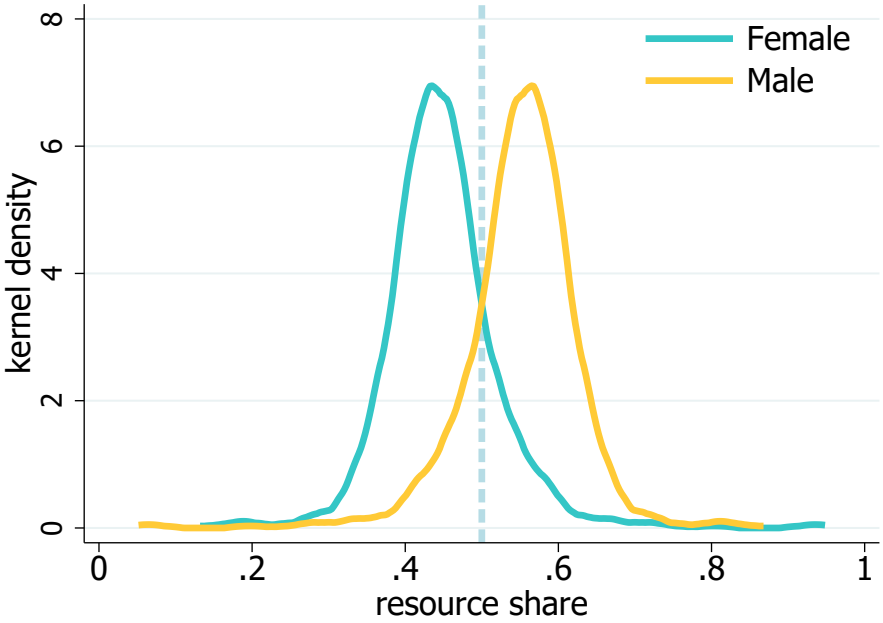

Figure 1: Baseline results, UKTUS. The distribution of male resource shares is a reflection of the distribution of female resource shares in the equal sharing (dotted) line. This is because each household's shares must sum to one.

4.2.1 Interpreting regression results and estimating marginal effects

Sharing rule estimates are obtained by running seemingly unrelated regressions of leisure expenditure equations for men and women. I allow resource shares to depend on the following characteristics: the hourly pay and educational attainment of both members, the age gap

Dependent variable	leisure expenditure	
Equation	male	female
Budget	0.243***	0.198***
	(0.00262)	(0.00236)
Budget * dev. fem. hourly pay	-0.00559***	0.00559***
	(0.000198)	(0.000198)
Budget * dev. mal. hourly pay	0.00215***	-0.00215***
	(6.87e-05)	(6.87e-05)
Budget * dev. fem. qualification	-0.00295	0.00295
	(0.00253)	(0.00253)
Budget * dev. mal. qualification	0.0154***	-0.0154***
	(0.00237)	(0.00237)
Budget * dev. average age	0.000905***	-0.000905***
	(0.000151)	(0.000151)
Budget * dev. age gap	0.000481	-0.000481
	(0.000338)	(0.000338)
Budget * dev. regional wealth p.c.	-5.44e-07**	5.44e-07**
	(2.19e-07)	(2.19e-07)
Observations (households)	711	711
R-squared	0.937	0.931

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 2: Regression results, baseline specification, UKTUS. The symmetry of coefficients on the interaction terms is imposed as a constraint during SUR estimation. Budget * dev. variable = budget * (household-specific value of variable - sample average of variable)

and average age of the couple[32] and regional wealth in the household's region. The results are reported in table 2.

The mean female resource share is calculated by dividing the coefficient on 'Budget' in the female regression by the sum of the 'Budget' coefficients across the two equations: $\hat{\eta_f^0} = 0.45$. The preference coefficient is estimated by dividing the coefficient on 'Budget' in the female regression by the estimated average female resource share: $\hat{\alpha^l} = 0.44$. This number is consistent with individuals choosing to take private leisure for a substantial proportion of their daily time endowment i.e. spending a large proportion of their budget on private leisure.[33]

To interpret the regression coefficients, consider an example household h. The starting

[32]The age gap is the difference in years between the man and the woman

[33]My baseline definition of private leisure includes sleep. haring rule estimates are similar when sleep is excluded, though the preference parameter falls substantially as expected.

point for their resource shares are the averages $\hat{\eta_f^0} = 0.45, \hat{\eta_m^0} = 0.55$. We then adjust for any deviations from the sample mean for determinants of sharing. For instance, consider a deviation on female hourly pay. The effect of a unit deviation from the mean of the female wage (w_f) is calculated by dividing the coefficient on the 'Budget * dev. fem. hourly pay' term by the preference coefficient: $\hat{\eta_f^{wf}} = 0.01$. We estimate the resource share for a couple where the woman earns £15 an hour, instead of the mean of £9.87 as: $\hat{\eta_{f,h}} = \hat{\eta_f^0} + \hat{\eta_f^{wf}} * (15 - 9.87)$, which is $\hat{\eta_{f,h}} = 0.51$ and hence $\hat{\eta_{m,h}} = 1 - 0.51 = 0.49$. Note that the impact of deviations from the mean female wage are very substantial. The change in predicted female resource share for changes in different determinants of sharing is summarised in table 3 .

variable		mean	sd	impact of..	on fem. share
Hourly pay (2020 GBP)	female	9.87	5.58	↑ 1 s.d.	0.0706
				↑ £1	0.0127
	male	11.63	8.74	↑ 1 s.d.	-0.0426
				↑ £1	-0.0049
Age (years)	average	42.93	12.82	↑ 1 s.d.	-0.0263
				↑ 10 years	-0.0205
	gap	2.06	4.72	↑ 1 s.d.	-0.0051
				↑ 1 year	-0.0011
Regional wealth p.c. (2020 GBP)		30,446	6,780	↑ 1 s.d.	0.0084
				↑ £5k	0.0062
Qualification (levels 0,1,2)	female	0.86	0.82	↑ 1 s.d.	0.0055
				↑ 1 level	0.0067
	male	0.82	0.81	↑ 1 s.d.	-0.0284
				↑ 1 level	-0.0349

Table 3: Interpretation of regression coefficients

The direction of estimated marginal effects is consistent both with bargaining theory and with previous findings in the literature. As shown in table 4, women's resource shares are higher in households with high female hourly pay and low male hourly pay. The impact of the former outweighs the latter, so that women in households with higher household budgets[34] have higher resource shares. Older couples (either because they are older or belong to more traditional generations) and couples with a larger age gap (older man relative to the woman) are characterised by a higher fraction of resources going to the man, although the latter effect is not statistically significant. Women have higher resource shares in regions with higher wealth per capita, potentially because of better outside options for women in

[34]The budget here is the full daily budget, i.e. the labour endowment of the couple (sum of male and female hourly pay, multiplied by 24 hours)

wealthier regions, or due to wealthier regions proxying more gender progressive regions. More educated men have higher resources shares, and the same goes for females, although for the latter the effect is not statistically significant. A possible explanation is that, in the UK, female qualifications have trended upwards more strongly than male qualifications over time and generations, substantially reducing the matching market returns of female education.

		male hourly pay	
		$<$ median	$>$ median
female	$<$ median	0.43	0.39
hourly pay	$>$ median	0.50	0.47

	mean	95% confidence interval	
		Lower bound	Upper bound
Private leisure	0.45	0.44	0.46
Non-market-work	0.49	0.48	0.49
Clothing (adjusted)	0.46	0.39	0.53

Table 4: Female resource shares vary with wages (left) and assignable good choice (right)

4.3 Empirical performance of the model

The model appears to fit the data well. Results are stable, and change in the theoretically consistent direction, when performing various robustness checks, including:

- Different definitions of private leisure, e.g. defining leisure in a much more narrow way, excluding time spent sleeping, eating, and work breaks.
- Estimation on different samples, e.g. excluding outliers with particularly low or high wages, and estimating the sharing rule separately on 2000 and 2014 data.
- Running alternative regression specifications, e.g. substituting the gap between male and female qualification for the levels of male and female qualification, and excluding the age gap and regional wealth, as determinants of bargaining power.

Moreover, the model imposes several testable restrictions on the sign and magnitude both of the regression coefficients and the structural parameters recovered from them. These restrictions are not imposed by the estimation method, and testing them suggests the model fits the data well. Importantly, all resource shares fall within the unit interval.[35] Additionally, in conformity to the way noise was incorporated in the model (see section 3.5.1), residuals are negatively correlated within the household, with a correlation coefficient of -0.09.[36]

The only restrictions that were imposed during estimation required that all coefficients apart from that on household budget sum to zero across equations. I run unconstrained SURE

[35] I note that this is very much not mechanical: estimates using clothing and non-market-work fall outside of the unit interval at the extremes.

[36] A statistical test of cross-equation independence rejects at the 5% significance level

regressions and find the magnitudes of coefficients are similar across the two equations, and the signs are opposites of each other. This suggests that imposing the restriction that these coefficients sum to zero across the equations is not far off correct, and is not strong-arming the results (although some of the estimated coefficients are statistically significantly different from each other).

4.3.1 Testing the private leisure C-D SAP assumption

To test the validity of the private leisure C-D SAP assumption, I use data on working singles using UKTUS pooled data. I focus on working singles to ensure comparability in the way the budget is calculated, and also since non-participation leads to different interpretations of time-use choices, including involuntary leisure for the unemployed.

For singles, $w_i l_i = \alpha_t^l y_i$. I run this regression for men and women, finding the coefficients are approximately equal: $\hat{\alpha_m^l} \simeq \hat{\alpha_f^l} \simeq 0.6$.[37] I test the null hypothesis that $\alpha_m^l = \alpha_f^l$ and cannot reject the private leisure CD-SAP assumption at any of the usual significance levels. These findings suggest single working men and women satisfy private leisure C-D SAP, and hence that private leisure C-D SAP is plausible for working couples without cohabiting children. It is unclear why the preferences of men and women would change differently in this specific respect between being single (and working) and being in a couple (and still working). As explained in section 4.4.2, this type of test appears to be highly informative for working couples. We may not be as sanguine about the validity of the test in other contexts, e.g. for couples with cohabiting children, since there are substantial gender differences in norms around childcare.

As an additional check, I also look at high-level patterns in the data on UK couples without cohabiting children. If private leisure C-D SAP holds then the proportion of household budget spent on overall private leisure (the sum of expenditure on female private leisure and expenditure on male private leisure) should not vary with the distribution of bargaining power. This is consistent with UKTUS data: the budget share spent on private leisure is mildly, and not statistically significantly, correlated with my estimated resource shares. I repeat the same check with the wage ratio, a frequently used proxy for bargaining power, yielding similar results. This provides additional reassurance that private leisure C-D SAP is a reasonable approximation in the context of this paper's application.

[37] Not only are these preference parameters similar to each other but, comfortingly, they are higher than the estimated preference parameter for private leisure for working couples. We would expect that to be the case because joint leisure is not an option for singles, so we would expect singles to have a higher relative preference (higher preference coefficient, under the normalising assumption that preference coefficients sum to 1) for private leisure.

4.3.2 Testing the public good C-D SAP assumption

Recall that, in order to estimate sharing from time-use data alone, I also assumed that preference parameters over all public goods sum to the same quantity for men and women. Equivalently, I can test whether the preference parameters over all private goods sum to the same quantity for men and women.[38]

Having already tested that private leisure preferences are similar across types, I only need to test that preferences for aggregate material private good consumption are the same for men and women. I test this with data from a widely used UK expenditure survey (Living Costs and Food Survey, LCF, see section 5) for working singles.[39] For singles, $\sum_{j\in\Omega^c} p^j c_{i,h} = \left(\sum_{j\in\Omega^c} \alpha_t^j\right) y_{i,h} = \alpha_t^c y_{i,h}$. The test therefore requires regressing aggregate expenditure on material private goods on the budget, and checking whether the coefficients are the same for men and women. The coefficients are very similar in magnitude ($\hat{\alpha_m^c} \simeq \alpha_m^c \simeq 0.07$) and a test of equality cannot reject at any usual significance level. This test is consistent with the public good C-D SAP assumption.

It is unclear why the preferences of single men and women would change differently with regard to private v public expenditure when forming a couple without cohabiting children. Therefore we may feel reasonably confident that the singles test provides good evidence in favour of public good C-D SAP in the context of this paper's application. As explained in section 4.4.2, this type of test appears to be highly informative for working couples.

I also perform additional high-level checks using data on couples, to provide additional evidence that public good C-D SAP does hold for UK couples. If public good C-D SAP holds then the share of household budget devoted to private goods should be independent of the distribution of bargaining power. Because private leisure C-D SAP entails that the share of household budget spent on private leisure is independent of the distribution of bargaining power, these assumptions together entail that the share of household budget devoted to private material goods is independent of the distribution of bargaining power. This is consistent with correlations in the LCF expenditure data. The proportion of budget spent on private material goods is mildly, and not statistically significantly, correlated with my estimated resource shares. This suggests internal consistency of my approach. As a further check, I also repeat this exercise with the ratio of female-to-male wages as a commonly used proxy of household sharing. Again, the correlation is mild and not significant, suggesting

[38] The latter implication is preferable for testing because data on private good consumption is less lumpy, and collected more accurately, in most expenditure surveys, including the one used here.

[39] For singles there isn't a distinction between private and public goods in practice, but we can still distinguish theoretically between goods which are non-rivalrous and those that aren't, categorising goods in the same way for singles and couples.

that public good C-D SAP is a reasonable simplification in the context of this paper's application.

As discussed for private leisure C-D SAP, while public C-D SAP appears to be a reasonable approximation for couples without cohabiting children in the UK, it is important to consider the context when evaluating this assumption. For instance, we may be less willing to make this assumption in contexts with cohabiting children as gender differences are more likely to emerge in that context.

4.3.3 Homotheticity tests

Cobb-Douglas is clearly a strong simplification of underlying preferences, and we may be particularly concerned about imposing homotheticity. I note that I am not imposing homotheticity on all goods, but only at a high level between private leisure and other consumption (and between overall private and public consumption if relying on public good SAP too).

I test homotheticity of private leisure by estimating singles' private leisure expenditure separately for higher and lower income singles. I cannot reject equality of coefficients at any of the usual significance levels. Similarly, for singles, the correlation between the budget share of private leisure and their budget is mild and not significant. As a further check, I estimate resource shares separately for higher and lower budget couples. Parameter estimates are broadly similar to the baseline, and the resulting estimated resource shares are highly correlated (correlation of 0.85)[40] with the baseline estimates, suggesting that the homotheticity assumption is not strong-arming results. Finally, I relax the homotheticity assumption by estimating the sharing rule under the assumption of Stone-Geary preferences.[41] Comparing the results to the baseline estimates, the direction of marginal effects is the same, and the estimated resource shares are very highly correlated (correlation of 0.97) with baseline estimates. Overall, it appears that, in this context, assuming homotheticity is not strong-arming results.

4.3.4 Internal consistency of estimates

An additional advantage of Cobb-Douglas preferences is that they have a simple, explicit utility representation which can be used to sense-check estimates. In Appendix F.6, I set out a

[40] This is for the sample excluding outliers. The correlation coefficient including outliers is still high but falls to 0.75 as each of the two sub-samples only contains outliers in one direction.

[41] In order to avoid colinearity issues due to the lack of accurate non-labour income data, this version of the estimates assumes all goods are private and that subsistence levels for private leisure are equal to zero (while they can be non-zero for other goods).

back-of-the-envelope method to check that the estimated sharing rule parameters are quantitatively, as well as qualitatively, consistent with bargaining theory. Consider the example household discussed in section 4.2.1, characterised by sample-averages for all characteristics apart from the female wage, which is £15 instead of £9.87 an hour, and hence by a female share of 0.51 as opposed to the average female share of 0.45. We wish to check that the implied utility is higher for the higher-earning female than for the average female, as she has a better outside option. This is confirmed by a back-of-the-envelope check: the higher-earning female has slightly lower-than-average hours of leisure, and more material consumption, with the latter effect outweighing the former.[42] The fact that leisure is lower for her means the price effect outweighs the income effect (which here is a double effect through the household budget and through the sharing rule). Similar findings hold for men, and for a reasonable range of possible wage changes.

A further check of internal consistency is discussed in section 4.4, where I am able to replicate the same gendered pattern in preferences for clothing observed in singles data by structurally estimating clothing preference parameters for couples, using my sharing estimates.

4.4 Comparison to other approaches

Reassuringly, my estimated resource shares are positively (correlation coefficient of 0.76) and very significantly correlated with the ratio of female to male wages,[43] which is often used as a proxy of sharing in the household. As further checks, I re-estimate resource shares using two alternative assignable goods: (i) non-market-labour time, and (ii) clothing, and compare the findings. Finally, I consider how my findings relate to existing estimates in the literature on UK data.

4.4.1 Non-market-work hours as the assignable good

I repeat my analysis using non-market-work as the assignable good. I construct it from a recall question on usual hours worked from the LCF expenditure data (see section 5).

As can be seen in table 1 and further discussed in section 5, while the expenditure and time-use data are comparable, the LCF sample is characterised by somewhat higher wages and education. Therefore, to construct the figures comparing the distribution of resource

[42] Different papers find different results on the relationship between wages and leisure, see Browning, Donni and Gørtz (2020). Those authors find that leisure, both private and public, is broadly stable across the wage distribution of both members of the couple. This is consistent with high-level correlations in UKTUS, though it is important to remember that there is significant assortative matching in the data and hence that these high-level correlations conflate the effects of female and male wages.

[43] Defined as the ratio of female wages to the sum of female and male wages.

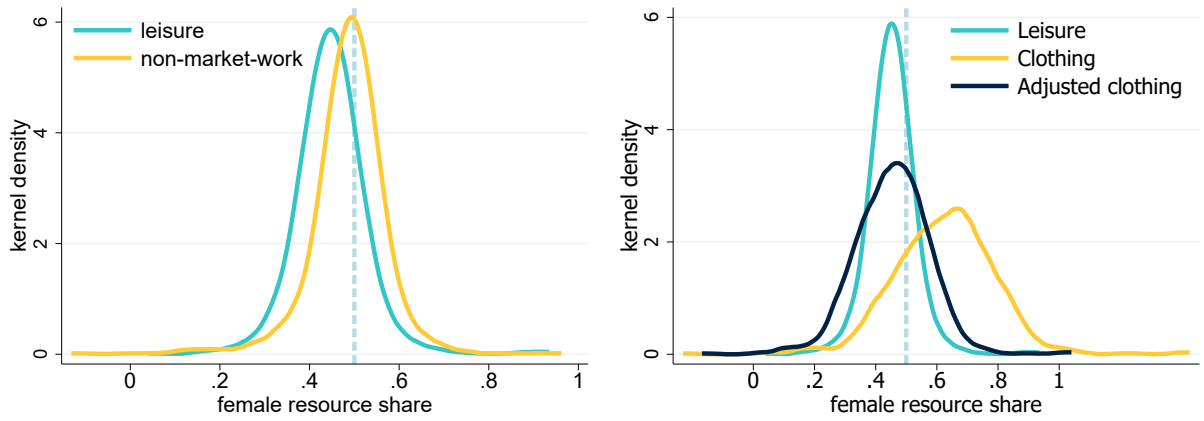

Figure 2: Estimated female shares using different assignable goods

shares for different assignable goods, and to estimate correlation coefficients between different sharing estimates, I compare estimates of sharing in the LCF obtained from different assignable goods. Non-market-work and clothing estimates are constructed directly in the expenditure survey, and for my baseline estimates I use my baseline sharing rule applied to LCF (baseline female shares are marginally higher than in UKTUS).

My baseline results and estimates using non-market-work as the assignable good are positively correlated, with a correlation coefficient of 0.76. As can be seen in figure 2, the whole distribution is shifted towards equal sharing relative to my baseline estimates, and the mean female share is estimated at 0.49, substantially higher than the 0.45 from my baseline results. As discussed in more detail in online appendix G, and shown in table 4, the difference between this estimate and my baseline estimate is statistically significant. This difference is consistent with the conceptual reasons set out in section 3.1 why non-market-work may lead to estimates biased towards equal sharing. This bias may be even greater in other applications, with cohabiting children and non-participation, where we may expect that women spend substantially more time than men on childcare and domestic work.

4.4.2 Clothing as the assignable good

I repeat my analysis using clothing as the assignable good, using the LCF expenditure data. Clothing is of interest because it has been used as the assignable good almost universally in this literature. I find a mean female resource share of 0.63, substantially higher than my baseline finding of 0.45. As can be seen in figure 2, the whole distribution of female resource shares is shifted upwards, and the extremes of the distribution violate the testable restriction that resource shares lie in the unit interval. Using singles data, I find that women's pref-

erence parameter for clothing is approximately twice as large as men's, and the identifying assumption (clothing CD-SAP) is rejected at the 5% significance level. As a further check, I also estimate clothing preference parameters *for couples*, by running structural regressions of clothing expenditure on estimated individual budget computed using my baseline sharing estimates. This also yields a female preference parameter for clothing which is approximately twice the male parameter, as for singles. Therefore, I adopt an adjusted identifying assumptions, assuming the female-to-male ratio of preference parameters estimated in singles data holds for working couples (clothing C-D SRAT). The resulting distribution of adjusted clothing resource shares is much closer to the baseline, as can be seen from figure 2, and the mean resource share for women falls to 0.46. This is encouraging, in that it is suggests that tests of identifying assumptions using singles data are highly informative for working couples.

As discussed in more detail in online appendix G, and shown in table 4, the confidence intervals for the clothing estimates are very wide, and partially overlap with the confidence intervals for my baseline estimates and non-market-work estimates. Leaving aside the noisiness of the clothing estimates, the fact that the estimated female share from clothing is higher than that the one from private leisure, even after the adjustment to the identifying assumption, may be explained by the conceptual reasons set out in section 3.1 However, a full comparison is left to future work - in particular, I note that my implementation with Cobb-Douglas preferences is not a good fit for clothing, which is not a homothetic good. In singles' LCF data I find that there is a significantly negative correlation between budget share spent on clothing and household budget, suggesting clothing is a necessity good. Potentially for this reason (or perhaps for other reasons, such as the high proportion of clothing zeros in my data), using clothing, the estimated marginal effects of household characteristics on sharing are at odds with bargaining theory (female hourly pay decreases female resource shares, and male hourly pay decreases male resource shares), the estimates are noisier, and the estimated shares are negatively correlated with the wage ratio (commonly used as a proxy of sharing).

4.4.3 Estimates from other papers with UK applications

Lise and Seitz (2011) use non-market-work as the assignable good, and their framework differs from this paper in several respects, including parametric specification, identifying assumption,[44] and estimation approach.

[44]Lise and Seitz (2011) make the 'symmetry' identifying assumption that women and men with the same potential earnings have the same resource shares. Empirically, this assumption is not supported by my findings. A woman with an hourly wage equal to the average male wage in the sample is estimated to have

Their estimated share for the last cohort in their data, born in the 1960s, has a female resource share of 44.2% on average. Their estimates include couples with non-participation, so are not directly comparable: on average, we would expect women to have lower resource shares in households with non-participation. We might expect a partial cancelling out of the opposite effects of (i) sample differences, and (ii) non-market-work leading to some degree of bias towards equal sharing. Hence, the similar magnitude of the estimate is reassuring as it suggests that the results can be reconciled based on sample differences and differences in the assignable good used, notwithstanding the other substantial methodological differences.

However, the results of Bargain, Donni and Hentati (2022), using clothing as the assignable good are substantially different. They find the average resource share for women in heterosexual couples without children (including non-participants) in 1978-2007 is 51.7%. This is an estimate for the pooled 1978-2007 sample, with an upward trend over time, implying a higher estimate for 2000-14. Therefore, it appears that the approach in Bargain, Donni and Hentati (2022) (which differs from this paper in several respects, including parametric specification and estimation approach) leads to much higher estimates than the adjusted clothing approach discussed above. Moreover, it differs substantially from my baseline estimates using private leisure as the assignable good. Qualitatively, my baseline result that, on average, women have a lower resource share than men is reversed. It would be surprising if women had higher resource shares than men on average in a society which, while comparatively gender progressive, still has a patriarchal tradition and norms. More systematic and thorough investigation of how different approaches compare is a priority for future work, to enable additional progress in this field.

5 Individual-level consumption for UK couples

Having estimated the sharing rule, it can be applied to other comparable datasets to investigate additional questions. A natural goal is to estimate individual-level consumption inequality. To do so, I apply my estimated sharing rule to a UK expenditure survey (LCF). Both my time-use and expenditure datasets are representative of the UK population, and they were collected at a similar time. Even so, as shown in table 1, the LCF sample of heterosexual working couples is characterised by somewhat higher wages and education than the UKTUS sample. This is not a problem, since when the sharing rule estimate is applied to the expenditure data the process explicitly takes into account differences from the sample averages in the time-use data. As expected, the distribution of my baseline sharing estimates

only a 47% resource share, substantially below the average male resource share of 55%.

are similar in UKTUS and LCF, with very slightly higher female shares in LCF. Applying the sharing estimates to recorded household expenditures, I find substantial gender gaps between men's and women's material consumption. I also estimate gender gaps in 'full', consumption, and discuss policy implications.

5.1 Methodology

I measure consumption inequality based on estimates of individual-level consumption, defined as the monetary market value of an individual's consumption.[45] Similarly to Lise and Seitz (2011), individual-level consumption is calculated as the sum of household material public expenditure and the individual's share of the household's material private expenditure. This metric allows me to compare the objective (preference-independent) value of material consumption of different individuals, and is a clearly policy-relevant measure.

Relative to existing approaches, I make four contributions. Firstly, I propose an approach to including large durables, such as housing and cars, by imputing rents for them to reconcile their dynamic nature with the static household model. Several papers in the literature exclude these goods from measures of individual consumption, but these are the largest expenditures in my UK data, making it desirable to take them into account. Secondly, I suggest drawing on usage data to estimate the degree to which partly public goods, such as cars, are private vs public. Thirdly, I propose standardising prices of goods across regions for goods with substantial geographic variation in prices, such as housing. Finally, this paper is the first to also provide estimates of 'full' consumption, inclusive of time-use as well as material consumption.

[45] This terminology is somewhat imprecise since I am not referring to the consumption bundles of different individuals. However, it captures the essence of this measure, which compares the value of different consumption bundles at their market prices. I avoid the term individual-level expenditure because this could be confused with a different approach, taking into account individual Lindahl prices for public goods. For instance, Chiappori, Meghir and Okuyama (2024) define the Money Metric Welfare Index (MMWI) as the expenditure that would be necessary for an individual to achieve the same level of utility they currently enjoy (i) without access to household economies of scale for public consumption, (ii) with the possibility of varying their chosen bundle of goods relative to what they are factually consuming, but (iii) keeping their preferences unchanged. We can conceptualise this approximately as the expenditure that would be necessary for an individual to achieve, as a single, the same level of utility they currently enjoy in a multi-person household (the caveat is that preferences are not adjusted to counterfactual single-hood preferences, and SAT is not assumed).

Similarly to this paper, Chiappori, Meghir and Okuyama (2024) work within a collective household model with Cobb-Douglas preferences. Unlike this paper, they do not estimate intra-household sharing but directly observe individual-level material consumption in the data. Depending on the available data, and the assumptions made to estimate the sharing rule, it may be possible to use the MMWI instead of, or in addition to, individual-level consumption measures. In the context of this paper, I focus on individual-level consumption because, being preference-agnostic, it is a more natural concept to measure individual-level poverty.

Individual-level consumption, defined as the monetary market value of an individual's material consumption, is the correct metric to measure consumption inequality only when prices of goods are reasonably similar for people in the sample. For material goods, individuals in the same country generally face similar market prices for the same goods (although the composition of consumption varies between individuals).[46] The exception is housing, which in a country like the UK is priced very differently across regions. That price heterogeneity is relevant for the purpose of estimating wealth inequality, but should be abstracted from in estimating consumption inequality. For instance, someone living in London may receive a higher salary than someone living in Manchester, spend a larger proportion of that salary on a home of the same underlying quality, and spend the same amount on other consumption. Then the person living in London would have higher wealth (if they own the home) but the same level of consumption as the person living in Manchester. Similarly, when thinking about 'full' consumption, we may wish to standardise the price of time by using the average wage, rather than individual wages.

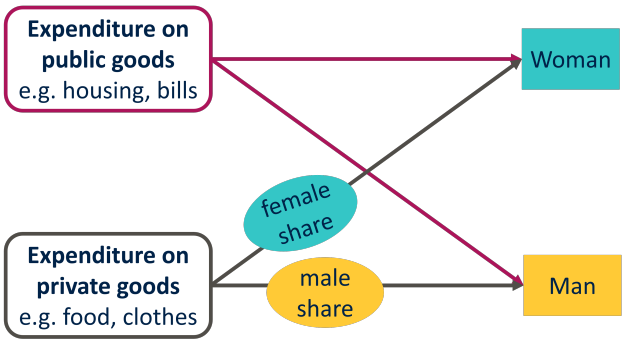

Figure 3: Individual-level consumption

Below, I summarise my proposed methodology to estimate individual-level material consumption, and how I implemented it using UK data. Further detail is provided in online appendix C. In section 5.2.1, I extend this to 'full' consumption, inclusive of time-use.

1. A cross-section household expenditure dataset is required.
    - I use the LCF (Living Costs and Food Survey)[47]; a high-quality, large-scale survey that is used to estimate official government statistics.
2. If the expenditure data is contained in a separate dataset from the estimating dataset (e.g. a time-use dataset) apply the estimated sharing rule to the expenditure dataset.

[46] For instance, see DellaVigna and Gentzkow (2019) on uniform pricing across the U.S.

[47] Office for National Statistics. (2019a). Living Costs and Food Survey [data series] 3rd Release. http://doi.org/10.5255/UKDA-Series-2000028

It is important that the dataset used to estimate the sharing rule and the expenditure dataset be comparable, e.g. nationally representative data for the same country in the same year, although they needn't have exactly the same distribution of relevant variables, as any such differences are explicitly accounted for in the methodology. The data should include any household characteristics and member's characteristics which influence the sharing rule (e.g. members' wages, ages, etc.)

- The LCF, like UKTUS, is nationally representative for the UK. I use data for the year 2014,[48] and focus on the sub-sample of working heterosexual couples without cohabiting children to ensure comparability with the UKTUS data used for resource share estimation. The LCF contains information on individual and household characteristics, as well as detailed income data, which allows me to estimate household-specific resource shares using my estimated sharing rule. The distribution of resource shares in the LCF is similar to that in the UKTUS, and the mean female resource share remains 45%.

3. In the expenditure dataset, divide household expenditure into public and private expenditures. Moreover, some outflows should be excluded if they are incompatible with the static nature of the model (e.g. saving, insurance and investments). To enable accurate classification of expenditures, the dataset should record household expenditure by sufficiently granular categories. Ideally, the data would allow inclusion both of durable and non-durable expenditure. Additionally, it is helpful if the data contains information on vehicles owned and the characteristics of the home the family lives in to enable imputing rent and lease prices consistently across the sample. This is important because expenditures on purchasing or renting a home, and purchasing vehicles, are extremely large in relative terms (especially in some contexts, including the UK).

    - The LCF contains household expenditure divided into very granular categories (COICOP codes). In addition, the LCF records a two-week expenditure data, recall questions on infrequent expenditures, data on the number of vehicles owned, and the characteristics of the home the family lives in.

4. In the expenditure dataset, estimate individual-level material consumption for each household member by summing (i) public good expenditure and (ii) private good expenditure weighted by the individual's resource share: $\hat{CC}_{i,h} = \hat{\eta_{i,h}} \left(\sum_{j \epsilon \Omega^c} p^j c_h^j \right) +$

[48] The sharing rule was estimated using data from 2000 and 2014. I do not additionally analyse LCF data from 2000 as there were substantial changes in the dataset which undermine the feasibility of accurately pooling the 2000 and 2014 LCF datasets.

$\left(\sum_{j\epsilon\Omega^X} r^j X_h^j\right)$ This yields a distribution of individual-level material consumption, which can then be used to estimate different measures of consumption inequality.

5.2 Findings and policy implications

For heterosexual working couples without cohabiting children, the average gender gap in consumption is 8.53%.[49] Note that this gap is substantial, and would have gone completely undetected by the standard 'per capita' approach to estimating consumption inequality because it is fully driven by intra-household inequality. The reason this gap is smaller than the average gender gap in resource shares is that the sharing rule only applies to private expenditures, while consumption is also inclusive of public expenditure. I also estimate two common measures of inequality: the Gini coefficient is 0.21, and 6.43% of the group live in relative poverty (consuming less than 60% of the median individual-level consumption). My baseline estimates of inequality are low relative to standard estimates of inequality, partly because I focus on the sub-sample of working couples without cohabiting children. This excludes sources of inequality such as non-participation. Moreover, Bargain, Donni and Hentati (2022) suggests that women's resource shares in the UK are lower in couples with children than without. Both of these factors suggest that my estimates are underestimates of inequality for the broader UK population.

A further driver of my inequality estimates being lower than some of the available estimates is that I attempt to fully account for public expenditure, and assign all public expenditure to each household member. Focusing only on private consumption, or treating voices of public expenditure as private, substantially overestimates inequality (especially absolute measures of poverty, as these mechanically increase if some expenditures are excluded). Excluding public expenditure, the Gini coefficient increases to 0.30 and the poverty rate to 15.95%. When treating public expenditure as private (similarly to the approach in LPW), the Gini coefficient increases to 0.24 and the poverty rate to 9.01%. This suggests it is important for us to properly take public expenditure into account to avoid over-estimating inequality.

Since women have lower resource shares than men, especially in lower income households, the poverty rate for working heterosexual couples is higher for women (7.03%) than for men (5.83%); a gender gap of 20.59%.[50] As previously noted, this is a lower-bound estimate for the population-wide gender gap in poverty, and an even greater underestimate of the level of intra-household inequality we are likely to find in other countries.

[49] This gap is defined as the difference between mean male consumption and mean female consumption, divided by mean male consumption.

[50] Defined as the female poverty rate minus the male poverty rate, divided by the male poverty rate

5.2.1 Individual-level full consumption (including time-use)

We may be interested in the monetary value of individual-level consumption including time-use. Following Becker (1965) and Lise and Seitz (2011) I term this 'full' consumption.

When we include private leisure expenditure, full individual-level consumption is given by:[51]

$$\hat{TCC}_{i,h} = w_{i,h} l_{i,h} + \hat{\eta_{i,h}} \left(\sum_{j \epsilon \Omega^c} p^j c_h^j \right) + \sum_{j \epsilon \Omega^X} r^j X_h^j$$

When time-use and expenditure datasets are separate, it may still be possible to estimate full individual-level consumption, as discussed in online appendix F.5 for the case of Cobb-Douglas preferences. I apply this methodology to the LCF data and find that the average gender gap in individual-level consumption is higher (11.3%) when taking into account time-use. While with material goods the prices of goods are generally the same for all households (even though the composition of consumption varies), the price of time varies at the individual level, making time-inclusive measures of individual-level consumption harder to interpret at face value. I re-estimate individual-level consumption using the average wage, instead of own-wage, as the price of time. This lowers the average gender gap inclusive of time because men's wages are higher than women's on average. However, the age gap estimated in this manner (10.1%) is still higher than the measure excluding time-use.[52]

These findings confirm the importance of including time-use to fully account for gender gaps in consumption. Further work is needed to carefully incorporate time-use into inequality measures, also considering public time-use and potentially accounting for complex issues such as any dis-utility from domestic work.

5.2.2 Policy implications in the UK setting

These findings suggest that policymakers should take into account intra-household inequality in targeting and evaluating policies. More accurate estimation of consumption inequality is relevant to a range of policy decisions. For instance, one of the United Nations' Sustainable Development Goals for 2030 is the eradication of extreme poverty. A recent enquiry in the UK concluded that foreign aid must be targeted to more closely align with this goal, and that as part of this is it is important for the impact of aid on poverty be measured and kept track of. Accurate estimates of poverty rates in this context can affect where foreign aid is

[51] Ideally, we would also include expenditure on joint leisure and public time $\hat{TCC}_{i,h} = w_{i,h} l_{i,h} + \hat{\eta_{i,h}} \left(\sum_{j \epsilon \Omega^c} p^j c_h^j \right) + \sum_{j \epsilon \Omega^X} r^j X_h^j + \sum_i w_{i,h} \left(d_{i,h} + j t_{i,h} \right)$. This exercise is left to future work as it requires careful analysis to disentangle time spent on joint leisure and time spent on domestic activities, which is beyond the scope of this paper.

[52] Note that this does not contradict the identifying assumption that men and women spend the same share of their individual budget on leisure. We detect more inequality when taking leisure into account because leisure is a private, rather than public, good, so that it is shared unequally between members.

targeted, and even whether the foreign aid budget is cut if it is seen as being ineffective. Moreover, as part of its commitment to the goal of eradicating poverty, the UK aims to halve its poverty rate by 2030. Accurate measurements of poverty would help both track progress with this goal, and evaluate which policies may be most impactful, for instance by targeting women specifically.

To reduce female poverty rates, policymakers may consider targeting income transfers to women specifically, especially in deprived households. Another way of targeting help to females is to expand policies subsidising products which are likely to disproportionately help women. This includes providing free or subsidised childcare for young children, especially for lower-earning households; something the UK has been moving towards, with the introduction of 30 hours of free childcare for children aged 9 months and above, from September 2025. Since women often take primary custody of children after divorce, free childcare improves the outside option for women relative to men, reducing asymmetries in bargaining power underlying unequal sharing. Free childcare can also play a role in narrowing gender wage gaps and can also equalise sharing through this channel. Another initiative that can help reduce female poverty is lowering the cost of female sanitary products. In the UK, the VAT rate on these products was dropped to zero in 2021, and as part of the 'Period Product Scheme', schools provide them for free since 2020. Expanding this policy to free provision for all women would be in line with the current UK policy of providing contraceptives, including the pill, free of charge to all. Moreover, policymakers may consider that an added advantage of pursuing policies to narrow gender pay gaps is their equalising effect on intra-household sharing (this paper, in line with the literature, finds that sharing is more equal in households with higher female wages and/or lower male wages).

5.2.3 Gender health gaps in the UK

An additional reason why this paper's findings are relevant to UK policymakers is the impact of poverty on health problems, and the fact that women suffer from worse health outcomes than men in the UK as measured by metrics such as the morbid obesity rate. Decreasing intra-household gender-based inequalities in sharing may also help reduce health inequalities at a national level. For instance, women living in poverty are more likely to suffer from anxiety disorders (Remes et al. (2017)) and to be obese (Griffith (2022)). As noted by Griffith (2022), there are several reasons why living in deprivation can lead to obesity, and in order to tackle high obesity rates it is important for policies to target the right people. Women in poverty are likely to have little time, little money, and live in 'food deserts' with lower availability of healthy foods. Hence they are likely to consume unhealthy, cheap,

store-bought foods, and not be able to exercise regularly.

Obesity is a common target for policymakers because of its wide-ranging impacts on physical and mental health, as well as mortality. For women specifically, obesity is also linked to gestational problems and adverse outcomes for children's health. Interestingly, in the UK, women are more likely than men to be morbidly obese. This is consistent with this paper's finding that women are more likely than men to be living in poverty. Moreover, in the UK, regional deprivation and household income are stronger predictors of female obesity than male obesity (The King's Fund (2021)). This is consistent with this paper's findings that (i) men have higher resource shares than women, and (ii) this is more marked in poorer households in poorer regions, so that household income and regional deprivation are better predictors of individual-level poverty for women than for men. Beyond obesity rates, while in the UK women have a longer life expectancy than men, the Women's Health Strategy for England policy paper notes that women "spend a significantly greater proportion of their lives in ill health and disability when compared with men" (Department of Health & Social Care (2022)). Targeting cash transfers to women in poor households, and labelling them as meant for expenditure on healthy foods and exercise for women may be particularly helpful in tackling obesity as well as poverty.[53]

6 Conclusion

My findings add to the growing literature on the importance of estimating intra-household inequality and the importance of considering the dimensions, such as gender, as well as the levels, of inequality. The approach set out in this paper is grounded in the well-established collective model of the household, has a clear source of identification, and is straight-forward to implement with widely available data. Moreover, while the approach is simple, tests of model fit in my application to UK data suggest that it is empirically sound. This approach is also well-suited to easily estimating a variety of measures of individual-level well-being, including individual-level material and 'full' consumption, allowing for important policy applications. These characteristics may contribute to the goal of facilitating adoption of individual-level inequality measures outside of academia. The empirical finding of substantial gender inequality within households, even in the context of the UK, one of the most gender equal countries in the world, strongly suggests that the widely used equal sharing assumption is not fit for purpose, and that is is empirically important for policy to be guided by measures of consumption inequality that take into account unequal sharing within households.

[53] See Beatty et al. (2014) for an example of the effectiveness of labelling cash transfers in the UK.

To further aid this goal, additional work is required to investigate the empirical performance of the different approaches that have been proposed in the literature. Bargain, Lacroix and Tiberti (2021) test a specific approach to estimating intra-household sharing against reported individual-level consumption. Future work might systematically compare the performance of different approaches using simulated and empirical data, as well as comparing their more general advantages in terms of applicability. This exercise would be very valuable in a literature which has been growing in several different directions, and may benefit from consolidation into something like a mainstream approach. In order to focus data collection efforts, it would be particularly useful to understand the partial discrepancy between estimates obtained from different assignable goods.

Another important direction for future work is to extend the approach in this paper to contexts with non-participation. Identifying the sharing rule from time-use data for non-participants requires estimating the unobserved price of their time (which is higher than their, unobserved, wage). This would allow one to appropriately take into account households with children and people who do not supply market work. For those households, the tests of identifying assumptions provided in this paper (which appear to be highly informative for working couples) may not be sufficiently convincing. As illustrated with clothing-based estimates of sharing in the UK application, the choice between different identifying assumptions can have a large impact on the magnitude of estimates and even on qualitative conclusions. Hence, it may also be important to work toward reduced reliance on identifying assumptions, or new approaches to testing them credibly for a wider range of household compositions.

Finally, this paper is the first to provide estimates of 'full' individual-level consumption inequality. These are supportive of the widely held view that gender inequality may be underestimated when only considering material consumption. Further work is required to consider how to best measure 'full' inequality, carefully overcoming the additional difficulty relative to material consumption that the price of time varies at an individual level.

References

Almås, I., Ringdal, C., & Sjursen, I. H. (2020). Understanding inequality within households. In K. F. Zimmermann (Ed.), *Handbook of labor, human resources and population economics* (pp. 1–27). Springer International Publishing. https://doi.org/10.1007/978-3-319-57365-6_222-1

Apps, P., & Savage, E. (1989). Labour supply, welfare rankings and the measurement of inequality. *Journal of Public Economics*, *39*(3), 335–364. https://doi.org/10.1016/0047-2727(89)90033-9

Bargain, O., & Donni, O. (2012). Expenditure on children: A rothbarth-type method consistent with scale economies and parents' bargaining. *European Economic Review*, *56*(4), 792–813. https://doi.org/10.1016/j.euroecorev.2012.02.003

Bargain, O., Donni, O., & Hentati, I. (2022). Resource sharing in households with children: A generalized model and empirical evidence from the UK. *Journal of the European Economic Association*, *20*(6), 2468–2496. https://doi.org/10.1093/jeea/jvac033

Bargain, O., Lacroix, G., & Tiberti, L. (2021). Intrahousehold Resource Allocation and Individual Poverty: Assessing Collective Model Predictions using Direct Evidence on Sharing. *The Economic Journal*, *132*(643), 865–905. https://doi.org/10.1093/ej/ueab085

Bastian, J., & Lochner, L. (2020, August). *The EITC and maternal time use: More time working and less time with kids?* (Working Paper No. 27717). National Bureau of Economic Research. https://doi.org/10.3386/w27717

Beatty, T. K. M., Blow, L., Crossley, T. F., & O'Dea, C. (2014). Cash by any other name? evidence on labeling from the UK winter fuel payment. *Journal of Public Economics*, *118*, 86–96. https://doi.org/10.1016/j.jpubeco.2014.06.007

Becker, G. S. (1965). A theory of the allocation of time. *The Economic Journal*, *75*(299), 493–517. https://doi.org/10.2307/2228949

Blundell, R., Chiappori, P.-A., Magnac, T., & Meghir, C. (2007). Collective labour supply: Heterogeneity and non-participation. *The Review of Economic Studies*, *74*(2), 417–445. http://www.jstor.org/stable/4626146

Blundell, R., Chiappori, P.-A., & Meghir, C. (2005). Collective labor supply with children. *Journal of Political Economy*, *113*(6), 1277–1306. https://doi.org/10.1086/491589

Brown, C., Calvi, R., & Penglase, J. (2021). Sharing the pie: An analysis of undernutrition and individual consumption in bangladesh. *Journal of Public Economics*, *200*, 104460. https://doi.org/10.1016/j.jpubeco.2021.104460

Browning, M., Bourguignon, F., Chiappori, P.-A., & Lechene, V. (1994). Income and outcomes: A structural model of intrahousehold allocation. *Journal of Political Economy*, *102*(6), 1067–1096. https://www.jstor.org/stable/2138780

Browning, M., Chiappori, P.-A., & Lewbel, A. (2013). Estimating consumption economies of scale, adult equivalence scales, and household bargaining power. *The Review of Economic Studies*, *80*(4), 1267–1303. https://doi.org/10.1093/restud/rdt019

Browning, M., Chiappori, P.-A., & Weiss, Y. (2014). *Economics of the family*. Cambridge University Press.

Browning, M., Donni, O., & Gørtz, M. (2020). Do you have time to take a walk together? Private and joint time within the household. *The Economic Journal*, *131*(635), 1051–1080. https://doi.org/10.1093/ej/ueaa118

Browning, M., & Gørtz, M. (2012). Spending time and money within the household. *The Scandinavian Journal of Economics*, *114*(3), 681–704. http://www.jstor.org/stable/41679527

Calvi, R. (2020). Why are older women missing in india? The age profile of bargaining power and poverty. *Journal of Political Economy*, *128*(7), 2453–2501. https://doi.org/10.1086/706983

Calvi, R., Penglase, J., & Tommasi, D. (2022). Measuring women's empowerment in collective households. *AEA Papers and Proceedings*, *112*, 556–560. https://doi.org/10.1257/pandp.20221054

Cherchye, L., De Rock, B., Lewbel, A., & Vermeulen, F. (2015). Sharing rule identification for general collective consumption models. *Econometrica*, *83*(5), 2001–2041. http://www.jstor.org/stable/43616995

Cherchye, L., De Rock, B., & Vermeulen, F. (2012). Married with children: A collective labor supply model with detailed time use and intrahousehold expenditure information. *American Economic Review*, *102*(7), 3377–3405. https://doi.org/10.1257/aer.102.7.3377

Chiappori, P.-A. (1992). Collective labor supply and welfare. *Journal of Political Economy*, *100*(3), 437–467. https://doi.org/10.1086/261825

Chiappori, P.-A., & Ekeland, I. (2009). The microeconomics of efficient group behavior: Identification1. *Econometrica*, *77*(3), 763–799. https://doi.org/10.3982/ECTA5929

Chiappori, P.-A., Meghir, C., & Okuyama, Y. (2024). Intrahousehold Welfare: Theory and Application to Japanese Data. *NBER Working Paper Series*. https://doi.org/10.3386/w32645

Deaton, A., & Muellbauer, J. (1980). An almost ideal demand system. *The American Economic Review*, *70*(3), 312–326. http://www.jstor.org/stable/1805222

DellaVigna, S., & Gentzkow, M. (2019). Uniform pricing in U.S. retail chains. *The Quarterly Journal of Economics*, *134*(4), 2011–2084. https://doi.org/10.1093/qje/qjz019

Department for Transport. (2021). National Travel Survey: 2020. https://www.gov.uk/government/statistics/national-travel-survey-2020/national-travel-survey-2020#trends-in-car-trips

Department for Transport. (2022). Car or van occupancy and lone driver rate by trip purpose, England: 2002 onwards. https://www.gov.uk/government/statistical-data-sets/nts09-vehicle-mileage-and-occupancy

Department of Health & Social Care. (2022). Women's health strategy for england [Policy paper].

Dunbar, G. R., Lewbel, A., & Pendakur, K. (2013). Children's resources in collective households: Identification, estimation, and an application to child poverty in Malawi. *American Economic Review*, *103*(1), 438–471. https://doi.org/10.1257/aer.103.1.438

Fong, Y., & Zhang, J. (2001). The identification of unobservable independent and spousal leisure. *Journal of Political Economy*, *109*(1), 191–202. https://doi.org/10.1086/318599

Geary, R. C. (1950). A note on "a constant-utility index of the cost of living". *The Review of Economic Studies*, *18*(1), 65–66. http://www.jstor.org/stable/2296107

Gitter, S. R., & Barham, B. L. (2008). Women's power, conditional cash transfers, and schooling in nicaragua. *The World Bank Economic Review*, *22*(2), 271–290. https://doi.org/10.1093/wber/lhn006

Griffith, R. (2022). Obesity, poverty and public policy. *The Economic Journal*, *132*(644), 1235–1258. https://doi.org/10.1093/ej/ueac013

Griffith, R., Jin, W., & Lechene, V. (2022). The decline of home-cooked food. *Fiscal Studies*, *43*(2), 105–120. https://doi.org/https://doi.org/10.1111/1475-5890.12298

HM Revenue & Customs. (2023). Percentile points from 1 to 99 for total income before and after tax. https://www.gov.uk/government/statistics/percentile-points-from-1-to-99-for-total-income-before-and-after-tax

Hubner, S. (2020). It's complicated: A Non-parametric Test of Preference Stability between Singles and Couples. *Bristol Economics Discussion Papers 20/735*. https://research-information.bris.ac.uk/en/publications/its-complicated-a-nonparametric-test-of-preference-stability-betw

Kalenkoski, C. M., Ribar, D. C., & Stratton, L. S. (2005). Parental child care in single-parent, cohabiting, and married-couple families: Time-diary evidence from the united kingdom. *American Economic Review*, *95*(2), 194–198. https://doi.org/10.1257/000282805774670176

Lechene, V., Pendakur, K., & Wolf, A. (2022). Ordinary Least Squares estimation of the intrahousehold distribution of expenditure. *Journal of Political Economy*, *130*(3), 681–731. https://doi.org/10.1086/717892

Lewbel, A., & Pendakur, K. (2008). Estimation of collective household models with engel curves. *Journal of Econometrics*, *147*(2), 350–358. https://doi.org/10.1016/j.jeconom.2008.09.012

Lise, J., & Seitz, S. (2011). Consumption inequality and intra-household allocations. *The Review of Economic Studies*, *78*(1), 328–355. https://www.jstor.org/stable/23015857

McElroy, M. B., & Horney, M. J. (1981). Nash-bargained household decisions: Toward a generalization of the theory of demand. *International Economic Review*, *22*(2), 333–349. https://doi.org/10.2307/2526280

Muellbauer, J. (1976). Community preferences and the representative consumer. *Econometrica*, *44*(5), 979–999. https://doi.org/10.2307/1911540

Office for National Statistics. (2019a). Living Costs and Food Survey [data series] 3rd Release. http://doi.org/10.5255/UKDA-Series-2000028

Office for National Statistics. (2019b). United Kingdom Time Use Survey [data series] 2nd Release. https://doi.org/http://doi.org/10.5255/UKDA-Series-2000054

Reiss, P. C., & Wolak, F. A. (2007). Chapter 64 Structural Econometric Modeling: Rationales and Examples from Industrial Organization. In J. J. Heckman & E. E. Leamer (Eds.), *Handbook of Econometrics* (pp. 4277–4415, Vol. 6). Elsevier. https://doi.org/10.1016/S1573-4412(07)06064-3

Remes, O., Wainwright, N., Surtees, P., Lafortune, L., Khaw, K.-T., & Brayne, C. (2017). Sex differences in the association between area deprivation and generalised anxiety disorder: British population study. *BMJ Open*, *7*, e013590. https://doi.org/10.1136/bmjopen-2016-013590

Stone, R. (1954). Linear expenditure systems and demand analysis: An application to the pattern of british demand. *The Economic Journal*, *64*(255), 511–527. http://www.jstor.org/stable/2227743

The King's Fund. (2021). Tackling obesity.

United Nations Development Programme. (2022). Human development report 2021-22. http://report.hdr.undp.org

Online Appendix

A Identification in a more general setting

It is possible to extend the approach in this paper to a more general setting. This appendix provides a discussion of how we might attempt identification and linear estimation under different functional form assumptions.

A.1 Assignable good

The assignable good $c^a_{i,h}$ may be clothing, as in most of the existing, private leisure, as in this paper, or some other assignable good. See online appendix D for a more detailed discussion of the considerations involved in choosing the assignable good.[54]

A.2 Linear approximation of resource shares

Resource shares will generally depend on household budget, prices (including wages of all household members) and Pareto weights of household members: $\eta_{i,h}(y_h, p, r, w_h, \mu_h)$. The form of an individual's resource share will depend on their type and household category so we can write $\eta_{t,g}(y_h, p, r, w_h, \mu_h)$.[55] Recall that Pareto weights are an unknown function of a vector of variables z_h that determine relative bargaining power in the household. Since z_h in general includes y_h, p, r, w_h we can write $\eta_{t,g}(z_h)$. The collective model only restricts bargaining to be efficient, so to estimate resource shares we must approximate the Pareto weights, or directly approximate the resource shares. For different household categories g, resource shares (and Pareto weights) will be different functions of household characteristics z_h. For this reason, the sharing rule $\eta_g(z_h)$, which assigns a resource share $\eta_{i,h}$ to each member of households h of category g, is estimated separately for households of different

[54] In practice, estimation is likely to proceed from individual-level demand for the same assignable good for everyone in the household. For this reason, the notation in the identification proof implies that the same assignable good c^a is observed for all members. However, it is worth noting that identification can also proceed from different assignable goods for different types of people, as long as the same assignable good is used for each person of the same type in the same household category. In this case, the identification result is unchanged, apart from indexing the assignable good for different members differently depending on their type.

[55] Where there are multiple individuals of the same type in a household, the notation must be amended to acknowledge the fact that an individual's share depends both on their type and on their own individual characteristics / Pareto weight. Everything goes through in the same way, so for simplicity of notation I avoid this extension of notation.

categories.[56]

To linearly estimate resource shares, we linearly approximate them as:

$\eta_{i,h} = \eta^0_{t,g} + \sum_z \eta^z_{t,g} (z_h - \bar{z})$

- $\eta^0_{t,g}$ is the average resource share of a type (living in a specific household category). This is the resource share evaluated at the average characteristics in the sample. By definition, $\sum_{t \epsilon h} \eta^0_{t,g} = 1$. In the context of heterosexual couples, the average resource share of men and women sum to one.

- $\eta^z_{t,g}$ captures the impact on sharing of a household's characteristic z_h deviating from the sample average $(z_h - \bar{z})$. For instance, a higher-than-average wage for the woman might increase the woman's resource share, so that she would have a higher-than-average-for-women resource share. Since resource shares must sum to one within the household, this implies her partner must have a correspondingly lower-than-average-form-men resource share: $\sum_{t \epsilon h} \eta^z_{t,g} = 0$. We can interpret $\eta^z_{t,g}$ as the marginal impact of characteristic z_h on the resource share.

A linear approximation does not guarantee estimates of resource shares which fall within the unit interval. However, by construction, only estimates in this range are theory-consistent. This provides a useful test of model fit. Reassuringly, in my application to UK data, my baseline resource share estimates from time-use data are all within the unit interval.

A.3 Parametric forms and estimating equations

The identification result holds for several families of preferences. Once we have chosen a parametric form for implementation we can derive the functional form of second-stage demands. We may directly use assignable good demands as estimating equations, or else choose some function f of demands: $f\left(c^a_{t,g}\left(p, w_{i,h}, Q_h, (y_h - R_h Q_h)\,\eta_{i,h}\right)\right)$. This could be for instance: demand for the assignable good, expenditure on the assignable good, the budget share spent on the assignable good, or expenditure on the assignable good relative to the expenditure on some sub-set of goods.[57] We may choose the most appropriate function depending on the functional form choices made for implementation, or depending on the data available.

[56] The approximation is more effective if person types and household categories are defined sufficiently granularly that the bargaining process, as a function of characteristics, would not differ too substantially within each household category sub-sample.

[57] Different assignable goods or functions of demand could be used for different types of people, but the same must be used for people of the same type in the same household category. The generalised notation is $f_{t,g}\left(c^{a,t,g}_{t,g}\left(p, w_{i,h}, Q_h, (y_h - R_h Q_h)\,\eta_{i,h}\right)\right)$ and the same result applies with minimal changes.

A.4 Identification

Previous literature has shown that resource shares are generally identified up to a constant in a context with the underlying model discussed in section 2 and the sort of data set out in section 3.6. This paper is concerned with a somewhat different goal: point-identifying the linear approximation of the sharing rule from coefficients estimated by linear regression.

The linear approximation of the sharing rule is semi-parametrically point-identified under one of three possible identifying assumptions restricting preference heterogeneity. In short, identification proceeds in two steps. First, a composite coefficient is identified by linear regression. This step relies on standard assumptions for linear regression. Then, the sharing rule parameters are identified from the composite coefficient by imposing an identifying assumption restricting preference heterogeneity. In section A.4.1 I set out the result and the conditions required for identification to hold. In section A.4.2 I discuss the three candidate identifying options. In section A.4.3 I prove the identification result with an argument by construction. In section A.4.4 I discuss the intuition behind the identification result. In section A.5, I exemplify the broader identification result with the Almost Ideal Demand System.

A.4.1 Identification result

Theorem A.1. *The linear approximation of the sharing rule* $\eta^0_{t,g} + \sum_z \eta^z_{t,g}\left(z_h - \bar{z}\right)$ *is fully identified for households of category* g *if (i) for all* N^t_g *household members we observe a function* $f\left(c^a_{i,h}\right) = f\left(c^a_{t,g}\left(p, w_{i,h}, Q_h, \left(y_h - R_h Q_h\right)\eta_{i,h}\right)\right)$ *of demand for assignable good* $c^a_{i,h}$*, and (ii) the following Assumptions hold.*

Assumption A.1.1. ***Linearity in resource shares.***

$f\left(c^a_{i,h}\right)$ *can be decomposed into a component* $f^0_{t,g}\eta_{i,h}$ *which is linear in the resource share:*

$$f\left(c^a_{i,h}\right) \;=\; \sum_{\varphi} f^{\varphi}_{t,g}\left(p, w_{i,h}, Q_h, \left(y_h - R_h Q_h\right)\eta_{i,h}\right) \;+\; f^0_{t,g}\left(p, w_{i,h}, Q_h, \left(y_h - R_h Q_h\right)\right)\eta_{i,h} \quad (1)$$

This assumption imposes that the assignable good demand functions (or some function of demand) have *a component* which is linear in the individual's second-stage budget. Roy's identity can be used to easily check whether a specific indirect utility function is consistent or not with this requirement. Some functional forms that are consistent with this assumption are: (i) the Linear Expenditure System (Cobb-Douglas or Stone-Geary, see Stone (1954) and Geary (1950)), (ii) Price-Independent Generalised Linear (PIGL) (indirect utility functions that are a function of $\rho^k_{i,h}, k > 0$, see Muellbauer (1976)) and (iii) Price-Independent

Generalised Logarithmic (PIGLOG) (indirect utility functions that are a function of $\ln \rho_{i,h}$ e.g. the Almost Ideal Demand System, see Muellbauer (1976) and Deaton and Muellbauer (1980)).[58] This assumption relaxes similar assumptions made in the related literature, allowing for broader families of preferences (e.g. DLP assumes budget share functions are linear in functions of expenditure, and LPW assumes that Engel curves be of the Almost Ideal Demand System, both of which are examples of functions of demand that have a component which is linear in the resource share).

Assumption A.1.2. ***Linearity in unknowns.***

Part A. *The vectors of unknown resource share parameters $\eta_{t,g}$ and preference parameters $\alpha_{t,g}$* [59] *enter $f\left(c^a_{i,h}\right)$ linearly in the sense that:*

$$f\left(c^a_{i,h}\right) = a_{t,g} + \sum_{\varphi} b^{\varphi}_{t,g} g^{\varphi}\left(p, w_{i,h}, Q_h, \left(y_h - R_h Q_h\right), \left(y_h - R_h Q_h\right)\left(z_h - \bar{z}\right)\right) + \gamma_{t,g}\left(\eta^0_{t,g} + \sum_{z} \eta^z_{t,g}\left(z_h - \bar{z}\right)\right) g^0\left(p, w_{i,h}, Q_h, \left(y_h - R_h Q_h\right)\right) \quad (2)$$

Where: (i) $a_{t,g}(\alpha_{t,g}, \eta_{t,g})$ and $b^{\varphi}_{t,g}(\alpha_{t,g}, \eta_{t,g})$ are unknown constants which may be (potentially composite) preference parameters, or composite parameters composed of preference parameters and unknown resource share parameters, (ii) $\gamma_{t,g}(\alpha_{t,g})$ is a potentially composite preference parameter and (iii) g^{φ} and g^0 are observed in the data.[60]

Part B. *The functions $a_{t,g}(\alpha_{t,g}, \eta_{t,g})$ and $b^{\varphi}_{t,g}(\alpha_{t,g}, \eta_{t,g})$ are linear in the vector of resource share parameters $\eta_{t,g}$.*

The assumption that unknowns enter linearly is required so that the estimating equations can be estimated by linear regression.

[58] Weaker parametric assumptions are also consistent with this assumption. For instance, preferences may be represented as Cobb-Douglas over the assignable good, an aggregate private and an aggregate public good (instead of Cobb-Douglas over each granular good). Alternatively, demand for the assignable good may be modelled as being linear in the budget without restricting the functional form of demands for other goods, and only requiring them to be jointly consistent with rational preferences.

[59] If modelling shareable goods, then read 'preference or shareability parameters' for 'preference parameters'

[60] Note that, as long as we assume the same parametric form for preferences of all individuals (of all types, and in all household categories) these fully observable objects have the same functional form for everyone. Identification also works if we assume different parametric forms for preferences of different types of individuals, or individuals in different household categories, and in that case we write $g^{\varphi}_{t,g}$ and $g^0_{t,g}$. Here I focus on the case where the same parametric form of preferences is assumed for all individuals as this is generally the case for implementation, and it leads to much more natural and realistic interpretations of the identifying assumptions.

If the resource share is modelled as a type-specific constant $\eta^0_{t,g}$ (the vector of characteristics affecting sharing, z_h, is zero) then Part A of this assumption is sufficient. Then, we have:

$$f\left(c^a_{i,h}\right) = a_{t,g}(\alpha_{t,g}, \eta^0_{t,g}) + \sum_{\varphi} b^{\varphi}_{t,g}(\alpha_{t,g}, \eta^0_{t,g}) g^{\varphi}\left(p, w_{i,h}, Q_h, (y_h - R_h Q_h)\right) + \gamma_{t,g}\eta^0_{t,g} g^0\left(p, w_{i,h}, Q_h, (y_h - R_h Q_h)\right)$$

In this case, the identification result holds exactly for broader families of preferences, such as the Almost Ideal Demand System.

If instead the resource share depends on a non-zero vector of characteristics, so that resource shares vary at the household level, we additionally require Part B of this assumption. In this case, the identification result holds exactly for narrower families of preferences, such as the Linear Expenditure System. Where Part B does not hold, it is possible to relax it by linearising whichever component(s) of $f\left(c^a_{i,h}\right)$ is problematic. Similarly to LPW, the identification result holds approximately for the Almost Ideal Demand System with covariates (see online appendix A.5).

Assumption A.1.3. ***Rank condition.***

No component of $g^0\left(p, w_{i,h}, Q_h, (y_h - R_h Q_h)\right)$ *or* $(z_h - \bar{z})\, g^0\left(p, w_{i,h}, Q_h, (y_h - R_h Q_h)\right)$ *is colinear with any other component of* $f\left(c^a_{i,h}\right)$

Substituting $\beta^0_{t,g} = \gamma_{t,g}\eta^0_{t,g}$ and $\beta^z_{t,g} = \gamma_{t,g}\eta^z_{t,g}$ into equation (2), we obtain the estimating equations:

$$f\left(c^a_{i,h}\right) = a_{t,g} + \sum_{\varphi} b^{\varphi}_{t,g} g^{\varphi}\left(p, w_{i,h}, Q_h, (y_h - R_h Q_h), (y_h - R_h Q_h)(z_h - \bar{z})\right) + \beta^0_{t,g} g^0\left(p, w_{i,h}, Q_h, (y_h - R_h Q_h)\right) + \sum_{z} \beta^z_{t,g}(z_h - \bar{z})\, g^0\left(p, w_{i,h}, Q_h, (y_h - R_h Q_h)\right) \quad (3)$$

We can see that the rank condition is required so that coefficients $\beta^0_{t,g}$ and $\beta^z_{t,g}$ can be identified separately from other coefficients in $f\left(c^a_{i,h}\right)$. This is necessary because we will identify $\eta^0_{t,g}$ and $\eta^z_{t,g}$ from $\beta^0_{t,g}$ and $\beta^z_{t,g}$. Depending on the functional form of choice, this assumption can imply restrictions on which variables z_h influence the resource share. A related, but less general, assumption which is often made in the literature is that resource shares are invariant to expenditure (see DLP for a discussion of why this assumption is reasonably non-restrictive both theoretically and empirically).

Assumption A.1.4. ***Invertibility condition.***

$\gamma_{t,g} \neq 0, \forall t, g$

Depending on the chosen functional form specification, parts of the general form of $f\left(c^a_{i,h}\right)$ may be absent, e.g. there may be no constant term $a_{t,g}$. However, the component which is linear in the resource share must be present for identification to work. The interpretation of this assumption depends on the functional form used in implementation. With Cobb-Douglas preferences, we can think of this as a requirement that the assignable good does not command a very small budget share. If using the Almost Ideal Demand System, then this assumption is equivalent to the requirement in LPW that the slope of the Engel curves for the assignable good isn't too close to zero.

Assumption A.1.5. ***Regression.***

Any noise in the observed data is uncorrelated with the right-hand-side variables. The variables have non-zero variance.

In order to run linear regressions we must introduce noise to our model, for example as outlined in section 3.5.1. In order to identify the coefficients of interest by linear regression, this noise must be uncorrelated with right-hand-side variables. Similarly, non-zero-variance is required in order to identify from linear regression.

Assumption A.1.6. ***SAP, SAT or SRAT.***

A suitable restriction on $\gamma_{t,g}$ heterogeneity applies.

The candidate preference heterogeneity restrictions are discussed in section A.4.2, and are in line with the identifying restrictions made in the literature.

A.4.2 Identifying assumption

Identification of the sharing rule from individual-level demand data for a single assignable good requires a strong identifying assumption. It is important to note that the chosen assumption has an important impact on estimates, and should be chosen carefully. Assumptions could be made on the bargaining process e.g. Lise and Seitz (2011) assume that women and men with the same potential earnings divide full income equally. Other papers have focused on restricting preference heterogeneity. For example, Lewbel and Pendakur (2008) and Bargain and Donni (2012) rely on the 'SAT' assumption (similarity across household types), that preferences for the assignable good are stable across household compositions, allowing for identification of the preference parameter from singles data. However, we may

be concerned by findings that, at least in some contexts, preference stability across household composition is rejected empirically (see Hubner (2020)). DLP and LPW use the alternative 'SAP' assumption (similarity across people) that women and men in the same household composition have the same preference parameter for the assignable good. Whether this is a good assumption depends on whether we have reason to suspect substantial gender differences for the assignable good. For example, as discussed in section 4.4, singles data suggest that women have much stronger preferences than men for clothing. In this case, we may choose to rely instead on a novel identifying assumption: SRAT.

Assignable good SRAT (similarity of ratios across types): the ratio of the preference parameter $\gamma_{t,g}$ between different types of people t remains stable across household compositions g: $\frac{\gamma_{t,g}}{\gamma_{st,g}} = \frac{\gamma_{t,single}}{\gamma_{st,single}}, \forall t, st \in h$. This assumption implies that preferences on the assignable good change in a similar way for different types of people in the transition from singlehood to other household categories. This means that the ratio of men's to women's preference parameters is identified from singles data. The SRAT assumption allows preference levels to vary between different people and different household categories, while requiring that the ratio of the parameters is constant across household compositions. For instance, single women's clothing preference parameter may be $\alpha^c_{w,single} = 0.2$ and men's $\alpha^c_{m,single} = 0.1$, while in a working couple it may be that $\alpha^c_{w,couple} = 0.16$ and $\alpha^c_{m,couple} = 0.08$. Note that the rest of the utility function is allowed to vary differently

While SRAT is not an innocuous assumption, it does not seem extraordinarily restrictive for working couples without cohabiting children. There is no particular reason to suspect women's and men's preferences change in a different manner when transitioning from living alone to living in a couple. The assumption is stronger for households with children, since the idea that single men and women's preferences change similarly when they have children is less realistic.

A.4.3 Identification proof

Proof. Firstly, by assumptions 3.1.1 - 3.1.5 and the usual arguments for identification from linear regression, the constants $a_{t,g}$ and $\beta^0_{t,g}$, and the vectors $b^{\varphi}_{t,g}$ and $\beta^z_{t,g}$ can be estimated by OLS (one for each person type and household category combination) and hence are identified.

Secondly, resource share parameters $\eta_{t,g}$ are identified from the β coefficients by assumption 3.1.6. Intuitively, the resource share parameters $\eta_{t,g}$ enter into the β coefficients multiplicatively with the (composite) preference parameter $\gamma_{t,g}$, so we require an identifying assumption to separately identify the resource share parameters. The proof is outlined separately for each of the three candidate identifying assumptions:

- Under SAT, the preference parameter $\gamma_{t,g}$ is identified from singles data, so that $\gamma_{t,g} = \gamma_{t,single} = \gamma_t$ is known. Therefore, we compute: $\eta^0_{t,g} = \frac{\beta^0_{t,g}}{\gamma_t}$ and $\eta^z_{t,g} = \frac{\beta^z_{t,g}}{\gamma_t}$.

- Under SAP, the average resource share of each type is identified by $\eta^0_{t,g} = \frac{\beta^0_{t,g}}{\sum_{st\in h}\beta^0_{st,g}}$. This is because under SAP $\frac{\beta^0_{t,g}}{\sum_{st\in h}\beta^0_{st,g}} = \frac{\gamma_g\eta^0_{t,g}}{\sum_{st\in h}\gamma_g\eta^0_{st,g}} = \frac{\eta^0_{t,g}}{\sum_{st\in h}\eta^0_{st,g}} = \eta^0_{t,g}$ since, by definition, resource shares sum to one within household. The marginal impact of characteristic z is identified by $\eta^z_{t,g} = \frac{\beta^z_{t,g}}{\beta^0_{t,g}}\eta^0_{t,g}$. This is because under SAP $\frac{\beta^z_{t,g}}{\beta^0_{t,g}}\eta^0_{t,g} = \frac{\gamma_g\eta^z_{t,g}}{\gamma_g\eta^0_{t,g}}\eta^0_{t,g} = \eta^z_{t,g}$.

- Under SRAT, the proof is similar to SAP with the difference that we identify $\frac{\gamma_{t,g}}{\gamma_{st,g}} = \frac{\gamma_{t,single}}{\gamma_{st,single}} = \Lambda_{t,st}$ from singles data and then identify the average resource share of each type as $\eta^0_{t,g} = \frac{\beta^0_{t,g}}{\sum_{st\in h}\Lambda_{st,g}\beta^0_{st,g}}$. The marginal impact of characteristic z is identified by $\frac{\beta^z_{t,g}}{\beta^0_{t,g}}\eta^0_{t,g} = \frac{\gamma_{t,g}\eta^z_{t,g}}{\gamma_{t,g}\eta^0_{t,g}}\eta^0_{t,g} = \eta^z_{t,g}$.

□

A.4.4 Discussion of the identification result

This identification result builds on the identification result in DLP and the estimation framework of LPW, extending them to a more general underlying structural model.

The extension to public goods[61] leads to a challenge for identification because recovering the sharing rule requires either data on public expenditure (both material and time-use) or an additional preference restriction. I discuss the latter in section 3, where I discuss an implementation with time-use data only.

I also note that the extension to contexts with price-variation (including individual-level wages) can complicate estimation. Firstly, it can greatly increase the number of regressors in the estimating equations, depending on how many prices are heterogenous within the sample and on the functional form of preferences. Moreover, additional data (on non-constant prices and wages) is required for estimation. Furthermore, where there is poor data on non-labour income, the wages of household members become colinear with the household budget, which can complicate or impede estimation under some functional form assumptions. However, allowing for price variation is necessary both to incorporating time-use in the underlying

[61] While all goods are non-public in the LPW setting, economies of scale from household size are modelled by letting the actual consumption vector of each member be equal to the vector of market purchases for that member multiplied by a matrix A. While I do not explicitly incorporate the sharing framework, the identification result in this section readily applies to a model with both public and shareable goods. The most notable effect of this inclusion, if there are cross-good economies of scale, is that estimating equations are likely to take on more complex functional forms and be less tractable.

model and to enabling using private leisure as the assignable good. Moreover, estimation with price variation is likely to yield more accurate estimates because it provides additional sources of identifying variation. Finally, as this identification result does not hinge on price variation, the result in this paper includes as a special case a model where all prices are constant.

In practice, I suggest that this identification result be implemented with Cobb-Douglas preferences where these don't strong-arm the data. They have substantial strengths in this context, as discussed in section 3. I also illustrate the identification result with the Almost Ideal Demand System in online appendix A.5. The intuition behind this general result is discussed for a specific implementation in section 3.

A.5 Almost Ideal Demand System

Consider preferences which are separable in public and private goods, and where the private good sub-utility corresponds to the Almost Ideal Demand System (Deaton and Muellbauer (1980)).[62] Then Engel curves for the assignable good take the form:

$$\frac{p^a_{i,h}c^a_{i,h}}{\rho_{i,h}} = \alpha_{t,g}+\sum_j \gamma^j_{t,g}\ln p^j_{i,h}+\beta_t \ln \rho_{i,h}-\beta_{t,g}\left(\alpha^0_{t,g}+\sum_j \alpha^j_{t,g}\ln p^j_{i,h}+\frac{1}{2}\sum_j\sum_k \gamma_{kj}\ln p^k_{i,h}\ln p^j_{i,h}\right)$$

Here the notation for material private goods is used to also include private leisure, so that $p^l_{i,h} = w_{i,h}$. Hence, the i subscript (some other material good prices may vary at the household level). Similarly, the prices of public goods are indexed by h: R_h.

We re-write the Engel curves in terms of observables and the object of interest, remembering that $\rho_{i,h} = \eta_{i,h}(y_h - R_hQ_h)$ and multiplying through by the resource share:

$$\begin{aligned}\frac{p^a_{i,h}c^a_{i,h}}{y_h - R_hQ_h} = \eta_{i,h}\left(\alpha_{t,g}-\beta_{t,g}\alpha^0_{t,g}\right)+\eta_{i,h}\sum_j\left(\left(\gamma^j_{t,g}-\beta_{t,g}\alpha^j_{t,g}\right)\ln p^j_{i,h}\right)\\ -\eta_{i,h}\beta_{t,g}\frac{1}{2}\sum_j\sum_k\gamma_{kj}\ln p^k_{i,h}\ln p^j_{i,h}+\eta_{i,h}\beta_{t,g}\ln\eta_{i,h}+\eta_{i,h}\beta_{t,g}\ln\left(y_h-R_hQ_h\right)\end{aligned}$$

If we model the resource share as a type-specific constant (rather than varying at the household-level based on covariates z_h) then the identification result holds exactly. How-

[62] This is an extension of the set-up in LPW to public goods. If also wishing to incorporate the shareable goods framework, this can be done as in LPW, with their assumption that A is block-diagonal so that there are no cross-good complementarities relating to the assignable good $A_g = \begin{matrix} A^X & 0 \\ 0 & A^c \end{matrix}$. This assumption ensures that the assignable good Engel curves have the usual form, instead of the more complex form which would result from cross-good complementarities with other goods.

ever, if we are interested in household-specific resource shares varying with z_h we need to linearly approximate the $\eta_{i,h}\beta_{t,g}\ln\eta_{i,h}$ term (similarly to LPW):

- As required by assumption 3.1.1, the estimating equation has a component which is linear in the resource share. We can write $\frac{p^a_{i,h}c^a_{i,h}}{y_h-R_hQ_h} = \sum_\varphi f^\varphi_{t,g} + f^0_{t,g}\eta_{i,h}$, where $f^0_{t,g} = \beta_{t,g}\ln(y_h - R_hQ_h)$ and

$$\sum_\varphi f^\varphi_{t,g} = \eta_{i,h}\left(\alpha_{t,g} - \beta_{t,g}\alpha^0_{t,g}\right) + \eta_{i,h}\sum_j\left(\left(\gamma^j_{t,g} - \beta_{t,g}\alpha^j_{t,g}\right)\ln p^j_{i,h}\right) - \eta_{i,h}\beta_{t,g}\frac{1}{2}\sum_j\sum_k\gamma_{kj}\ln p^k_{i,h}\ln p^j_{i,h} + \eta_{i,h}\beta_{t,g}\ln\eta_{i,h}$$

  .

- As required by part A of assumption 3.1.2, the estimating equation is linear in unknowns if we consider resource shares to be type-specific constants. We can write $\frac{p^a_{i,h}c^a_{i,h}}{y_h-R_hQ_h} = a_{t,g} + \sum_\varphi b^\varphi_{t,g}g^\varphi + \gamma_{t,g}\eta^0_{t,g}g^0$ where (with some slight abuse of notation):
  - † $a_{t,g} = \eta^0_{t,g}\left(\alpha_{t,g} - \beta_{t,g}\alpha^0_{t,g}\right) + \eta^0_{t,g}\beta_{t,g}\ln\eta^0_{t,g}$ is a constant which is a function of unknown preference and resource share parameters
  - † $\sum_\varphi b^\varphi_{t,g}g^\varphi = \eta^0_{t,g}\sum_j\left(\left(\gamma^j_{t,g} - \beta_{t,g}\alpha^j_{t,g}\right)\ln p^j_{i,h}\right) - \eta^0_{t,g}\beta_{t,g}\frac{1}{2}\sum_j\sum_k\gamma_{kj}\ln p^k_{i,h}\ln p^j_{i,h}$
    - $b^1_{t,g} = \eta^0_{t,g}\sum_j\left(\left(\gamma^j_{t,g} - \beta_{t,g}\alpha^j_{t,g}\right)\right)$ is a constant which is a function of unknown preference and resource share parameters
    - $b^2_{t,g} = \eta^0_{t,g}\beta_{t,g}\frac{1}{2}\sum_j\sum_k\gamma_{kj}$ is a constant which is a function of unknown preference and resource share parameters
    - $g^1 = \sum_j\left(\ln p^j_{i,h}\right)$ is observed in the data (price data is required for this)
    - $g^2 = \sum_j\sum_k\ln p^k_{i,h}\ln p^j_{i,h}$ is observed in the data (price data is required for this)
    - $g^0 = \ln(y_h - R_hQ_h)$ is observed in the data (requires expenditure on all public goods including both time-use and material)
    - $\gamma_{t,g} = \beta_{t,g}$ and hence is a preference parameter

- However, part B of the assumption is not satisfied because of the non-linear $\eta_{i,h}\beta_{t,g}\ln\eta_{i,h}$ term.
  - † We can linearly approximate $\beta_{t,g}\ln\eta_{i,h}$ as $\kappa^0_{t,g} + \sum_\omega\kappa^z_{t,g}\omega_h$, where ω_h could coincide with the characteristics z_h used in the resource share approximation, or be

transformations of z_h. Substituting this in and rearranging we obtain:

$$\frac{p^a_{i,h}c^a_{i,h}}{y_h - R_hQ_h} = \eta_{i,h}\left(\alpha_{t,g} - \beta_{t,g}\alpha^0_{t,g} + \kappa^0_{t,g}\right) + \eta_{i,h}\sum_j\left(\left(\gamma^j_{t,g} - \beta_{t,g}\alpha^j_{t,g}\right)\ln p^j_{i,h}\right)$$
$$- \eta_{i,h}\beta_{t,g}\frac{1}{2}\sum_j\sum_k \gamma_{kj}\ln p^k_{i,h}\ln p^j_{i,h} + \eta_{i,h}\left(\sum_\omega \kappa^z_{t,g}\omega_h\right) + \eta_{i,h}\beta_{t,g}\ln\left(y_h - R_hQ_h\right)$$

- The rank condition (assumption 3.1.3) is satisfied if
  - † We are interested in estimating resource shares as type-specific constants. In this case we just need to check that $\ln\left(y_h - R_hQ_h\right)$ is not colinear with the other observables i.e. $g^1 = \sum_j\left(\ln p^j_{i,h}\right)$ and $g^2 = \sum_j\sum_k \ln p^k_{i,h}\ln p^j_{i,h}$, which it is not.
  - † We are interested in estimating household-specific resource shares (with covariates), and $(y_h - R_hQ_h)$ is not an element of ω_h. A related assumption in LPW is that resource shares don't depend on the household budget.
- The invertibility assumption 3.1.4 requires that assignable good Engel curves are sufficiently responsive to household budget net of public expenditure. This has to be assessed in a specific context for application. Assumptions 3.1.5 and 3.1.6 should also be examined in a specific context for application. The identifying assumption restricting preference heterogeneity (SAP, SAT or SRAT) is made on parameter $\beta_{t,g}$ and hence is to be interpreted with regard to the responsiveness of assignable good Engel curves to household budget net of public expenditure.

The Almost Ideal Demand System Engel curves are not very tractable if all prices vary in the sample. This is both because it becomes highly multi-dimensional and because estimating it requires high-quality, detailed price data. As a result, DLP and LPW assume no price variation and drop the price index terms into the constant. This was possible because time-use was not modelled in those papers. In a model with time-use, there will be some price variation in the form of individual-level wages, even if we can assume all other prices to be constant in the sample.[63] If this is the case, we can simplify the estimating equations to:
$\frac{p^a_{i,h}c^a_{i,h}}{y_h - R_hQ_h} = a^0_{t,g}\eta_{i,h} + a^1_{t,g}\eta_{i,h}\ln w_i + a^2_{t,g}\eta_{i,h}\left(\ln w_i\right)^2 + \sum_\omega a^\omega_{t,g}\omega_h\eta_{i,h} + \eta_{i,h}\beta_{t,g}\ln\left(y_h - R_hQ_h\right)$
Even so, as soon as we add covariates affecting intra-household bargaining this exponentially increases the number of regressors (because of the ineraction between ω and η, which both depend on these covariates). Without yet more approximation, this is not a tractable

[63] To avoid dealing with this, if the assignable good is material, we could assume preferences that are separable in material goods and time-use.

specification, especially for estimation with small samples. This is problematic since small samples are almost inevitable in this literature since estimation must be conducted separately for households of different categories. Yet, more approximation is also unpalatable, as it undermines the link between the structural model and the estimated regression. A more restrictive, but more parsimonious functional specification for preferences may be a preferable approach.

B Time-use data

Activity list from UKTUS 2014
Unspecified personal care
Visiting and receiving visitors
Correspondence
Unspecified organisational work
Other specified ball games
Gymnastics
Visiting a wildlife site
Celebrations
Work for an organisation
Fitness
Unspecified TV video or DVD watching
Watching a film on TV
Telephone conversation
Unspecified childcare as help to other households
Other specified or unspecified arts and hobbies
Volunteer work through an organisation
Unspecified water sports
Other specified TV watching
Unspecified food management
Watching a film on video
Watching sport on video
Other specified water sports
Other specified video watching
Food preparation and baking
Disposal of waste

Activity list from UKTUS 2014
Other specified physical exercise
Physical care and supervision of own child as help to other household
Study: Unspecified activities related to school or university
Unspecified productive exercise
Dish washing
Study: Classes and lectures
Information searching on the internet
Hunting and fishing
Preserving
Other specified social life
Other specified information by computing
Study: Homework
Communication on the internet
Main job: unspecified main job
Unspecified entertainment and culture
Other specified communication by computing
Unspecified other computing
Skype or other video call
Travel escorting to/ from education
Main job: Working time in main job
Cinema
Other specified computing
Second job: Working time in second job
Travel related to household care
Main job: Coffee and other breaks in main job
Unspecified theatre or concerts
Plays musicals or pantomimes
Opera operetta or light opera
Concerts or other performances of classical music
Unspecified informal help to other households
Dance performances
Unspecified listening to radio and music
Other specified theatre or concerts
Sleep
Sleep: In bed not asleep

Activity list from UKTUS 2014
Food management as help to other households
Other specified food management
Listening to music on the radio
Sleep: Sick in bed
Borrowing books records audiotapes videotapes CDs VDs etc. from a library
Reference to books and other library materials within a library
Using internet in the library
Household upkeep as help to other households
Reading newspapers in a library
Other specified radio listening
Unspecified household upkeep
Other specified library activities
Sports events
Unspecified games
Gardening and pet care as help to other households
Cleaning dwelling
Travel related to voluntary work and meetings
Study: other specified activities related to school or university
Construction and repairs as help to other households
Other specified productive exercise
Cleaning yard
Unspecified games and play with others
Billiards pool snooker or petanque
Shopping and services as help to other households
Heating and water
Other specified parlour games and play
Free time study
Swimming
Travel escorting a child other than education
Unspecified sports related activities
Activities related to sports
Arranging household goods and materials
Unspecified video watching
Other unspecified entertainment and culture
Visiting a historical site

Activity list from UKTUS 2014
Gambling
Visiting a botanical site
Visiting a leisure park
Physical care and supervision of child as help to other household
Teaching non-coresident child
Reading playing & talking to non-coresident child
Accompanying non-coresident child
Other or unspecified entertainment or culture
Reading playing & talking to own non-coresident child
Accompanying own non-coresident child
Other specified childcare as help to other household
Travel related to organisational work
Physical care and supervision of an adult as help to another household
Accompanying an adult as help to another household
Other specified help to an adult member of another household
Unspecified help to an adult of another household
Resting - Time out
Other specified informal help to another household
Other specified informal help
Second job: Coffee and other breaks in second job
Outdoor team games
Travel related to informal help to other households
Unspecified participatory activities
Eating
Meetings
Other or unspecified household upkeep
Other specified games
Religious activities
Unspecified making and care for textiles
Laundry
Ironing
Handicraft and producing textiles
Travel related to religious activities
Activities related to employment: Unspecified activities related to employment
Travel to visit friends/relatives in their homes not respondents household

Activity list from UKTUS 2014
Activities related to employment: Lunch break
Travel related to other social activities
Other personal care: Unspecified other personal care
Visiting an urban park playground designated play area
Travel related to entertainment and culture
Other personal care: Wash and dress
Computing - programming
Other specified making and care for textiles
Travel related to participatory activities other than religious activities
Gardening
Tending domestic animals
Caring for pets
Activities related to employment: Other specified activities related to employment
Activities related to employment: Activities related to job seeking
Walking the dog
Unspecified information by computing
Activities related to employment: Other unspecified activities related to employment
Travel related to other leisure
Other personal care: Other specified personal care
Travel related to physical exercise
Travel escorting an adult other than education
Travel related to hunting & fishing
Travel related to productive exercise other than hunting & fishing
Other specified gardening and pet care
Unspecified construction and repairs
Unspecified communication by computer
House construction and renovation
Repairs of dwelling
Travel to work from home and back only
Picking berries mushroom and herbs
Making repairing and maintaining equipment
Woodcraft metalcraft sculpture and pottery
Other specified making repairing and maintaining equipment
Vehicle maintenance
Travel related to gambling

Activity list from UKTUS 2014
Travel related to hobbies other than gambling
Other specified construction and repairs
Unspecified shopping and services
Unspecified shopping
Shopping mainly for food
Shopping mainly for clothing
Shopping mainly related to accommodation
Shopping or browsing at car boot sales or antique fairs
Window shopping or other shopping as leisure
Other specified shopping
Commercial and administrative services
Personal services
Other specified organisational work
Travel related to changing locality
Unspecified household and family care
Travel to holiday base
Travel for day trip/just walk
Other specified shopping and services
Live music other than classical concerts opera and musicals
Household management not using the internet
Shopping for and ordering clothing via the internet
Unspecified household management using the internet
Shopping for and ordering unspecified goods and services via the internet
Shopping for and ordering food via the internet
Shopping for and ordering goods and services related to accommodation via the internet
Shopping for and ordering mass media via the internet
Shopping for and ordering entertainment via the internet
Banking and bill paying via the internet
Other specified household management using the internet
Art exhibitions and museums
Other specified travel
Unspecified radio listening
Unspecified library
Punctuating activity
Unknown: at home

Activity list from UKTUS 2014
Unspecified childcare
Filling in the time use diary
Study: Unspecified study school or university
Unspecified physical care & supervision of a child
Feeding the child
Other specified participatory activities
No main activity no idea what it might be
Using computers in the library other than internet use
Other and unspecified physical care & supervision of a child
Teaching the child
No main activity some idea what it might be
Reading playing and talking with child
Illegible activity
Accompanying child
Listening to recordings
Unspecified time use
Queryable
Other specified performing arts
Listening to sport on the radio
Travel related to unspecified time use
Other or unspecified childcare
Travel related to personal business
Travel related to services
Unspecified mass media
Unspecified help to a non-dependent eg injured adult household member
Physical care of a non-dependent e.g. injured adult household member
Accompanying a non-dependent adult household member e.g. to hospital
Other specified help to a non-dependent adult household member
Unspecified help to a dependent adult household member
Physical care of a dependent adult household member e.g. Alzheimic parent
Accompanying a dependent adult household member e.g. Alzheimic
Solo games and play
Unspecified hobbies games and computing
Other specified help to a dependent adult household member
Watching sport on TV

Activity list from UKTUS 2014
Travel related to shopping
Unspecified sports and outdoor activities
Unspecified employment
Unspecified social life and entertainment
Travel to/from work
Travel in the course of work
Chess and bridge
Unspecified volunteer work and meetings
Unspecified reading
Travel to work from a place other than home
Reading periodicals
Reading books
Unspecified arts
Unspecified visual arts
Painting drawing or other graphic arts
Making videos taking photographs or related photographic activities
Not applicable
Computer games
Other specified visual arts
Unspecified performing arts
Singing or other musical activities
Unspecified physical exercise
Help to other households in employment and farming
Literary arts
Walking and hiking
Taking a walk or hike that lasts at least miles or 1 hour
Other specified arts
Other walk or hike
Jogging and running
Unspecified social life
Unspecified hobbies
Activities related to productive exercise
Biking skiing and skating
Biking
Skiing or skating

Activity list from UKTUS 2014
Indoor team games
Socialising with family
Collecting
Travel related to education
Unspecified ball games
Indoor pairs or doubles games
Other specified reading
Outdoor pairs or doubles games

B.1 Definition of private leisure

In the 2014 data, I select the following activities in my baseline definition of private leisure. I also conduct robustness analyses removing some of the activities which are borderline between leisure and domestic work, such as gardening, and activities which, like sleep, may be more likely to have externalities on other family members. Both in my baseline and robustness checks, I focus only on leisure time which was spent 'NOT with other household members' (there are several binary variables coding co-presence information), with the exception of sleep, where I allow other household members to be co-present.

Leisure activities
Unspecified personal care
Visiting and receiving visitors
Other specified ball games
Gymnastics
Visiting a wildlife site
Celebrations
Fitness
Unspecified TV video or DVD watching
Watching a film on TV
Other specified or unspecified arts and hobbies
Unspecified water sports
Other specified TV watching
Watching a film on video
Watching sport on video
Other specified water sports

Leisure activities
Other specified video watching
Other specified physical exercise
Other specified social life
Communication on the internet
Unspecified entertainment and culture
Other specified communication by computing
Skype or other video call
Cinema
Main job: Coffee and other breaks in main job
Unspecified theatre or concerts
Plays musicals or pantomimes
Opera operetta or light opera
Concerts or other performances of classical music
Dance performances
Unspecified listening to radio and music
Other specified theatre or concerts
Sleep
Sleep: In bed not asleep
Listening to music on the radio
Reading newspapers in a library
Other specified radio listening
Sports events
Unspecified games
Unspecified games and play with others
Billiards pool snooker or petanque
Other specified parlour games and play
Swimming
Unspecified sports related activities
Activities related to sports
Unspecified video watching
Other unspecified entertainment and culture
Visiting a historical site
Gambling
Visiting a botanical site
Visiting a leisure park

Leisure activities
Other or unspecified entertainment or culture
Resting - Time out
Second job: Coffee and other breaks in second job
Outdoor team games
Unspecified participatory activities
Eating
Other specified games
Activities related to employment: Lunch break
Other personal care: Unspecified other personal care
Other personal care: Wash and dress
Gardening
Other personal care: Other specified personal care
Unspecified communication by computer
Woodcraft metalcraft sculpture and pottery
Window shopping or other shopping as leisure
Travel for day trip/just walk
Live music other than classical concerts opera and musicals
Art exhibitions and museums
Unspecified radio listening
Other specified participatory activities
Other specified performing arts
Listening to sport on the radio
Unspecified mass media
Solo games and play
Unspecified hobbies games and computing
Watching sport on TV
Unspecified sports and outdoor activities
Unspecified social life and entertainment
Chess and bridge
Unspecified reading
Reading periodicals
Reading books
Unspecified arts
Unspecified visual arts
Painting drawing or other graphic arts

Leisure activities
Making videos taking photographs or related photographic activities
Computer games
Other specified visual arts
Unspecified performing arts
Singing or other musical activities
Unspecified physical exercise
Literary arts
Walking and hiking
Taking a walk or hike that lasts at least miles or 1 hour
Other specified arts
Other walk or hike
Jogging and running
Unspecified social life
Unspecified hobbies
Biking skiing and skating
Biking
Skiing or skating
Indoor team games
Collecting
Unspecified ball games
Indoor pairs or doubles games
Other specified reading
Outdoor pairs or doubles games

Activities are defined in a similar manner in UKTUS 2000.

C Expenditure data and individual-level consumption estimation

C.1 The dataset

The LCF (Living Costs and Food Survey) (previously FES) is a UK survey containing information on individual and household characteristics, individual labour supply, detailed income data, and very detailed expenditure data. It is a repeated cross-section available

yearly since 1978.

It is a high-quality, nationally representative, large-scale survey that is used to estimate official government statistics. The FES/LCF data has been widely used both for academic and policy applications due both to its scale and high quality. For instance, Bargain, Donni and Hentati (2022) and Lise and Seitz (2011) use this data.

The survey has multiple components: (i) a household survey recording household characteristics and retrospective questions on irregular expenses (rent, vehicles...); (ii) an individual questionnaire with individual characteristics, including demographic characteristics, hours worked and sources of income;[64] (iii) a detailed two-week expenditure diary for all members older than 7 (simplified diary for people aged 7-15, full diary for people aged 16 or above). The household questionnaire is answered by the reference person either alone, or together with other household members. Individual surveys, and expenditure diaries, are answered by the relevant person. The expenditure diaries are kept for two weeks by all household members.

The expenditure diary records the type of good in detail, and receipts are attached. Clothing and footwear is divided into male, female, and children.[65] Household expenditure is obtained by summing expenditure over all members. Additional information on expenditure on large infrequent expenses, such as house repairs, and regular expenses, such as rent, is obtained during the household survey. These expenditures are transformed to an equivalent weekly value to make them comparable to other categories.

After restricting the LCF 2014 sample to heterosexual working couples and cleaning the data, the final sample comprises 583 households (i.e. 1,166 individuals). Key variables are summarised in table 1.

C.2 Categorisation of expenditures into private and public

The next step towards estimating individual-level inequality is to divide the LCF household-level expenditure data into (i) private expenditure, (ii) public expenditure, and (iii) expenditure to be excluded from consideration. In deciding how to do so, it is important to consider the goal of the exercise. In the case of this paper, the aim is to compare the material standard of living of different individuals in the UK.

[64] In rare instances, income is top-coded. I adjust top-coded values using data on after-tax income percentiles from HM Revenue & Customs. HM Revenue & Customs. (2023). Percentile points from 1 to 99 for total income before and after tax. https://www.gov.uk/government/statistics/percentile-points-from-1-to-99-for-total-income-before-and-after-tax

[65] Note that any particular good may be purchased by a household member but consumed by any combination of them and/or other members, so personal expenditures do not measure personal consumption.

C.2.1 Excluded categories

I exclude expenditure categories which have almost no immediate consumption value and cannot easily be squared with the static model underlying the methodology in this paper. This includes: savings, insurance, investments, major house works including renovation (minor repairs are included), financial gifts, bets, gambling and expenditure on education (the latter is minor for working couples without cohabiting children). Future work considering dynamic aspects would enable incorporating these categories into overall resources in a theory-consistent manner. Again with in mind the goal of measuring material well-being, I focus on expenditures gross of any government refund or subsidy . Sometimes expenditures are partly funded by the government and this is visible in the expenditure data for some goods. Where this is the case, I do not detract any subsidies or refunds from expenditure, since these still contribute to material well-being.[66]

C.2.2 Private vs public consumption

Based on the detailed COICOP plus codes into which expenditure is divided in the LCF, I divide expenditure between private and public. While it would be possible to categorise expenditure based on less granular data, the granularity aids accuracy. For instance, most house-cleaning products are categorised as public (as they contribute to the public cleanliness of the house) but washing powders are categorised as private as more is needed to wash the clothes of more members.

Discretion is needed in categorising goods, as most goods have at least some public element, including externalities of consumption on other household members. In some cases this categorisation could be made conditionally on household characteristics. For instance, holiday accommodation is a public good for couples, who will generally stay in the same room. For families with older children, one might consider holiday accommodation as partly private if teenagers are likely to e.g. have a separate hotel room.

In some cases, it might be possible to use other data (e.g. data on car occupancy) to estimate to what degree a good (e.g. cars) is public and to what degree it is private. We can think of car-related expenditures (insurance, fuel, etc.) as separate goods depending on the type of trip, where some of them are private and some of them are public. A car and related expenses could be purely public if everyone in the household only used it together (e.g. to drive to a holiday home). Alternatively it could be purely private if only one household

[66]For other applications, e.g. to estimate what proportion of the population has access to a minimal standard of living, we might not consider the consumption of 'bads' to contribute to that standard of living, and hence also exclude those.

member used it to drive to work. As discussed in more detail below, I approximate the likely economies of scale of car-related expenditures using UK car occupancy data.

C.2.3 Treatment of durables, including homes and vehicles

While some papers in this literature focus on non-durable consumables, I suggest it is important to also consider durables. In particular, housing is a very durable and infrequently purchased, but it is important to take it into account since it is such a large expenditure for many households, and since it is a very important component of public consumption and household economies of scale.

For less expensive durables such as clothing and phones, I use the LCF expenditure data without adjustments. While for any specific category this is likely to lead either to over-estimates or under-estimates, the overall expenditure across categories is likely to be a reasonably accurate estimate of usual expenditure. For very large expenses (buying a home or a vehicle), a different approach is needed, since the magnitude of these expenditures dwarves regular weekly purchases. Moreover, these expenditures are often diluted over some periods of time, with a mortgage or loan, with repayments depending not only on the quality of the good being purchased but also on factors which are not directly relevant to material well-being, such as macroeconomic conditions at the time of purchase and individual credit score. Keeping in mind the goal of measuring material well-being, we wish to estimate the value of the good being consumed (e.g. a home in a certain area with a certain number of rooms) while abstracting from extraneous considerations (e.g. whether the home is owned or rented). Of course the household may financially benefit from owning outright instead of renting, through decreased monthly expenditures on housing. However, this will translate into increased expenditures on other goods, and hence will still be taken into account insofar as it affects material well-being.

Imputed rents for homes Housing is a particularly complex good from the viewpoint of measuring inequality in the UK. The prices of homes with similar attributes in different regions differs vastly, but households generally have limited choice as to their location, especially in the short-run (due to jobs, relationships, and coordination between members). While it is approximately true that goods like food are similarly priced across the UK, assuming price homogeneity for homes across the UK would be too unrealistic (e.g. homes in the London area command a very large premium). Another source of difficulty is that homes are sometimes owned outright, sometimes they are purchased with a mortgage (the interest on which varies vastly by year and credit score of the purchasing party), sometimes they

are rented privately, and sometimes publicly (e.g. council housing provided as a benefit). To avoid both the issue of infrequency and the problem of comparability between renters, outright owners, and owners paying back a mortgage, I impute a weekly standardised rental price. This price increases in the quality of the home (as measured by the number of bedrooms, centrality of location, etc.) but is standardised across different purchasing conditions, as well as for regional house price differences (the latter being mostly something that affects the investment value of the property rather than reflecting the underlying quality of the home).

I suggest taking the following approach to balance between the two opposing goals of comparability between households and granularity of household-specific expenditure:

- Using data on private renters, I regress rent expenditure on
    1. Number of bedrooms. This is the best proxy for home size available in the data.
    2. Area type. The 2011 Area Classification for Output Areas (OAC) categorises postcodes into types e.g. areas dominated by 'urban professional and families'. Using granular OAC codes, I define four aggregated area types: rural, more desirable urban, less desirable urban, and suburban.
    3. Council tax band. Council tax bands are available for homes in England, Wales, and Scotland (I impute them for Northern Ireland). Council tax bands are based on legacy valuations of homes, and provide a good signal of the quality of the home.
    4. Region (Wales, Scotland, Northern Ireland, and the nine regions of England). There are substantial regional disparities in the prices of homes in the UK.
- I impute the rent for all households in the data (including those who own, rather than rent) based on these characteristics.
- Next, I standardise imputed rents across regions by indexing them to the region with the lowest median imputed rent (the North East of England). I deflate the imputed rents for homes in other regions by the ratio of the median imputed rent in their region to the median imputed rent in the North East of England. This procedure can be thought of as uncovering the fundamental quality of the consumption obtained from the house, disentangling it from other considerations like investment value of property. We can also motivate this choice by noting that households often cannot move region (at least in the short-run), but can choose the specific location of their home (with different associated OAC code), its size (proxied by the number of bedrooms) and broader quality (proxied by council tax band).

- For each household I record the standardised imputed rent.
- A small number of households have a second home, but the data contains no information on the second home. Where this is the case, I double the standardised imputed rent of the main home, since the value of the main home is the best available signal in the data of the likely value of the second home.
- I add the standardised imputed rents to public household expenses.

Imputed lease price for vehicles Car purchases are the second largest expenditure items after homes for many UK households. For this reason, it seems important not to exclude this expenditure. Following a principle similar to the one outlined above for housing, I propose a rental approach to vehicles (cars, vans and motorcycles):

- I calculate the median weekly lease price paid by households that lease a vehicle.
- For each household, I observe how many vehicles they own, and estimate their vehicle lease expenditure by multiplying the average lease price by the number of vehicles owned.
- I add 54% of this imputed lease to private expenditures and the remaining 46% to public expenditures. This approximation is based on data from the National Travel Survey[67] on the proportion of car trips by purpose, and data on the occupancy rate of cars by trip purpose.[68] This method aims to capture the likely economies of scale of cars, although of course further granularity could be achieved if the data contained information on how the household uses cars. We can think of this as dividing car-related expenditures into multiple goods, some of which are private (e.g. car trips for solo work trips) and some of which are public (e.g. car trips for family holidays).[69]

D Considerations on choosing the assignable good

D.1 Availability and accuracy of assignable good data

Ideally we would have accurate, widely available data for our assignable good.

[67] Department for Transport. (2021). National Travel Survey: 2020. https://www.gov.uk/government/statistics/national-travel-survey-2020/national-travel-survey-2020#trends-in-car-trips

[68] Department for Transport. (2022). Car or van occupancy and lone driver rate by trip purpose, England: 2002 onwards. https://www.gov.uk/government/statistical-data-sets/nts09-vehicle-mileage-and-occupancy

[69] I do not attempt to adjust this split based on the numbers of vehicles owned as I am not aware of data that would enable such an adjustment.

For clothing, the breakdown between men, women and children is often (though not always) available. Where it is available, the accuracy of clothing expenditure data may vary depending on whether it was derived from expenditure diaries or recall questions. Where expenditure by men, women and children is not available, some papers e.g. Calvi (2020) estimate these expenditure categories from expenditure on specific types of clothing e.g. assigning pyjamas to men, which may add measurement error. Narrowing down to specific clothing types such as skirts and ties is not a viable alternative as these purchases are very infrequent and command a very small budget share (clothing itself commands a low budget share).

Similarly, time-use data may be more or less accurate depending on whether it is based on recall questions or on a detailed time-diary. High-quality time-use data is available for many countries, and generally includes information both on very detailed activities and who was co-present during the activity (e.g. the UK Time Use Data used in the application in this paper). Using the detailed activity information it is possible to construct accurate measures of time spent on leisure activities, and using the co-presence information it is possible to exclude leisure time which was joint rather than private. Where this is possible, private leisure is likely measured accurately, especially where the time diary was taken for a longer period of time. Measurement error may be higher for proxies of leisure obtained from recall questions on hours of leisure in a typical week (non-market-work).

Individual-level food expenditure is a good candidate assignable good, but it is resource-intensive to collect accurate data on it, and hence it is rarely available in practice.

D.2 Credibility of the assignable good being private

Identification relies on the assignable good being private. In general, using an assignable good with a substantial public element (externalities) will bias resource share estimates towards equality. It is likely that, in reality, any assignable good will have some externalities. Ideally we would choose a good which is close to being fully private.

Conceptually, clothing is shareable, even between different types of people. Especially in poorer households, we may worry that this can introduce a substantial public element of clothing. Moreover, even when each person wears their own clothing, people often have strong views about the way their family members, especially their partner, should dress in public.

Non-market work includes a substantial public element due to its inclusion of domestic work and joint leisure. Where high-quality time-use data is available, private leisure is more credibly private. Of course, we may still worry that people care about what their family

members do in their spare time, but it is hard to imagine a good for which we would have weaker concerns about externalities than for private leisure (individual-level food expenditure also has similarly small conceptual externalities).

D.3 Frequency of purchases and magnitude of budget share

Ideally, our assignable good will command a high budget share and there will be a very low proportion of people hitting a corner solution on it.

Clothing is an infrequent purchase, and the proportion of zeros is very high in many datasets used in the literature (most expenditure surveys are taken over short periods of time, e.g. two weeks). In the UK expenditure data used in the application in this paper, 73.20% of households have zero recorded expenditure on at least one of male and female clothing.[70] This is problematic because identification relies on demand functions which treat observations as interior solutions. Therefore, using an assignable good with a high proportion of zeros adds substantial inaccuracy to the estimation approach, as corner solutions are treated as interior solutions. Moreover, since zeros are not merely driven by random infrequency but are also systematically related to low resource shares, sharing rule estimates are likely to be biased towards equality.

By contrast, households spend a large proportion of their resources on individual-level food expenditure, non-maket-work and private leisure, with virtually no zeros in the data, so using it as the assignable good.

D.4 Further considerations

Clothing durability. Clothing is highly durable and often passed down through generations (e.g. older to younger cousins), especially for small children. This means that clothing consumption can be substantially different from clothing purchases. This may lead to additional inaccuracies arising when using clothing as the assignable good. It might potentially also contribute to biasing estimates if e.g. clothes hand-me-downs are more frequently enjoyed by certain sub-groups, such as small children or young women.

Clothing expenditure is not recorded at the individual level. Clothing expenditure is almost never measured at the individual level in expenditure data, and is only available in broad break-downs such as men's, women's and children's clothing. In contexts with larger households and multiple members of each type, common especially in developing countries,

[70] Some surveys may ask recall questions on clothing expenditure to alleviate this problem, but the accuracy of responses to such questions is more doubtful.

this reduces the granularity of estimates to type-level, rather than individual-level, estimates of resources.

E The issue of non-participation

The difference between the case of participation and non-participation is illustrated in figure 4. The left panel shows the case of an interior solution with market participation, while the right panel exemplifies non-participation. To simplify exposition, I group domestic work and joint leisure into a single public time category. This is to be understood as the optimal combination of domestic work and of joint leisure. In the interior solution case, there is an internal market for the individual's time, so that the price of time is the observed market wage. In the case of market non-participation, the (unobserved) market wage is lower than the returns from optimal non-market-work time-use. This substantially complicates the problem. The price of public time-use is not the (unobserved) market wage, so we would need to estimate this unobserved price to use private leisure as the assignable good (and more generally for other assignable goods if separability is not assumed). This is one of two reasons why this paper focuses only on households where all members participate in market work. The second reason is that we can more credibly test the identifying assumptions for the sub-sample of working couples. As discussed in section 4.4.2, tests from data on working singles appear to be highly informative for this sub-sample, but may be less convincing for other household compositions.

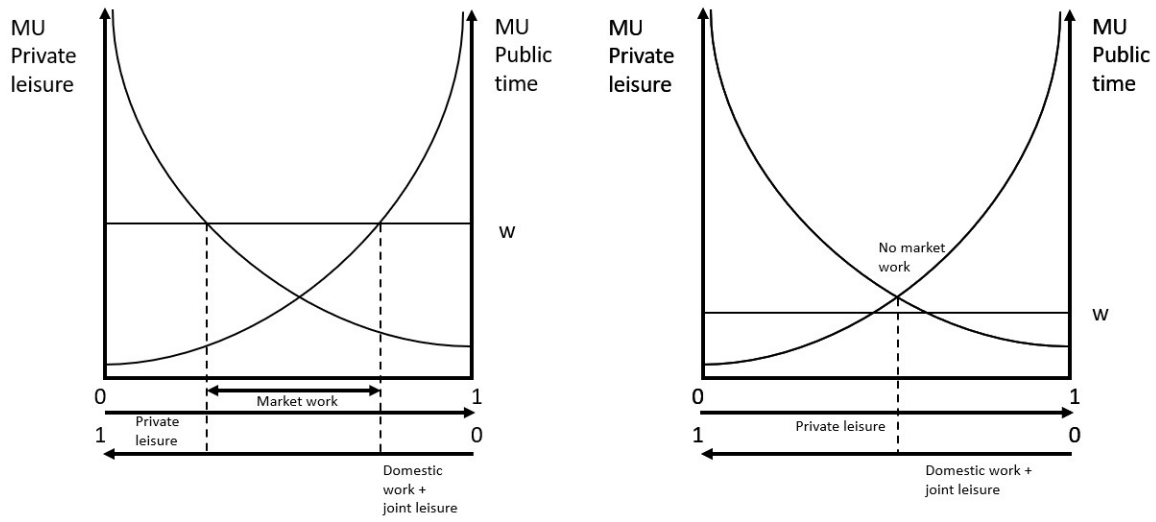

Figure 4: Interior solution vs. corner solution for time-use

E.1 Why excluding couples with non-participation is unlikely to bias estimates

In my application to UK data, I focus only on couples without cohabiting children. In this setting, I restrict my sample only to couples where both members supply some market work. It is important to note that this is unlikely to bias the estimates, for the following reason. In the UK, for couples without dependent children, men and women have very similar, and high, participation rates. Non-participation is due to reasonably exogenous drivers such as long-term illness, disability or temporary unemployment. Importantly, non-participation is not driven by a member having a particularly high, or low, resource share. Hence, excluding non-participating couples should not bias resource share estimates for participating couples.

For couples with dependent children, it would be much more problematic to exclude non-participating couples. This is because, in the UK, there is a substantial gap in participation between men and women with dependent (especially very young) children. It is realistic to think that part of this phenomenon is driven by women with very low potential wages, and low resource shares, taking on a full-time childcare and domestic work commitment, leaving them too little leisure time to additionally take on market work. In my application, I do not incur the latter difficulty because I focus only on couples without cohabiting children. While the having of children may itself be endogenous, it would require a very different, dynamic, model to take this into account appropriately, and difficulties would arise in modelling the preference changes associated with changes in household composition.

Of course it is important to remember that my estimates are not likely to be externally valid to other household compositions, which is why I do not attempt to use my estimated sharing rule for couples with children.

Finally, I note that, if using clothing as the assignable good, excluding households with zero expenditure on clothing may bias results because zero expenditure on clothing may be driven either by infrequency of purchase or, problematically, by actual zeros due to low resource shares.

F Cobb-Douglas: additional material

F.1 Cobb-Douglas Domestic production functions

Household economics, despite its emphasis on households, frequently does not involve modelling domestic production because it introduces identification issues. However, it is clear that domestic production does play an important role in understanding individual-level resources

(see for instance Apps and Savage (1989)). One way to model domestic production within a collective model with relative ease is to assume that the domestic good is marketable - it can be bought and sold. This assumption has the advantage of leading to separability between the production and consumption functions of the household as the price of the domestic good is exogenously determined by the market, and not endogenously within the household (see e.g. Browning, Chiappori and Weiss (2014) section 4.6.2).

However, this assumption cannot easily be reconciled with empirical facts. If market work has a constant return of $w_{i,h}$ and domestic work has a constant return $w_{i,h}^{dm}$, then each individual would either supply market work or domestic work, and not both. This is in contrast with empirical evidence that a very substantial proportion of the population do both.

One way of trying to reconcile marketability and this empirical fact is if market work is constrained in terms of hours, a fact which is consistent with some of the literature on elasticity of labour supply. This assumption is sensible in some contexts, but is less likely realistic for low skilled work, where one is more likely to be able to ask to do overtime shifts, or have multiple jobs alongside each other to make ends meet. Moreover, this assumption would still not be reconcilable with the empirical fact that many people work part-time and also do domestic work. These people are clearly not constrained in terms of number of hours doing market work (especially as part-time hours are quite heterogeneous) and yet they do both types of work.

This is suggestive that we should not think of domestic work as having constant returns. If we wish to maintain the assumption of constant returns to market work, the most coherent way forward is to avoid modelling domestic work as marketable, as it would then become hard to justify decreasing returns to one type of marketable work and constant returns to another. Moreover, it is simply not realistic to model domestic work as being marketable. We can substitute between it and market purchased goods to some degree, but they are not the same good. Cleaning is perhaps the example where the boundary is most blurred, but making a home-cooked meal while looking after one's own child are clearly not the same as buying take-out food and sending the child to a nursery. Empirically, even people with high wages spend some time on domestic work - the reason is that it cannot be purchased, and it is desirable.

Therefore, we are left with the problem of modelling domestic production. Modelling domestic production is a complex task, and one that has historically not received nearly as much attention as aspects relating to market work. The literature which does exist has used a variety of production functions, depending on the aims of the analysis and the key features from the data of interest. For instance, Griffith et al. (2022) employ a Leontief production function for home cooked food where the inputs are market purchased ingredients

and domestic time. They further assume constant returns to domestic time, and perfect substitution between different household members' time (time spent cooking is just the sum of individual time spent cooking, and household leisure is just the sum of individual time spent on leisure).

For the purpose of this paper, it is instead important to model decreasing returns to domestic work to avoid the model being irreconcilable with the empirical facts described above. It is also important to model the domestic time of different members as not being perfect substitutes, again to explain patterns of behaviour with market and domestic work. I propose using a simple Cobb-Douglas production function. The productivity of each type of household member (and of the same types in different household categories) is heterogeneous. The concavity of the production function ensures that everyone in the household will do some domestic work.

An advantage of this approach is that the same functional form also seems appropriate to model non-private leisure. We want everyone in the household to have at least some time together (e.g. many households insist on a dinner with everyone, even those who are very busy), and there are higher returns to public leisure if others are able to join in. The Cobb-Douglas production function captures this concavity, and also allows us to model possible heterogeneity in the returns to joint leisure (e.g. the quality of joint leisure might be higher if children are involved, as it avoids the negative externalities of children being away from their parents). The model lends itself to greater granularity where relevant, for instance separating out leisure that is joint between strict subsets of household members.

However, for the purpose of this paper, consider the following production functions:

$D_h = \delta_g \prod_{i \epsilon h} \left(d_{i,h}\right)^{\delta t,g}$

$T_h = \phi_g \prod_{i \epsilon h} \left(jt_{i,h}\right)^{\phi t,g}$

The Cobb-Douglas production functions, substituted into the Cobb-Douglas utility functions, yield a Cobb-Douglas functional form, preserving the advantages of the Cobb-Douglas preferences:

$$u_{t,g} = \sum_{j \epsilon \Omega^c} \left(\alpha_{t,g}^{cj} \ln(c_{i,h}^j)\right) + \sum_{j \epsilon \Omega^X} \left(\alpha_{t,g}^{Xj} \ln(X_h^j)\right) + \alpha_{t,g}^l \ln(l_{i,h}) + \alpha_{t,g}^T \ln(T_h) + \alpha_{t,g}^D \ln(D_h)$$

$$u_{t,g} = \sum_{j \epsilon \Omega^c} \left(\alpha_{t,g}^{cj} \ln(c_{i,h}^j)\right) + \sum_{j \epsilon \Omega^X} \left(\alpha_{t,g}^{Xj} \ln(X_h^j)\right) + \alpha_{t,g}^l \ln(l_{i,h}) + \alpha_{t,g}^T \ln(\phi_g \prod_{s \epsilon h} \left(jt_{s,h}\right)^{\phi st,g}) + \alpha_{t,g}^D \ln(\delta_g \prod_{s \epsilon h} \left(d_{s,h}\right)^{\delta st,g})$$

The above can be re-written as a standard Cobb-Douglas utility function as a direct function

of time-use:

$$u_{t,g} = \sum_{j\epsilon\Omega^c} \left(\alpha^{cj}_{t,g} \ln(c^j_{i,h})\right) + \sum_{j\epsilon\Omega^X} \left(\alpha^{Xj}_{t,g} \ln(X^j_h)\right) + \alpha^l_{t,g} \ln(l_{i,h}) \\ + \left(\alpha^T_{t,g} \ln(\phi_g) + \alpha^D_{t,g} \ln(\delta_g)\right) + \alpha^T_{t,g} \sum_{s\epsilon h} \phi_{st,g} \ln jt_{s,h} + \alpha^D_{t,g} \sum_{s\epsilon h} \delta_{st,g} \ln d_{s,h}$$

We can work with this utility function and drop the production function constraints. We may wish to re-normalise the utility functions by setting the constant terms $\left(\alpha^T_{t,g} \ln(\phi_g) + \alpha^D_{t,g} \ln(\delta_g)\right)$ to 0, and define some new notation to simplify the coefficients on domestic time and joint leisure:

$u_{t,g} = \sum_{j\epsilon\Omega^c} \left(\alpha^{cj}_{t,g} \ln(c^j_{i,h})\right) + \sum_{j\epsilon\Omega^X} \left(\alpha^{Xj}_{t,g} \ln(X^j_h)\right) + \alpha^l_{t,g} \ln(l_{i,h}) + \sum_{s\epsilon h} \left(\alpha^{Tst,g}_{t,g} \ln jt_{s,h}\right) + \sum_{s\epsilon h} \left(\alpha^{Dst,g}_{t,g} \ln d_{s,h}\right)$

F.2 Normalising assumptions

Observationally, household behaviour is equivalent up to (i) positive affine transformations of individual utility functions, and (ii) any positive monotonic function of the sum of the individual utilities weighted by their respective Pareto weights.[71] I normalise the model as follows:

- the constant term in the utility functions is set to zero (omitted in the equations above)
- the sum of each person's preference parameters over all goods (material and time-use, public and private) is set to one $\sum_j \alpha^j_{t,g} = 1$
- the Pareto weights are set to sum to one: $\sum_{i\in h} \mu_{i,h} = 1$

Heterogeneity extension

Note that these normalisations are carried out under the assumption that productivity in domestic work and joint leisure, like preference parameters, are type-specific constants. If types are men and women, this means we are assuming that productivity (as well as preferences) are homogeneous within women and within men. To extend the model to contexts where preferences and/or productivity vary based on characteristics like age or wages, two approaches can be taken. The first is simply to define types more granularly and then

[71] Note that there is no uncertainty in this model so that the overall optimisation problem is unchanged by a positive monotonic transformation. The same cannot be said for the individual utilities because the household's optimisation problem is not to maximise a single utility but the weighted sum of all of the utilities, so that each individual utility can only be transformed up to positive affine transformations.

conduct estimation separately for different household categories (households with different combinations of types as members). Alternatively, drop the normalising assumptions and explicitly define preference and productivity parameters as functions of characteristics.

F.3 Public good C-D SAP

F.3.1 Under public good C-D SAP, the resource share and Pareto weight coincide

Note that $\rho_{i,h} = y_h \mu_{i,h} \left(\sum_{j \in \Omega^c} \alpha_{t,g}^j + \alpha_{t,g}^l \right)$. Also recall that:

$\eta_{i,h} = \frac{\rho_{i,h}}{\sum_{s \in h} \rho_{s,h}} = \frac{y_h \mu_{i,h} \left(\sum_{j \in \Omega^c} \alpha_{t,g}^j + \alpha_{t,g}^l \right)}{y_h \sum_{s \in h} \mu_{s,h} \left(\sum_{j \in \Omega^c} \alpha_{st,g}^j + \alpha_{st,g}^l \right)} = \frac{\mu_{i,h} \left(\sum_{j \in \Omega^c} \alpha_{t,g}^j + \alpha_{t,g}^l \right)}{\sum_{s \in h} \mu_{s,h} \left(\sum_{j \in \Omega^c} \alpha_{st,g}^j + \alpha_{st,g}^l \right)}$.

Now assume public good SAP so that $\left(\sum_{j \in \Omega^X} \alpha_{t,g}^j + \sum_{s \epsilon h} \alpha_{t,g}^{Tst,g} + \sum_{s \epsilon h} \alpha_{t,g}^{Dst,g} \right) = a_g^Q$

Equivalently, $\left(\sum_{j \in \Omega^c} \alpha_{t,g}^j + \alpha_{t,g}^l \right) = a_g^{c,l}$

Having made this assumption, we can write $\eta_{i,h} = \frac{\mu_{i,h} a_g^{c,l}}{a_g^{c,l} \sum_{s \in h} \mu_{s,h}}$. By definition, $\sum_{s \in h} \mu_{s,h} = 1$ and the $a_g^{c,l}$ terms cancel out, so that $\eta_{i,h} = \mu_{i,h}$.

In this case, finding that resource shares vary with market variables implies that Pareto weights do too, and hence is evidence in favour of the collective model over the unitary model.

F.3.2 Under public good C-D SAP, second-stage demands do not depend on public expenditure

Note that $l_{i,h} w_{i,h} = \frac{\mu_{i,h} \alpha_{t,g}^l}{\sum_{s \in h} \mu_{s,h} \sum_j \alpha_{st,g}^j} y_h$, where $\sum_j \alpha_{st,g}^j$is the sum of preference coefficients over all different goods, and was normalised to 1, and $\sum_{s \in h} \mu_{s,h} = 1$. Therefore, $l_{i,h} w_{i,h} = \alpha_{t,g}^l \mu_{i,h} y_{i,h}$. Without assuming public good C-D SAP, re-writing this equation in terms of resource shares, rather than Pareto weights, (this is necessary to be able to estimate the resource shares, which are our object of interest) requires including public expenditure $R_h Q_h$ as a term (or writing the latter in terms of bargaining fundamentals).

With public good C-D SAP, as discussed above, $\mu_{i,h} = \eta_{i,h}$ and hence we can write $l_{i,h} w_{i,h} = \alpha_{t,g}^l \eta_{i,h} y_h$. In this case, we are able to estimate resource shares from data that does not contain public good expenditure. Intuitively, this is driven by the assumption that changing bargaining power may affect how the household divides expenditure on specific public goods, but not on the aggregate public budget, so that we can ignore public good expenditure for the purpose of estimating resource shares.

F.4 Structural approach to the error term

It is good practice to include sources of noise in the structural model (see for instance Reiss and Wolak (2007) for an excellent discussion of this point). I suggest doing this in two ways here.

Firstly, it is unlikely that we can control for all characteristics that affect bargaining within the household. Even if we have many characteristics in our data, there are likely unobserved factors. Moreover, recall that we are using a linear approximation of the Pareto weight, and hence there is likely an approximation error. We can write:

$w_{i,h}l_{i,h} = \alpha^l_{t,g} y_h \left(\eta^0_{t,g} + \sum_z \eta^z_{t,g} \left(z_h - \bar{z}\right) + e_{i,h}\right)$

Where by definition $e_{i,h}$ is mean-zero. As long as and we assume $e_{i,h}$ is independent of y_h and z_h (or, more weakly, that it is uncorrelated with $y_h, y_h^2, z_h, y_h z_h$) then $\alpha^l_{t,g} y_h e_{i,h}$ is uncorrelated with the other terms. Note that, by definition, the shocks sum to zero within each household: $\sum_{i \in h} e_{i,h} = 0$. Hence in a two-person household this implies a correlation of -1 between the errors of members of a couple.

Secondly, there may be some optimisation error $u_{i,h}, u^l_{i,h}$ so that:

$w_{i,h}l_{i,h} = \alpha^l_{t,g} \left(y_h \left(\eta^0_{t,g} + \sum_z \eta^z_{t,g} \left(z_h - \bar{z}\right) + e_{i,h}\right) + u_{i,h}\right) + u^l_{i,h}$

We can think of it in these terms:

1. In the first stage of the household problem, the household makes optimisation errors in the division of resources between public goods and individual budgets. This means that a member's individual budget will deviate by $u_{i,h}$ relative to the optimal budget. These errors needn't sum to zero across household members' individual budgets since there is also scope for error in public good expenditure. Note that $\sum u_{i,h} + u_{X,h} = 0$ so that the optimisation errors in the first stage must sum to zero. The $u_{i,h}$ errors will be negatively correlated across household members, with a correlation weakly smaller in magnitude than -1.

2. In the second stage of the household problem, each individual member makes optimisation errors in the division of their budget between private goods for their personal consumption. This means that each private expenditure may deviate from optimal by $u^j_{i,h}$ where $\sum_{j \in \Omega_c} u^j_{i,h} = 0$. Specifically, leisure expenditure may deviate from optimum by $u^l_{i,h}$. These errors could potentially be correlated across members, but there is no clear reason why they would be.

For both sources of optimisation error, as is standard, assume they are mean-zero and uncorrelated with each other and all other variables.

Taking these three sources of error, the final model can be written as:

$w_{i,h}l_{i,h} = \alpha^l_{t,g} y_h \left(\eta^0_{t,g} + \sum_z \eta^z_{t,g} \left(z_h - \bar{z}\right)\right) + \left(\alpha^l_{t,g} y_h e_{i,h} + a^l_{t,g} u_{i,h} + u^l_{i,h}\right)$

Define $\epsilon_{i,h} = \alpha^l_{t,g} y_h e_{i,h} + a^l_{t,g} u_{i,h} + u^l_{i,h}$ so that $w_{i,h}l_{i,h} = \alpha^l_{t,g} y_h \left(\eta^0_{t,g} + \sum_z \eta^z_{t,g} \left(z_h - \bar{z}\right)\right) + \epsilon_{i,h}$ where $\epsilon_{i,h}$ is mean-zero and uncorrelated with all other regressors. The errors are negatively correlated within household (with a correlation that is negative but smaller in magnitude than -1), so that a SURE estimation approach is recommended.

F.5 Individual-level full consumption when expenditure data is separate from time-use data

Under Cobb-Douglas preferences, we can estimate time-inclusive individual-level consumption even when expenditure and time-use data are separate, as long as we observe hours worked $m_{i,h}$ and hourly wage $w_{i,h}$ in the expenditure data.[72] This is the case in many expenditure datasets which do not contain any other information relating to time-use. The procedure is simple:

- Recall that leisure expenditure is $w_{i,h}l_{i,h} = a^l y_h \eta_{i,h}$ and that we have estimated $\hat{\eta_{i,h}}$ and $\hat{\alpha^l}$ from the time-use data
- Hence we can estimate $l_{i,h}$ in the expenditure data as $\hat{l_{i,h}} = \frac{\hat{a^l} y_h \hat{\eta_{i,h}}}{w_{i,h}}$
- We therefore obtain expenditure on private leisure $w_{i,h}\hat{l_{i,h}}$

F.6 Back-of-the-envelope check

An advantage of working with preferences which have a simple direct utility representation is that we can substitute parameter estimates into indirect utility functions to check that their magnitudes (as well as their signs) are consistent with bargaining theory. Specifically, we can check whether an increase in own-wage implies an increase in own-utility.

First, I illustrate three example households: one characterised by average characteristics, one which deviates in terms of the female wage (to £15 an hour), and one which deviates in terms of the male wage (to £15 an hour). Comparison between the first and, in turn, the second and third, allows us to think about the impact of a change in female wage on female utility and a change in male wage on male utility. This impact is a combination of

[72] If the data has income from work $m_{i,h}w_{i,h}$ and hours worked $m_{i,h}$ then we can estimate $w_{i,h} = \frac{m_i w_{i,h}}{m_{i,h}}$. Where expenditure data only has higher level information, such as part-time or full-time work, employment status and sector, we can estimate hours worked based on those variables.

multiple channels: (i) a price effect on own private leisure, (ii) an income effect through overall household budget, (iii) an income effect through sharing, (iv) a price effect on joint leisure and domestic work, and (v) increased weight on own-preferences in the household's choices of the composition of the budget share devoted to public goods. Because I do not estimate the whole model, I cannot compute exact utility changes. Instead, I conduct a back-of-the-envelope check where I assume away the last two effects (which have opposite effects on utilities). Private leisure falls (the price effect dominates), but overall utility increases through increased material private consumption and increased expenditure on public goods.

Then I consider the general case and show that estimates are consistent with bargaining theory for reasonable variations in wages (I do not consider more extreme variations in wages since the linear approximation of the sharing rule entails that for very extreme wages things break down).

F.6.1 Three example households

The average household

- The daily budget is $y_h = (9.87 + 11.6) * 24 = 515.28$
- Under C-D SAP, a fixed proportion of household budget a^Q is spent on (material and time-use) public goods and enjoyed by both household members
  - We can conceptualise this expenditure in terms of a composite public good. The composition of this good varies with the distribution of bargaining power as more weight is assigned to goods preferred by the member with more bargaining power. Moreover, the price of public time (domestic work and joint leisure) depends on individual wages. Hence, we can think of the price of the aggregate good as $R(w_f, w_m)$, which for the average household would be $R(9.87, 11.6)$[73]
  - To illustrate, consider the example of $a^Q = 0.2$ (as explained further below, the results hold under almost any theory-consistent value of a^Q)
  - Then expenditure on public goods is $R(9.87, 11.6)Q_h = 0.2 * 515.28 = 103.06$
- Under the estimated preference parameter on private leisure (which is the same for men and women), the household spends a budget share equal to $\alpha^l = 0.44$ on private leisure.

[73] R depends on wages both as prices and as determinants of sharing. I omit other factors affecting sharing because they are kept constant at sample averages throughout this exercise.

- Hence the household spends a total on private leisure of: $w_m l_m + w_f l_f = 0.44 * 515.28 = 226.72$
- Because this is a private expenditure, it is assigned to each member according to the sharing rule
- For the average household, we estimate that the woman has a share of 0.44 and the man has a share of 0.55
- Therefore, expenditure on the woman's private leisure is: $w_f l_f = 0.45 * 226.72 = 102.03$
- Evaluated at the mean female wage, $l_f = \frac{102.03}{9.87} = 10.34$
- Expenditure on the man's private leisure: $w_m l_m = 0.55 * 226.72 = 124.70$
- Evaluated at the mean male wage, $l_m = \frac{124.70}{11.6} = 10.75$

- Total material private expenditure falls out of the budget, private leisure expenditure and public expenditure: $p_f c_f + p_m c_m = (1 - 0.2 - 0.44) * 515.28 = 185.50$
  - Similarly to the composite public good, we can think of a composite material private good for women and a composite material private good for men. Unlike public goods, the composition of each of these material private composite goods does not vary with wages because of the properties of Cobb-Douglas preferences, and hence the price is constant for the purpose of this exercise (the price of the woman's composite material private good may differ from the man's because they may have different preferences on different private material goods).
  - We assign total material private expenditure between members according to the sharing rule
  - Therefore, $p_f c_f = 185.50 * 0.45 = 83.48$
  - And $p_m c_m = 185.50 * 0.55 = 102.03$
  - Hence, $c_f = \frac{83.48}{p_f}$
  - And $c_m = \frac{102.03}{p_m}$

- Back-of-the-envelope simplification:
  - When a member's wage increases, this leads to three effects on their utility from public expenditure:
    - A price effect (the price of public time has increased) which lowers utility

    - ∗ A composition effect (the composition of the public good becomes more aligned with their preferences on public goods) which increases utility
    - ∗ An income effect (household budget increases) which increases utility
  - Since we do not estimate the whole model, we cannot quantify the first two effects. As they are opposite in direction, as a back-of-the-envelope simplification we assume them away (we can think of the price of the composite public good as a constant R, and of the man and woman having the same preference parameter for it).

- Substituting all of the above into Cobb-Douglas utilities over private leisure, an aggregate private material good, and an aggregate public (time-use and material) good, yields the following utilities:
  - The woman's utility is: $u_f^0 = 0.44\ln(10.34) + 0.2\ln(\frac{103.06}{R}) + 0.36\ln(\frac{83.48}{p_f})$
  - And the man's is: $u_m^0 = 0.44\ln(10.75) + 0.2\ln(\frac{103.06}{R}) + 0.36\ln(\frac{102.03}{p_m})$

The household characterised by average characteristics excepting that the woman earns £15 an hour instead of £9.87

To check consistency with bargaining theory, we evaluate the woman's utility in this scenario and compare it to her utility in the average household. We should find it is higher for the woman that earns a higher-than-average wage.

- Household budget: $y_h = (15 + 11.6) * 24 = 638.40$
- Public expenditure: $RQ_h = 0.2 * 638.40 = 127.68$
- Sharing rule: $\eta_f = 0.51$ (see section 4.2.1 for how to calculate the sharing rule based on household characteristics)
- Expenditure on total private leisure: $w_m l_m + w_f l_f = 0.44 * 638.40 = 280.90$
  - Expenditure on the woman's leisure is: $w_f l_f = 0.51 * 280.90 = 143.26$
  - Her private leisure is: $l_f = \frac{143.26}{15} = 9.56$
- Expenditure on material private goods: $p_f c_f + p_m c_m = (1 - 0.2 - 0.44) * 638.40 = 229.82$
  - Expenditure on the woman's private material goods is: $c_f = \frac{0.51 * 229.82}{p_f} = \frac{117.21}{p_f}$
- The woman's utility now is: $u_f^{HF} = 0.44\ln(9.56) + 0.2\ln(\frac{127.68}{R}) + 0.36\ln(\frac{117.21}{p_f})$

- Then $\Delta u_f = u_f^{HF} - u_f^0 > 0$
- Consistently with bargaining theory, the woman's wage increase has increased her utility

The household characterised by average characteristics excepting that the man earns £15 an hour instead of £11.6

Similarly, to check consistency with bargaining theory, we evaluate the man's utility in this scenario and compare it to his utility in the average household. We should find it is higher for the man that earns a higher-than-average wage.

- Household budget: $y_h = (9.87 + 15) * 24 = 596.88$
- Public expenditure: $RQ_h = 0.2 * 596.88 = 119.38$
- Sharing rule: $\eta_m = 0.57$
- Expenditure on total private leisure: $w_m l_m + w_f l_f = 0.44 * 596.88 = 262.63$
  - Expenditure on the man's leisure is: $w_m l_m = 0.57 * 262.63 = 149.70$
  - Her private leisure is: $l_m = \frac{149.70}{15} = 9.98$
- Expenditure on material private goods: $p_f c_f + p_m c_m = (1 - 0.2 - 0.44) * 596.88 = 214.88$
  - Expenditure on the man's private material goods is: $c_m = \frac{0.57 * 214.88}{p_m} = \frac{122.48}{p_m}$
- The man's utility now is: $u_m^{HM} = 0.44 \ln(9.98) + 0.2 \ln(\frac{119.38}{R}) + 0.36 \ln(\frac{122.48}{p_m})$
- Then $\Delta u_m = u_m^{HM} - u_m^0 > 0$
- Consistently with bargaining theory, the man's wage increase has increased his utility

F.6.2 General result

I now generalise the examples above to general changes in female and male wages under general values of a^Q

General result for women's utility

- $u_f = 0.44 \ln(\frac{0.44 \eta_f (w_f + w_m)}{w_f}) + a^Q \ln(\frac{a^Q (w_f + w_m)}{R}) + (1 - a^Q - 0.44) \ln(\frac{\eta_f (1 - a^Q - 0.44)(w_f + w_m)}{p_f})$
  - $\eta_f = 0.45 + 0.01(w_f - 9.87)$
    - $\frac{d\eta_f}{dw_f} = 0.01$

- We can re-write this as:

$$u_f = \ln(w_f + w_m) + (1 - a^Q)\ln(\eta_f) - 0.44\ln(w_f) + \left(0.44\ln(0.44) + a^Q\ln(\frac{a^Q}{R}) + (1 - a^Q - 0.44)\ln(\frac{(1 - a^Q - 0.44)}{p_f})\right)$$

- $\frac{du_f}{dw_f} = \frac{1}{w_f + w_m} + \frac{1-a^Q}{\eta_f}\frac{d\eta_f}{dw_f} - \frac{0.44}{w_f}$
- substituting in $\frac{d\eta_f}{dw_f}$ and η_f,
- $\frac{du_f}{dw_f} = \frac{1}{w_f + w_m} + 0.01\frac{1-a^Q}{0.45+0.01(w_f - 9.87)} - \frac{0.44}{w_f}$
- The condition we are interested in checking is: $\frac{1}{w_f + w_m} + 0.01\frac{1-a^Q}{0.45+0.01(w_f - 9.87)} - \frac{0.44}{w_f} \lesseqgtr 0$
- Recall we are thinking of a household with all other characteristics at average, including the male wage of $w_m = 11.6$
- $\frac{1}{w_f + 11.6} + 0.01\frac{1-a^Q}{0.45+0.01(w_f - 9.87)} - \frac{0.44}{w_f} \lesseqgtr 0$
- We evaluate this for three options of w_f to find which values of a^Q are consistent with bargaining theory in each case:
  - the lower quartile of the female wage (£6.6)
  - the mean female wage (£9.87)
  - the upper quartile of the female wage (£11.6)
- For a woman with a lower quartile wage:
  - $\frac{1}{6.6 + 11.6} + 0.01\frac{1-a^q}{0.45+0.01(6.6 - 9.87)} - \frac{0.44}{6.6} = \frac{1}{18.2} + \frac{1-\beta}{41.73} - \frac{2.2}{33}$
  - This value is consistent with bargaining if it is positive, which occurs for values of a^Q such that $\frac{1}{18.2} + \frac{1-a^Q}{41.73} - \frac{2.2}{33} > 0$
  - $a^Q < 0.51$
  - This parameter range almost certainly contains the true value of a^Q. Since the estimated share of household budget spent on private leisure is 0.44, if the household were to spend more than 51% of its budget on (time-use and material) public goods, then it would have less than 5% of the budget available to spend on material private goods, which is unrealistically low.
- For a woman with the mean wage of £9.87

- $\frac{1}{9.87+11.6} + 0.01\frac{1-a^Q}{0.45} - \frac{0.44}{9.87} \lessgtr 0$
- this is consistent with bargaining theory if
- $\frac{1}{21.47} + \frac{1-a^Q}{45} - \frac{0.44}{9.87} > 0$
- $a^Q < 1.09$
- This is definitionally the case

- For a woman with an upper quartile wage (£11.6):
  - $\frac{1}{11.6+11.6} + 0.01\frac{1-a^Q}{0.45+0.01(11.6-9.87)} - \frac{0.44}{11.6} \lessgtr 0$
  - This is consistent with bargaining theory if:
  - $\frac{1}{23.2} + \frac{1-a^Q}{46.73} - \frac{0.44}{11.6} > 0$
  - $a^Q < 1.24$
  - This is definitionally the case

General result for men's utility

- $u_m = 0.44\ln(\frac{0.44\eta_m(w_f+w_m)}{w_m}) + a^Q\ln(\frac{a^Q(w_f+w_m)}{R}) + (1-a^Q-0.44)\ln(\frac{\eta_m(1-a^Q-0.44)(w_f+w_m)}{p_m})$
  - $\eta_m = 0.55 + 0.005(w_m - 11.6)$
    - $\frac{d\eta_f}{dw_f} = 0.005$
  - simplifying assumption that $\frac{dR}{dw_m} = 0$
- $\frac{du_m}{dw_m} = \frac{1}{w_f+w_m} + \frac{1-a^Q}{\eta_m}\frac{d\eta_m}{dw_m} - \frac{0.44}{w_m}$
- substituting in $\frac{d\eta_m}{dw_m}$ and η_m,
- $\frac{du_m}{dw_m} = \frac{1}{w_f+w_m} + 0.005\frac{1-a^Q}{0.55+0.005(w_m-11.6)} - \frac{0.44}{w_m}$
- The condition we are interested in checking is: $\frac{1}{w_f+w_m} + 0.005\frac{1-a^Q}{0.55+0.005(w_m-11.6)} - \frac{0.44}{w_m} \lessgtr 0$
- Recall we are thinking of a household with all other characteristics at average, including the female wage of $w_f = 9.87$
- $\frac{1}{9.87+w_m} + 0.005\frac{1-a^Q}{0.55+0.005(w_m-11.6)} - \frac{0.44}{w_m} \lessgtr 0$
- As for women, we also check this for the lower quartile, mean, and upper quartile values of the male wage
- Lower quartile wage for men: £7.51

- $\frac{1}{9.87+7.51} + 0.005\frac{1-a^Q}{0.55+0.005(7.51-11.6)} - \frac{0.44}{7.51} \lesseqgtr 0$
- To be consistent with bargaining theory, we require:
- $\frac{1}{17.38} + \frac{1-a^Q}{105.91} - \frac{0.44}{7.51} > 0$
- $a^Q < 0.89$
- This is definitionally the case since the estimated budget share spent on private leisure is 0.44 and $0.89 + 0.44 > 1$

- Men's mean wage: £11.6
  - $\frac{1}{9.87+11.6} + 0.005\frac{1-a^Q}{0.55} - \frac{0.44}{11.6} \lesseqgtr 0$
  - To be consistent with bargaining theory, we require:
  - $\frac{1}{21.47} + \frac{1-a^Q}{110} - \frac{0.44}{11.6} \lesseqgtr 0$
  - $a^Q < 1.95$
  - This is definitionally the case

- Upper quartile wage for men: £13.1
  - $\frac{1}{9.87+13.1} + 0.005\frac{1-a^Q}{0.55+0.005(13.1-11.6)} - \frac{0.44}{13.1} \lesseqgtr 0$
  - To be consistent with bargaining theory, we require:
  - $\frac{1}{22.97} + \frac{1-a^Q}{111.5} - \frac{0.44}{13.1} \lesseqgtr 0$
  - $a^Q < 2.11$
  - This is definitionally the case

G Estimation and hypothesis testing

Under the usual assumptions, our regression coefficients (α_f^0, α_m^0) are consistent estimators of their population counterparts, i.e.

$$N \to \infty : (\alpha_f^0, \alpha_m^0) \to (\mu_{\alpha 0 f}, \mu_{\alpha 0 m})$$

The average female share is estimated as a function of these objects:

$$\eta_f^0(\mu_{\alpha 0 f}, \mu_{\alpha 0 m}) = \frac{\mu_{\alpha 0 f}}{\mu_{\alpha 0 f} + \mu_{\alpha 0 m}}$$

By the Continuous Mapping Theorem:

$$N \to \infty : \eta_f^0(\alpha_f^0, \alpha_m^0) \to \eta_f^0(\mu_{\alpha 0f}, \mu_{\alpha 0m})$$

The asymptotic distribution of the regression coefficients (α_f^0, α_m^0) is found by the Central Limit Theorem:

$$\sqrt{N}\begin{bmatrix} \alpha_f^0 - \mu_{\alpha 0f} \\ \alpha_m^0 - \mu_{\alpha 0m} \end{bmatrix} \xrightarrow{D} \mathcal{N}\big(0, \Sigma\big)$$

Where N is the sample size and $\Sigma = \begin{bmatrix} \sigma_{\alpha 0f}^2 & \sigma_{\alpha 0fm}^2 \\ \sigma_{\alpha 0fm}^2 & \sigma_{\alpha 0m}^2 \end{bmatrix}$

As long as the first two moments of this distribution are finite, and that the Jacobian of the resource share as a function of the regression coefficients exists and is non-zero valued, we can apply the delta method to find the asymptotic distribution of the estimated resource share:

$$\eta_f^0(\alpha_f^0, \alpha_m^0) \xrightarrow{D} \mathcal{N}\left(\eta_f^0(\mu_{\alpha 0f}, \mu_{\alpha 0m}), \frac{\delta \Sigma \delta'}{N}\right)$$

Where:

$$\delta = \left[\frac{\delta \eta_f^0(x,y)}{\delta x}\bigg|_{x=\mu_{\alpha 0f}, y=\mu_{\alpha 0m}} \quad \frac{\delta \eta_f^0(x,y)}{\delta y}\bigg|_{x=\mu_{\alpha 0f}, y=\mu_{\alpha 0m}}\right] = \left[\frac{\mu_{\alpha 0m}}{\big(\mu_{\alpha 0f}+\mu_{\alpha 0m}\big)^2} \quad \frac{-\mu_{\alpha 0f}}{\big(\mu_{\alpha 0f}+\mu_{\alpha 0m}\big)^2}\right]$$

We can construct the confidence interval as:

$$\left[\eta_f^0(\mu_{\alpha 0f}, \mu_{\alpha 0m}) \pm c\sqrt{\frac{\delta \Sigma \delta'}{N}}\right]$$

Since we do not know the population parameters of the distributions, we substitute consistent estimators for the parameters in the δ vector and in the variance-covariance matrix. Hence, we use the following estimates to construct the confidence interval:

$$\hat{\delta} = \left[\left. \frac{\delta \eta_f^0(x,y)}{\delta x} \right|_{x=\alpha_f^0, y=\alpha_m^0} \quad \left. \frac{\delta \eta_f^0(x,y)}{\delta y} \right|_{x=\alpha_f^0, y=\alpha_m^0} \right] = \left[\frac{\alpha_m^0}{\left(\alpha_f^0+\alpha_m^0\right)^2} \quad \frac{-\alpha_f^0}{\left(\alpha_f^0+\alpha_m^0\right)^2} \right]$$

And we use the variance-covariance matrix for our regression coefficients:

$$\frac{\hat{\Sigma}}{N} = \begin{bmatrix} \hat{\sigma_{\alpha 0 f}^2} & \hat{\sigma_{\alpha 0 f m}^2} \\ \hat{\sigma_{\alpha 0 f m}^2} & \hat{\sigma_{\alpha 0 m}^2} \end{bmatrix}$$

We compute the 95% confidence interval for the average female resource share as follows:

$$\left[0.4487294 \pm 1.96\sqrt{1.971e-05} \right] = \begin{bmatrix} 0.4400278 & 0.4574310 \end{bmatrix}$$

To estimate confidence intervals for resource shares in Stata it is possible to use the command rnlcom as an alternative to coding the confidence intervals based on the methodology shown in this appendix.