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Focal pricing and pass-through



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Abstract

The degree of pass-through of input cost changes is relevant in several contexts, including estimating antitrust damages and evaluating the impact of changes in taxation. In recent antitrust cases it has been alleged that the adoption of focal pricing by firms reduces the degree of pass-through in an industry. I show that this claim is not theoretically sound by outlining a simple model where focal pricing leaves expected pass-through unaffected. However, focal pricing does lead to more lumpiness in the distribution of pass-through. This paper reinforces the importance of context-specific empirical analysis to determine the degree of pass-through in an industry, regardless of the presence of focal pricing.

1 Introduction

Focal pricing is a widely observed phenomenon, consisting in firms only charging prices with specific characteristics. These are often prices with 9s in the last digits, as noted e.g. by Levy et al (2011), and Snir, Levy and Chen (2017). Authors have offered different explanations for this phenomenon, including behavioural e.g. Strulov-Shlain (2019); rational inattention, e.g. Basu (1997); and tacit collusion, e.g. Scherr (1981). Knotek (2008, 2011) shows 'convenient' prices (multiples of cash denominations) are widespread for frequently purchased goods paid in cash in high-traffic transactions. Moreover, virtually all firms employ focal pricing to some degree, because money denominations constrain most prices to being multiples of pence.

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The presence of focal pricing has been used in high-profile antitrust cases to argue against estimated pass-through rates, based on the notion that if an industry adopts focal pricing that in itself implies there will be very little, if any, pass-through of input cost changes. The argument is that if firms round to certain special prices, they are unlikely to adjust their prices in response to small input cost changes.

In re Lithium Ion Batteries antitrust litigation, the Court struck out the case, amongst other reasons, because the defendants argued that the plaintiffs' expert analysis of pass-through did not take into account focal pricing. The defendants alleged that "focal point pricing is prevalent in the pricing of products within the class definition, and will result in no passthrough when a small cost change—such as the estimated \$2.16 overcharge for a notebook computer battery here—in presence of focal points that are wider apart than the cost difference itself." A similar argument has been made by Qualcomm in antitrust litigation in several jurisdictions.¹

While this argument has intuitive appeal, it fails to recognise the other side of the coin: in the presence of focal pricing, some prices will be overadjusted if the input cost change leads to a jump from one focal price to another. This is confirmed empirically, e.g. Levy et al (2011) find price changes are less frequent, but bigger in magnitude, with focal pricing. Also, see Conlon and Rao (2020) for examples of high pass-through in industries which adopt focal pricing.

While there is empirical evidence that pass-through can still be high in contexts with focal pricing, little attention has been devoted to theoretical modelling of this issue. The likely reason is that many models of competition become highly complex when pricing is discrete. Filling this theoretical gap is important to complement the existing empirical evidence and show that it is not safe to assume that the adoption of focal pricing will reduce passthrough relative to a similar context without focal pricing.

In this paper I illustrate a simple model of pass-through of input cost changes. This framework applies to several models of competition which are frequently employed in practical settings, including monopoly, perfect competition, undifferentiated Bertrand, and tacit or explicit oligopolistic collusion. I show that in this model average pass-through is the same regardless of the adoption of focal pricing ('the Irrelevance Theorem'). Where this framework is considered unrealistic, it is still not safe to assume that

¹e.g. United States District Court Northern District of California San Jose Division (2018), In re: Qualcomm Antitrust Litigation, Defendant Qualcomm Incorporated's Opposition to Plaintiff's Motion for Class Action, September 27, Case No. 5:17-md-02773-LHK-NMC.

focal pricing reduces pass-through, as the degree of pass-through may be affected in complex ways by the adoption of focal pricing, depending on the precise industry structure and type of competition. There is no general theoretical reason to believe average pass-through will be lower with, than without, focal pricing.

I then turn to the distribution of pass-through across products, which becomes lumpy with focal pricing. I characterise the drivers of this lumpiness and the contexts in which it may be important to explicitly consider whether an industry is characterised by focal pricing.

These contributions have numerous applications, in particular to tax incidence and antitrust damages. Where possible, the degree of average pass-through should be assessed empirically for the specific industry of interest. Additionally, where the distribution of pass-through is important, it will be useful to consider the context-specific degree of likely pass-through lumpiness. Where it is likely to be high, additional analysis is warranted to estimate more granular pass-through rates, e.g. for specific categories of products or groups of individuals.

2 A simple model

Consider a stylised model of the market of interest. There is a single firm selling a single good. The monopolist can only change the price of the good, not any non-price characteristics. The firm faces a constant marginal cost c and a linear, continuous latent demand function q(p), where q is quantity sold. Hence the monopolist's optimisation problem is:

$$argmax_p(q(p)(p-c))$$

The profit function is maximised at the optimal price and decreases symmetrically in both directions, so that:

$$\pi(p^* + x) = \pi(p^* - x)$$

We can imagine that in the presence of frictions, such as currency denominations and cognitive costs, the latent continuous demand function is mapped onto a step-wise realised demand function (focal demand function), as illustrated in Figure 1. Reducing frictions leads to a smoother demand function, so that the concept of the latent continuous demand function is not empty, but represents the demand function that consumers tend towards as frictions decrease. Under focal demand, consumers faced by a price below a focal price but above the next one demand the same quantity as if the higher focal price was charged.² In this case, the monopolist has no incentive to charge the latent optimal price³ because it could charge the higher focal price without demand being affected. The firm will either do this, or select the next lower focal price, depending on the trade-off between charging a higher price and facing lower demand. Since profits are symmetric around the latent optimal price, the firm will choose to round to the closest focal price.

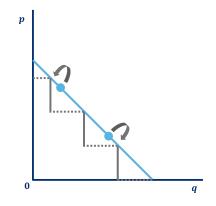


Figure 1: A monopolist facing a focal demand function

2.1 Other stylised models

The monopoly case also applies to oligopolistic settings with tacit or explicit collusion. Let us now consider the polar opposite in terms of market concentration: perfect competition. This framework also applies to oligopolistic settings with undifferentiated Bertrand competition.

In this case, faced with a smooth demand function, the firms set price equal to marginal cost. If faced with a focal demand function, firms will round up to the closest focal price above marginal cost. They cannot sustainably charge the next focal price below marginal cost, because they would incur negative profits. They have no incentive to deviate to any non-focal price because it does not alter the demand for their product and simply lowers their profit.

 $^{^2\}mathrm{Results}$ are similar if we consider consumers who behave as if the lower focal price was charged.

 $^{^3 \}rm{Unless}$ it happens to coincide with a focal price, which happens with probability 0 since the firm optimises over a continuous domain.

2.2 A focal pricing framework

Under any of the aforementioned industry structures, we can think of the firm's optimisation problem as a two-step process. In the first step, the firm faces the 'standard' continuous profit-maximisation problem. In this step, the firm chooses a latent optimal price $p^*(c)$. In the absence of frictions, this is the price it sets. Instead, in the presence of frictions, it proceeds to a second step; focal pricing. This consists in a set of cut-off rules whereby the outcome of the first step is mapped onto a value from a discrete range. If $p^*(c)$ is above a threshold price (t) but below the successive one, a specific focal price (f) is charged:

$$t_i < p^*(c) \le t_{i+1} \implies f^*(c) = f_{i+1}$$

The outcome of the two-step optimisation problem is therefore the optimal focal price $f^*(c)$. This is exemplified in Figure 2 below. In this example, prices are rounded to the nearest price ending in 9.

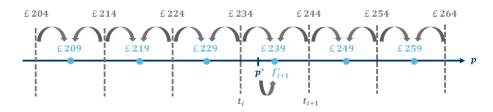


Figure 2: Example of the framework

We are interested in comparing pass-through in the latent underlying problem and in the case characterised by focal pricing. In the case of perfect competition (or Bertrand competition) the latent optimal price is the marginal cost, so a unit increase in marginal cost leads to a unit increase in the latent optimal price. Instead, in the monopolistic (or collusive) setting, a unit increase in the marginal cost leads to a 0.5 increase in the latent optimal price. Regardless of the pass-through from marginal cost to latent optimal price, here the question of interest is whether, in the presence of focal pricing, a unit increase in latent optimal price is mapped onto a unit increase in charged focal price, or to a higher or lower increase.

3 Expected pass-through: an Irrelevance Theorem

3.1 Assumption Set A

Under these mild assumptions, expected pass-through is unaffected by the presence and extent of focal pricing:

- 1. Regularity condition on focal prices. The focal prices are at regular intervals, so that the distance between each consecutive focal price is consistent: $f_{i+1} f_i = G$. This is likely to cover the vast majority of real-life cases which, as discussed above, involve consistently rounding to prices ending in specific digits, generally 9s. Moreover, the discretisation of prices due to currency limitations is also an example of regular focal pricing.
- 2. Regularity condition on thresholds. The thresholds are also spaced out at the same regular intervals as the focal prices, so that: $t_{i+1} - t_i = f_{i+1} - f_i = G$. The thresholds could be symmetric within an interval (equivalent to rounding to the nearest focal price), at the lower or upper bound (equivalent to rounding up or down to the nearest focal price), or anywhere else within their interval. This assumption is more restrictive, but still applies in many stylised IO models. It always holds for perfect competition or undifferentiated Bertrand. In a monopoly (or collusive) context it applies as long as demand is approximately linear.⁴
- 3. Uniformity condition. If we observe a firm charging the focal price f_{i+1} we can infer that $f^*(c) = f_{i+1}$ and therefore that $t_i < p^*(c) \le t_{i+1}$. However, we do not know where the optimal latent price lies within this interval. This is important because if we had detailed information on firm decision-making we would know exactly how a specific input cost change affected the charged downstream price. In general this type of information is not available, so our reasoning is based on observed, charged focal prices, and a mapping between these and unknown latent optimal prices. We can formalise this uncertainty by assuming that the latent optimal price underlying observed focal price f_{i+1} is uniformly

⁴Recall that in the simple monopoly model set out above, profits are symmetric around the latent optimal price. This is driven by the linearity in demand. With non-linear demand profits could be asymmetric around the latent optimal price in a manner which varies with the latent optimal price.

distributed over the interval (t_i, t_{i+1}) :⁵

 $p^* \sim U(t_i, t_{i+1})$

I prove the Irrelevance Theorem in section 3.3. Prior to that, in section 3.2, I I illustrate an example of the Irrelevance Theorem, to sketch out the intuition behind the formal proof.

3.2 An example

Consider a firm that rounds latent optimal prices to the nearest price ending in 9. This might be driven by consumer inattention to the tens digit, so that demand is the same whether the price is 244 or 249, and hence the firm has no incentive to charge any prices that do not end in 9. In this example, the firm rounds to the nearest focal price, as illustrated in Figure 3. Note that the regularity assumptions are met. Let us also assume that the firm can only adjust prices, so we can assume the uniformity condition.

In this example, focusing on the interval [239, 249) for illustration:

- $p^* \in [234, 244) \implies f^* = 239$
- $p^* \in [244, 254) \implies f^* = 249$

By the uniformity assumption, $p^* \sim U(239, 249)$. Hence, following a unit increase in the latent optimal price (due to an input cost change), i.e. $p2^* = p1^* + 1$, there are three possible scenarios:

• Scenario 1: the price was below the threshold and remains below the threshold, so that the focal price charged is unchanged:

$$p1^* \in [239, 243) \implies p2^* \in [240, 244)$$

The focal price charged is still $f2^* = f1^* = 239$, i.e. £0 pass-through. Because of the uniformity assumption, this scenario materialises with $prob(p1^* \in [239, 243)) = \frac{4}{10}$

⁵The uniformity assumption is realistic when firms cannot easily adjust non-price characteristics (such as quality, pack-size, components included in a bundle..), or cannot do so in a near-continuous way. When firms are able to adjust non-price characteristics in a near-continuous way, we might expect them to adjust them so that the latent optimal price is very close to, or equal to, a focal price, to reduce the loss due to focal-point-pricinginducing frictions. In this case, we would expect the latent price distribution to be bunched around focal prices, rather than uniform. However, by the same logic, following a change in input costs, a firm would be able to re-optimise both price and non-price characteristics, so that we would expect the holistic pass-through, capturing characteristic-adjusted prices, to be the same regardless of the presence of focal pricing. Hence, when non-price adjustments are possible, it is likely that the Irrelevance Theorem still holds.

• Scenario 2: the price was below the threshold but then crosses, so that the charged price jumps to the next focal price:

$$p1^* \in [243, 244) \implies p2^* \in [244, 245)$$

The focal price jumps from $f1^* = 239$ to $f2^* = 249$, i.e. £10 pass-through.

This case happens with $prob(p1^* \in [244, 245)) = \frac{1}{10}$

• Scenario 3: the price was above the threshold and remains above the threshold, so that the focal price charged is unchanged:

$$p1^* \in [244, 249) \implies p2^* \in [245, 250)$$

The focal price charged is still $f2^* = f1^* = 249$, i.e. £0 pass-through. This case happens with $prob(p1^* \in [244, 249)) = \frac{5}{10}$

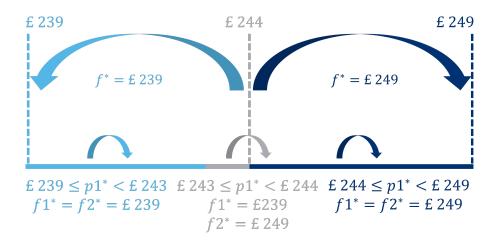


Figure 3: Impact of a £1 increase in the latent optimal price on the focal price charged

There is a $\frac{9}{10}$ probability of a £0 jump, and a $\frac{1}{10}$ chance of a £10 jump. Therefore, the expected change in the charged focal price is equal to the change in the underlying latent optimal price: $E[f2^* - f1^*] = p2^* - p1^* =$ 1. The amount passed-through is the same, in expectation, regardless of whether focal pricing is adopted or not.

Now consider the same context, but an input price change of $\pounds 11$. With certainty, the focal price paid will increase by at least $\pounds 10$ (the gap between

thresholds) because the latent optimal price has increased by more than that gap. Then, similarly to before, there is a $\frac{9}{10}$ chance of no additional increase in the focal price charged, and a $\frac{1}{10}$ chance of an additional £10 jump.

3.3 Proof

More generally, define G as the gap between thresholds (and focal prices). Then we can characterise the expected increase in the charged focal price due to a Δ increase in the latent optimal price as follows:

- $\Delta < G \implies E[f2^* f1^*] = \frac{\Delta}{G}G = \Delta$
- $G < \Delta < 2G \implies E[f2^* f1^*] = G + \frac{\Delta}{G}G = \Delta$ i.e. there is at least one jump to a higher focal price, and potentially two
- $2G < \Delta < 3G \implies E[f2^* f1^*] = 2G + \frac{\Delta}{G}G = \Delta$ i.e. there are at least two consecutive jumps to higher focal prices, and potentially three
- etc.

Therefore, the expected pass-through is the same irrespective of whether there is focal pricing or not, and irrespective of the magnitude of the gaps between focal prices: $E[f2^* - f1^*] = p2^* - p1^* = \Delta$. More formally:

Theorem 1. Under assumption set A, if the input cost increases from c to c + x,⁶ the expected increase in focal price is the same as the expected increase in the latent optimal price, i.e. focal pricing is irrelevant to the expected pass-through of the input cost change.

Let the firm's input cost increase from c to c+x. Consider that this leads to an increase in the optimal latent price from $p^*(c)$ to $p^*(c+x) = p^*(c) + \Delta$, where typically $\Delta < x$. Hence, in the absence of focal pricing, the amount of input cost passed through is $p^*(c+x) - p^*(c) = \Delta$.

By the Regularity Assumption, the price charged will jump by $\lfloor \frac{\Delta}{G} \rfloor$ focal prices with certainty where $\lfloor \rfloor$ is the floor operator. Still by the Regularity Assumption, each of these jumps entails a price change of G, i.e. the size of the gap between focal prices. Therefore, the focal price will increase by $\lfloor \frac{\Delta}{G} \rfloor G$ with certainty.

Additionally, by the Uniformity Assumption and Regularity Assumption, there is a $\frac{\Delta modG}{G}$ probability of a further jump in focal price (where *mod* is

⁶This is a decrease in cost where c < 0.

the modulo function). By the Regularity Assumption, this further jump, if it occurs, would lead to an additional increase of G in the charged price.

In expectation, the input price increase therefore leads to the following change in the focal price charged:

$$E[f^*(c+x)] - E[f^*(c)] = \lfloor \frac{\Delta}{G} \rfloor G + \frac{\Delta modG}{G}G = G(\lfloor \frac{\Delta}{G} \rfloor + \frac{\Delta modG}{G})$$

By definition of the modulo and floor operators:

$$E[f^{*}(c+x)] - E[f^{*}(c)] = G(\frac{\Delta}{G}) = \Delta = p^{*}(c+x) - p^{*}(c)$$
 QED

3.4 Average pass-through

In empirical contexts, we are often interested in estimating the *average* pass-through in an industry following an input cost change. I have shown that, in expectation, pass-through is unaffected by the presence and extent of focal pricing. Hence, *in expectation*, average pass-through is the same regardless of the presence and extent of focal pricing. If we are interested in a context where several different prices adjusted to input cost changes over time, then the law of large numbers is likely to apply, and the average pass-through is likely to be close to this expected pass-through.⁷

The nuance is that, with focal pricing, the realisation of average passthrough is more lumpy than in its absence. In the extreme, if we are interested in a single firm and a single product, then returning to the example in section 3.2, pass-through would, with certainty, be £1 in the absence of focal pricing. However, with focal pricing, it might be £0 or £10. If we are interested in an empirical context characterised by a small number of price changes, the average pass-through will be between £0 and £10. It may happen to be close to £1, but it may also happen to be closer to one of the extremes.

Hence in these cases average pass-through is not equal to what it would have been without focal pricing. If we have the possibility of conducting empirical analysis specific to the context of interest, then it is likely the

⁷This holds under two conditions. The first is that we are interested in a context where several different prices adjusted to input cost changes. The second is that these various prices were independently distributed - which is unlikely for a cross-section of prices of different products at one point in time, but more likely if considering different products over time.

most appropriate way of estimating average pass-through. If we do not have that possibility, we can rely on the expected pass-through based on similar contexts, irrespective of the degree to which they are characterised by focal pricing.

Note that if we knew the magnitude of the change in the latent optimal price Δ , and we knew it to be an exact multiple of the gap between thresholds G then we would know with certainty that the charged focal prices will also increase by the same amount Δ so that there is no lumpiness even if we are interested in a single price change. However, in practice, we are unlikely to know what the change in the underlying latent optimal price is.

4 Distribution of pass-through: focal lumpiness

Although focal pricing does not affect expected pass-through, it introduces lumpiness in the distribution of pass-through. In the example in section 3.2, without focal pricing, pass-through was £1 for any initial latent price, while with focal pricing it was £10 for some, and £0 for other, initial latent prices.⁸

Heterogeneity in pass-through across consumers can be of interest for different reasons. For instance, policy-makers may be concerned with VAT incidence differing widely between consumers, especially if there is reason to think lower-income consumers may be those shouldering a disproportionate share. Similarly, in consumer class actions, fairly awarding damages from anti-competitive behaviour involves considering whether the degree of pass-through is likely to have been approximately homogenous within the consumer group, or whether it is possible to estimate who suffered more damage relative to others. This application is of particular interest since a class action could be thrown out on the basis that the consumer group is not sufficiently homogenous.⁹

⁸Note that even in the absence of focal pricing, pass-through rates may differ for different products - my focus is whether focal pricing substantially adds to this heterogeneity.

 $^{^9{\}rm For}$ instance, focal pricing was one of two key arguments Qualcomm presented in its defence in the 2018 US Class Certification.

See: United States District Court Northern District of California San Jose Division (2018), In re: Qualcomm Antitrust Litigation, Defendant Qualcomm Incorporated's Opposition to Plaintiff's Motion for Class Action, September 27, Case No. 5:17-md-02773-LHK-NMC.

4.1 The degree and drivers of lumpiness

Firstly, note that focal pricing introduces at most a difference of G (the gap between focal prices) between high and low pass-through cases, as the lumpiness is driven by whether an additional jump in focal price was reached or not. Hence, the closer focal prices are, the smaller the degree of heterogeneity in pass-through introduced by focal pricing.

Consider the example of a large marginal cost increase which increases optimal latent prices by £10.50 in an industry where prices are rounded to numbers ending in 99 pence. Here the gap between thresholds, and focal prices, is: G = 1. The expected pass-through is equal to the change in latent optimal price: $\Delta = 10.50$. The smallest possible increase in charged price is £10 while the largest possible increase is £11. Then the maximum variation in impact on prices between different products, relative to the average impact, is $\frac{1}{10.50}$.

We can capture pass-through heterogeneity with the G-to- Δ ratio (where Δ is the expected pass-through). The higher this metric, the more important to explicitly model focal pricing. For instance, the impact of a very small change in marginal cost, in an industry where focal prices are far apart from each other, is likely to be very unevenly distributed. Instead, when a substantial change in marginal cost occurs in an industry with focal prices which are close to each other, the heterogeneity in impact will be small relative to the average impact.

Secondly, when consumers purchase a variety of products at different times, the expected pass-through is likely a close estimate for each individual consumer. This is because, assuming independent draws, the law of large numbers applies for multiple purchases, so that the likelihood of a consumer having several below-average (or above-average) pass-through purchases decreases with the number of purchases. Hence, even if the G-to- Δ metric is high, if the number of purchases is large, most consumers are likely to have experienced pass-through close to the average. For instance, average pass-through is likely to be a sufficient metric when assessing how to change a value-added tax (as this affects a very large number of goods over a long time period).

4.2 Non-price adjustments

Finally, sometimes firms can adjust other product characteristics (quality, pack-size, components included in bundles...), as well as prices, in response to input cost changes. In this case, the relevant measure of pass-through is

holistic, capturing both the monetary pass-through I have focused on, and other changes in product characteristics. This holistic pass-through is less lumpy than monetary pass-through, as firms have additional dimensions over which to optimise profits, some perhaps near-continuous, other than the discrete price set. If firms optimise over non-price dimensions, they are likely to adjust those product characteristics to ensure their latent optimal price is close to the charged focal price. Hence, in contexts where firms are able to optimise over a range of non-price dimensions, as well as price, and where any of these dimensions is approximately continuous, concerns about focal pricing leading to lumpy pass-through will be less pressing.

5 A note on empirical estimation

Empirical estimation of average pass-through can be conducted similarly regardless of the extent to which focal pricing affects an industry. If price points are far from each other, then it may be preferable to use regression approaches for discrete dependent variables. Allegations that the presence of focal pricing undermines the reliability of regressions estimating passthrough are not grounded in economic theory. In the presence of focal pricing there is more heterogeneity underlying the average pass-through, but that average can still be estimated by the usual econometric methods. Where the heterogeneity in pass-through is of interest, that can also be estimated, for instance by quantile regression.

6 Conclusions

This paper shows that expected pass-through is unaffected by the adoption or extent of focal pricing. This result holds for several models of competition often used in practical contexts, including antitrust cases. This theoretical finding complements existing empirical evidence that pass-through is not necessarily decreased by the adoption of focal point pricing strategies.

Therefore, it is not safe to assume that, simply because an industry adopts focal point pricing, pass-through will be low. It is crucial to estimate pass-through on a case-by-case basis, and to come to data-driven conclusions on the degree of pass-through in a specific context.

This paper also speaks to the question of the extent to which focal pricing introduces additional lumpiness in the distribution of pass-through for different products or consumers. The distribution of pass-through is likely to be approximately smooth, so that the average pass-through is the key metric, when: the G-to- Δ ratio is low; and/or firms can adjust non-price characteristics; and/or with multiple purchases. When none of these conditions obtain, pass-through is lumpy. If pass-through distribution, beyond the average, is important to the question of interest, explicitly modelling focal pricing may be beneficial to differentiate between high and low passthrough cases. In practice, the informational requirements for this type of analysis may be high, so it may still be appropriate to consider the average pass-through the best feasible estimate.

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