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Working paper

24/30

Intrahousehold welfare: Theory and application to Japanese data

Economic and Social Research Council

Intrahousehold Welfare: Theory and Application to Japanese Data*

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June 25, 2024

Abstract

In this paper we develop a novel approach to measuring individual welfare within households, recognizing that individuals may have both different preferences (particularly regarding public consumption) and differential access to resources. We construct a money metric measure of welfare that accounts for public goods (by using personalized prices) and the allocation of time. We then use our conceptual framework to analyse intrahousehold inequality in Japan, allowing for the presence of two public goods: expenditures on children and other public goods including housing. We show empirically that women have much stronger preferences for both public goods and this has critical implications for the distribution of welfare in the household.

*Acknowledgements: Pierre-André Chiappori gratefully acknowledges financial support from the NSF (grant 1124277). Costas Meghir is grateful for financial support by the Cowles foundation and the Institution for Social and Policy Studies at Yale. Moreover, we thank Tony Atkinson, Francois Bourguignon, Marc Fleurbaey and Frederic Vermeulen for useful comments on a previous version. The usual disclaimer applies.

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1 Introduction: individual and household welfare

When studying gender disparities in standards of living the literature has appropriately focused on the pay gap and on differences in educational and professional opportunities. However, a neglected but key element for understanding the gap between men and women is the distribution of welfare within the household. This explains the way differences in labor market opportunities translate to differences in consumption and utility.

In this paper we specify and estimate a collective model of consumption, labor supply and public goods aimed at understanding and measuring how welfare is distributed in Japanese households. The focus on Japan is important because it significantly lags behind peer industrialized nations in terms of gender equality, with one of the largest pay gaps and one of the toughest glass ceilings in the OECD. Indeed, data show married women consume about half the private consumption than men, and appear to be almost entirely responsible for household work; and yet they still marry and the divorce rate is particularly low.[1]

What is missing from this discussion are household public goods: it turns out that these are key to obtaining a better picture of the distribution of welfare. Whether public goods improve the dismal picture one gets from the distribution of private consumption and housework crucially depends on whether preferences for the household public goods are aligned within the couple; consequently identifying these preferences is an important focus of empirical work. The other central element is bargaining power and the way this varies with economic conditions.

To capture these elements we build on the Collective approach of Chiappori (1988, 1992) and its extension to public goods in Blundell, Chiappori, and Meghir (2005) and Chiappori and Ekeland (2009a). The approach assumes intrahousehold allocations are efficient. Based on relatively weak assumptions on preferences and on the observability of a private good, we are able to recover the sharing rule for private goods, once public goods have been decided and accounted for. Importantly we are also able to recover individual preferences, including for the public goods, which are of key

[1] According to the Ministry of Health, Labour, and Welfare, the number of divorces per 1,000 population is 1.57 in 2020.

importance for identifying how individual welfare is distributed. Our model includes private consumption, labor supply and two public goods, namely expenditures on children and expenditures on other household related public goods. The distinction between the two is important because preferences may be aligned for one public good but not for the other. For example, mothers may place a greater weight on children than fathers and consequently an increase in expenditure on children may increase her relative welfare. Our paper thus also speaks to the literature that focuses on how parental decisions are made for allocations to children, with implications for child development (see Attanasio, Cattan, Fitzsimons *et al.*, 2020; DelBoca, Flinn, and Wiswall, 2014).

For our analysis we use the Japanese Panel Survey of Consumers (JPSC). This is a panel over the years 1993-2020. The panel includes detailed information on household consumption, labor supply, income and demographic characteristics. The time and spatial dimension of the data allows us to use price variation to identify preferences over goods. And the detailed demographic and earnings information allows the estimation of wage equations, which are central to identifying labor supply effects.[2]

Existing literature Our paper lies at the intersection of two strands of the literature. On the one hand, a recent literature analyzes intrahousehold allocation of welfare. Browning, Chiappori, and Lewbel (2013) introduced the notion of indifference scales. These are defined, for each individual within the household, as the income level this person would need, as a single, to reach the same welfare level as they currently achieve within the household. Empirical applications include Dunbar, Lewbel, and Pendakur (2013), Calvi, Penglase, Tommasi *et al.* (2023), Lechène, Pendakur, and Wolf (2022) and Calvi (2020) among many others. A crucial feature of this approach, however, is that it relies on the assumption that each commodity is privately consumed within the household – although some consumptions may display economies of scale, that the approach allows to identify.

[2] Necessary and sufficient conditions for a demand function to stem from a collective framework are in (Chiappori and Ekeland, 2006); Identification is discussed in (Chiappori and Ekeland, 2009b). To the best of our knowledge, the collective model is the only model of the household for which such results have been derived. Browning, Chiappori, and Lechène (2010), Lechène and Preston (2011) and Chiappori and Naidoo (2020) provide a set of necessary conditions for non cooperative models. However, whether these conditions are sufficient is not known; moreover, no general identification result has been derived so far.

Our approach, on the contrary, explicitly recognizes that a large fraction of household expenditures relates to public commodities, whether purchased or produced in the household - i.e., goods that are jointly consumed by the household, without anyone being excluded. Crucially, spouses may have different preferences regarding public goods; therefore, the fraction of household expenditures devoted to public consumption has a potentially important (and differentiated) impact on individual welfare that cannot be ignored.

We introduce the notion of *Money Metric Welfare Index* (MMWI), a direct generalization of indifference scales that allows for public consumption. Specifically, an individual's MMWI is again defined as the the income level this individual would need, as a single, to reach the same welfare level as they currently achieve within the household. The difference, however, is that in the hypothetical, counterfactual situation of singlehood, individuals would have to pay for the full amount of public consumption they choose *at market price* - whereas public consumption within the household only requires each individual to pay for the public consumption *at their own Lindahl price* (and efficiency requires individual prices to add up to the market price of the commodity under consideration). In other words, the MMWI approach not only recognizes that household consumption is equivalent to a reduction in the price of some commodities, but allows these reductions to be *individual-specific* (and therefore to depend on individual preferences for public consumptions). In particular, if women care more than men for a specific public good, then any increase in the consumption of that good has a differential effect on husband's and wife's welfare that is not captured by the sole analysis of private consumption.

In this paper we present an approach to measuring the distribution of intrahousehold welfare that is explicitly based on the notion of MMWI. Non parametric identifiability of a model of this type is established by Chiappori and Ekeland (2009a). However, this paper is, to the best of our knowledge, the first to explicitly apply these notions to data.

Another related paper, due to Lise and Yamada (2019), specifies a dynamic model of labor supply, private consumption and a public good, produced by expenditure on a market good and by time inputs. Intrahousehold allocations are governed by a dynamic collective limited commitment model (see Mazzocco, 2007; Voena, 2015). The authors also use the JPSC; they exploit a unique feature of this data that reports how much each household spends on each of its members. This provides researchers with direct observation of intrahousehold resource allocation, which would not otherwise be observed. This allows direct identification of the sharing rule and the way it changes with external shocks.

Our paper differs from Lise and Yamada (2019) in some important ways. Our model is static, ignoring dynamic aspects of marriage but relying only on an ex post efficiency assumption; we are therefore agnostic about commitments issues. Moreover, we place more emphasis on public goods, by distinguishing expenditures on children from other household public goods and identifying individual preferences over these goods, consumption and leisure. Last but not least, our approach allows for public goods to affect welfare in an individual-specific manner. Our money metric welfare index (MMWI) can provide a measure of relative welfare among the partners, allowing for labor supply, housework, public and private goods. This turns out to be key to assessing the relative welfare of men and women in Japan.

We produce three key empirical results. First, estimates reveal that the spouses typically have different preferences: the wife values household public consumption more than her husband does. This discrepancy has a clear impact on intrahousehold inequality. Second, the wife's Pareto weight increases as the couple's wage gap decreases. This captures the notion of women's empowerment: women's greater earnings potential in the labor market translates into greater power over household resource allocations. Finally, the wife's mean MMWI ratio is 0.44. This reveals that while intrahousehold welfare inequality remains substantial, it is less severe than what the sole analysis of private consumptions would suggest (0.36). A significant fraction of total budget is allocated to public consumption; public consumption tends to reduce inequality, particularly when it benefits the wife more than the husband.

The remainder of the paper is organized as follows. Section 2 provides definitions for the basic collective household model, concepts, and axioms that we use throughout the paper. Building on them, we further overviews individual welfare measurements, and we introduce the Money Metric

Welfare Index (MMWI). Then in Section 3, we take the MMWI to data. Section 4 describes our data sources. In Section 5, we explain our empirical design and some estimation issues that we address. Section 6 presents and discusses our findings. Finally, Section 7 concludes.

2 Intrahousehold welfare: basic issues

Our basic structure draws upon the collective model for households developed in Chiappori (1988, 1992) and as extended by Blundell, Chiappori, and Meghir (2005) to allow for the consumption of public goods. The key defining characteristic of the collective model is that intrahousehold allocations are efficient. And while individuals can have caring preferences, in the sense that they value their partners welfare overall, they do not derive utility from the patterns of consumption of their partner, i.e. they do not have paternalistic preferences. We refer the reader to the articles above for a full theoretical analysis of the collective model and its empirical content. Here we consider individual welfare issues. We first consider a special case in which all commodities are privately consumed, then move to the general case.

2.1 The case of private goods

When all commodities are privately consumed, the household can be considered as a small economy without externalities or public goods. From the second welfare theorem, any Pareto efficient allocation can be decentralized by adequate transfers.

Hence, in a private goods setting, any efficient decision can be described as a two-stage process. In the first stage, household members (say the two cohabiting partners) jointly decide on the allocation of household aggregate full income y between them (and member a gets ρ^a). Since the vector of private goods includes leisure, full income for each person is defined as $w^aT + y^a$ where w^a is the individual hourly wage, T is total time available and y^a is the person's a non-labor income. In stage two, agents freely spend the share they have received. The decision process (bargaining, for instance) takes place in the first stage.

From a welfare perspective, the crucial point is that there exists a one-to-one, increasing correspondence between Pareto weights and the sharing rule (at least for any cardinal representation of individual preferences such that the Pareto set is strictly convex). When prices and incomes are constant, increasing the weight of one individual (reducing the other's weight proportionally in order to maintain the normalization) always results in a larger share for that individual and conversely.

This result has two consequences. First, given each person's preferences, the sharing rule is a sufficient statistic for the entire decision process. Indeed, since all agents face the same prices, the sharing rule fully summarizes intrahousehold allocation of resources. As such, it is directly relevant for intrahousehold inequality. Second, and more importantly for our present purpose, the sharing rule is a money metric measure of individual utility. For given prices, ρ^a is an increasing transform of the collective indirect utility of person a; moreover, and unlike the indirect utility V^a, it is always measured in monetary units.

2.2 Public and private commodities

Convenient as the previous notions may be, they still rely on a strong assumption - namely that all commodities are privately consumed. Relaxing this assumption is indispensable; after all, the existence of public consumption is one of the main motives for household formation. So now we address the question of defining individual welfare within a collective household, in the presence of public goods. In what follows we denote by K the vector of public goods and by q^a the vector of private goods consumed by individual a in the household. In general there may be any number S of decision makers within the household, although in our empirical example there will always be two (husband and wife).

2.2.1 Public goods and Lindahl prices

Blundell, Chiappori, and Meghir (2005) define the notion of the conditional sharing rule, which reflects the allocation of resources made to each household member for their *private* consumption, given public goods expenditure. Contrary to the case with just private goods, this measure does not reflect the intrahousehold allocation of welfare because it does not take into account that the choices over public goods also affect the distribution of welfare, particularly if preferences over public goods differ across partners. In the approach below we address this issue, which leads us to a money-metric measure of overall distribution of welfare.

One approach to public consumption relies on the notion of Lindahl prices. A key result in public economics states that, in the presence of public goods, Pareto efficient allocations can be decentralized using personal (Lindahl) prices that add up to the market price of the commodity. Formally, we have the following result:

Proposition 1. *Assume an allocation* $\left(\bar{K}, \bar{q}^1, ..., \bar{q}^S\right)$ *is Pareto efficient. Then there exists* S *non-negative functions* $\left(\rho^{*1}, ..., \rho^{*S}\right)$*, with* $\sum_k \rho^{*k} = y$*, and for each* $a = 1, ..., S$*,* N *non-negative functions* $\left(P_j^a\right), j = 1, ..., N$ *(where* P^a *is* a*'s N-vector of personal prices), with* $\sum_a P_j^a = P_j$ *for all* j*, such that for all* a *the vector* $\left(\bar{K}, \bar{q}^a\right)$ *solves:*

$$\max_{K, q^a} u^a \left(K, q^a\right) \tag{DP}$$

under the budget constraint

$$\sum_{i=1}^{n} p_i q_i^a + \sum_{j=1}^{N} P_j^a K_j = \rho^{*a}$$

Conversely, for any non-negative functions $\left(\rho^1, ..., \rho^S\right)$ *such that* $\sum_a \rho^a = y$ *and* P_j^a *such that* $\sum_a P_j^a = P_j$ *for all* j*, an allocation that solves (DP) for all* a *is Pareto-efficient.*

The vector $\rho^* = \left(\rho^{*1}, ..., \rho^{*S}\right)$ defines a generalized sharing rule (GSR). From an inequality perspective, this notion raises interesting issues. One could choose to adopt ρ^* as a description of intrahousehold welfare allocation; indeed, agents now maximize utility under a budget constraint in which ρ^* describes available income. In particular, ρ^* is a much better indicator of the distribution of resources than the conditional sharing rule $\tilde{\rho}$, because it takes into account both private and public consumptions.

However, the welfare of agent a is not fully described by ρ^{*a}; one also needs to know the vector P^a of a's personal prices. Technically, the collective indirect utility of a is:

$$V^a(p, P, y) = v^a(p, P^a, \rho^{*a}(p, P, y))$$

which depends on both ρ^{*a} and P^a. This implies that the sole knowledge of the GSR is not sufficient to recover the welfare level reached by a given agent, even if her preferences are known; indeed, one also needs to know the prices, which depend on all preferences and on the decision process and hence differ in general between members of the household.

In particular, welfare within the household cannot be analyzed from the sole knowledge of the generalized sharing rule. Agents now face different personal prices, and this should be taken into account. This simply reflects a basic but crucial insight - namely that if agents 'care differently' about the public goods (as indicated by personal prices, which reflect individual marginal willingnesses to pay), then variations in the quantity of these public goods have an impact on the intrahousehold distribution of welfare.

2.2.2 The Money Metric Welfare Index

This leads us to the basic concept of Money Metric Welfare Index (MMWI) of agent a. Formally:

Definition 2. *The Money Metric Welfare Index (MMWI) of agent a, $m^a(p, P, y, z)$, is defined by:*

$$v^a(p, P, m^a(p, P, y)) = V^a(p, P, y) \tag{1}$$

Equivalently, if c^a denotes the expenditure function of agent a, then:

$$m^a(p, P, y) = c^a(p, P, V^a(p, P, y)) \tag{2}$$

In words, m^a is the monetary amount that agent a would need to reach the utility level $V^a(p, P, y)$, if she was to pay the full price of each public good (i.e., if she faced the price vector

P instead of the personalized prices P^a). Unlike the GSR, the Money Metric Welfare Index fully characterizes the utility level reached by the agent. That is, knowing an agent's preferences, there is a one-to-one relationship between her utility and her MMWI, and this relationship does not depend on the partner's characteristics.

Some remarks can be made at this point. First, in the absence of public goods, the MMWI coincides with the sharing rule. In other words, the MMWI is a fully general measure of individual welfare, which coincides with the natural concept (i.e. the sharing rule) in the (largely explored) case of private consumptions, and extends it to allow for public expenditures within the household.

A second remark is that in the presence of public goods, the MMWI depends on the price vector used as a reference. While using the market price as a benchmark is a natural solution, it is by no means the only one. Even more striking is the fact that even the direction of intrahousehold inequality may be affected by this choice; i.e., one can easily construct examples in which the MMWI of member A is larger than B's for some prices but smaller for others.[3]

Third, the previous definition compares the utility currently reached by a married individual with the utility the *same* individual (i.e., with the *same preferences*) would reach in the hypothetical situation where she would have to purchase the public goods at market prices (in which case the chosen consumption bundle would obviously be quite different). It is tempting to think of this hypothetical situation as the individual being single. But this interpretation is by no means needed, and may sometimes be misleading: it requires the assumption that marriage does not change preferences, which is far from obvious.

Lastly, there is a direct relationship between the MMWI and the standard notion of *equivalent income*.[4] Both approaches rely on the notion that referring to a common price vector can facilitate interpersonal comparisons of welfare. However, to the best of our knowledge, equivalent income has exclusively be applied so far to private goods. Our point, here, is that using the concept of Lindahl prices allows to extend it to the case of public consumption, thus providing a natural solution to a recurrent and somewhat difficult problem.

[3]We thanks Frederic Vermeulen for pointing out this result..

[4]See for instance Fleurbaey, Kanbur, and Snower (2023) for a recent survey.

Finally, the previous construct can readily be extended to domestic production - although we do not consider it in the empirical application below.

2.2.3 An example

As an illustration of the previous concepts, we consider a two person household each with Cobb-Douglas preferences over a private good, leisure and a public good. In what follows γ^1 (resp. γ^2) is the preference weight for the public good for person 1 (resp. 2). Moreover, $1-\mu$ is the Pareto weight of individual 1 and μ that of individual 2. In Appendix A.1 we provide all the calculations for this case. Here we summarize the key insights from this.

The Lindahl prices for the public good take the form

$$P^1 = \frac{(1-\mu)\,\gamma^1}{(1-\mu)\,\gamma^1 + \mu\gamma^2} P, \quad P^2 = \frac{\mu\gamma^2}{(1-\mu)\,\gamma^1 + \mu\gamma^2} P$$

and are increasing in the preference for the public good, reflecting an increased willingness to pay. This results in the Generalized Sharing Rules:

$$\rho^{*1} = q^1 + w^1 L^1 + P^1 K = (1-\mu)\,Y$$
$$\rho^{*2} = q^2 + w^2 L^2 + P^2 K = \mu Y$$

where Y is the aggregate full income of the household. In this case the ratio of 2's to total GSR is

$$r_{GSR} = \frac{\rho^{*2}}{\rho^{*1} + \rho^{*2}} = \mu$$

We can now derive the the MMWI for each member which are given by

$$\bar{y}^1 = \left(\frac{(1-\mu)\,\gamma^1 + \mu\gamma^2}{(1-\mu)\,\gamma^1}\right)^{\gamma^1} (1-\mu)\,Y \text{ and similarly } \bar{y}^2 = \left(\frac{(1-\mu)\,\gamma^1 + \mu\gamma^2}{\mu\gamma^2}\right)^{\gamma^2} \mu Y$$

leading to an index of relative welfare for person 2

$$I = \frac{\bar{y}^2}{\bar{y}^1 + \bar{y}^2} = \frac{\mu \left(\frac{(1-\mu)\gamma^1 + \mu\gamma^2}{\mu\gamma^2}\right)^{\gamma^2}}{(1-\mu)\left(\frac{(1-\mu)\gamma^1 + \mu\gamma^2}{(1-\mu)\gamma^1}\right)^{\gamma^1} + \mu\left(\frac{(1-\mu)\gamma^1 + \mu\gamma^2}{\mu\gamma^2}\right)^{\gamma^2}}$$

This ratio is our preferred measure of welfare allocation within the household. Unlike r_{GSR}, it depends not only on the Pareto weight μ, but also on individual preferences for the public good.

The case of identical preferences To better understand the underlying mechanisms, it is useful to consider the particular case of identical preferences, where $\gamma^1 = \gamma^2$. For our Cobb-Douglas preferences, this results in all sharing rules (private goods only, conditional sharing rule and Generalized sharing rule) being identical

$$r_p = r_{CSR} = r_{GSR} = \mu \tag{3}$$

whereas the ratio of the MMWIs I is different

$$I = \frac{\bar{y}^2}{\bar{y}^1 + \bar{y}^2} = \frac{\mu^{1-\gamma}}{(1-\mu)^{1-\gamma} + \mu^{1-\gamma}}$$

which coincides with the previous measures only when $\gamma = 0$. In other words the MMWI depends on the presence of the the public good, even when the various alternative definitions of the sharing rule do not, because the expenditure on the public good is equalizing in this case.

Thus, taking public consumption into account reduces the measure of intrahousehold inequality; the larger the share of expenditures devoted to public goods, the more important the dampening effect.[5] Note that even the Generalized Sharing Rule fails to detect this impact; only the MMWI ratio provides an effective measure of actual intrahousehold inequality. Technically, access to (implicitly) 'cheaper' public goods benefits both spouses. Even with identical preferences, the corresponding gain is added to both welfare measures, thus reducing welfare inequality. In addition, an individual's Lindahl price of the public good increases with the individual's Pareto weight; unequal

[5] For instance, in the extreme case where consumption is exclusively public ($\gamma = 1$), our index indicates equal allocation of welfare, which is the only economically meaningful conclusion.

distribution is therefore partly compensated by an even 'cheaper' access to public consumption for the disadvantaged party.[6]

An important point is that individual MMWIs add up to more than total household income, precisely because public consumptions generate an economic gain. It is therefore interesting to consider the ratio

$$S = \frac{\bar{y}^1 + \bar{y}^2}{Y}$$

which provides a measure of the benefits generated by public consumption. Here:

$$S = (1-\mu)\left(\frac{(1-\mu)\,\gamma^1 + \mu\gamma^2}{(1-\mu)\,\gamma^1}\right)^{\gamma^1} + \mu\left(\frac{(1-\mu)\,\gamma^1 + \mu\gamma^2}{\mu\gamma^2}\right)^{\gamma^2}$$

Interestingly, when the bargaining power of the wife is less than that of the husband ($\mu < 0.5$), this benefit reaches its maximum for some $\gamma^2 > \gamma^1$, i.e. when her preference for the public good is stronger than his.

Finally, this simple example does not take domestic work into account. In the empirical estimation, given the characteristics of most Japanese households (in which domestic work is almost entirely female), we consider it as a fixed imposition on the wife's time that (also) benefits the husband; de facto, there is thus a transfer of resources from the wife to the husband that should also be taken into account. Another interpretation of our empirical work is that we condition on the amount of housework we observe, very much like Browning and Meghir (1991). In other words, we acknowledge that housework is a choice we do not explicitly model, but on which we condition.

2.3 Indifference scales

A related approach to these issues, initially introduced by Browning, Chiappori, and Lewbel (2013, from now on BCL) and then extended by Dunbar, Lewbel, and Pendakur (2013), relies on the notion of *Indifference Scales (IS)*. It posits that agents, when they get married, keep the same preferences but can access a different (and generally more productive) technology. That is, while

[6] All these ideas generalize to the case where some of the private goods carry individual specific prices, such as the (opportunity cost of) leisure.

the basic rates of substitution between consumed commodities remain unaffected by marriage (or cohabitation), the relationship between purchases and consumption does not; therefore, the structure of demand, including for exclusive commodities (consumed only by one member) is different from what it would be for singles. In practice, the technology available to singles is normalized to be the identity, in the sense that single individuals consume exactly their market purchase. Within households, commodities are assumed to be privately consumed, time inputs are being disregarded, and the technology is assumed to be linear, so that the n-vector of consumption q can be produced given a n-vector of market purchases x if:

$$x = A.q \tag{4}$$

where A is a $n \times n$ matrix. Moreover, the matrix is taken to be diagonal, so that (4) becomes:

$$x_i = \lambda_i q_i$$

Here, parameters λ_i represent economies of scale generated by the household technology; in particular, $\lambda_i < 1$ means that the amount purchased to provide the household with a total consumption in good i equal to q_i is less, by a factor λ_i, than the sum of purchases that would be needed to provide an equal number of singles with the same total consumption. As a result, household members de facto face different prices than singles; technically, the *within household* price of commodity i becomes $\pi_i = \lambda_i p_i$ for all i. Indifference scales refer precisely to the income that an individual would, as a single (i.e. facing the market prices π), need to achieve the same utility level as he or she does within the couple.

The basic intuition of the MMWI is, in many respects, close to that of the indifference scale literature. In both cases, the household generates an economic gain by enlarging the consumption space available to agents; and in both cases the practical translation is that intrahousehold prices differ from market prices. The main difference, however, is that commodities, in the IS setting, are privately consumed. In particular, while intrahousehold prices may differ from market prices, *they are identical for all agents within the household* : the price 'rebate' simply reflects the more efficient consumption technology in marriage. On the contrary, the MMWI approach relies on Lindahl prices that are individual-specific (and add up to market prices). In other words, both models capture a fundamental, economic intuition behind marital gains - namely that the household provides its members with an access to the same commodities as if they were alone, but at a lower price. However, the indifference scale additionaly assumes that intrahousehold prices are identical across individuals, while the MMWI setting allows for individual-specific valuations.

All in all, the two approaches are complementary, and their respective scope depends on the question under consideration. If the goal is to analyze individual poverty in developing countries, where most of the budget is spent on essential goods (food, basic clothing) that are privately consumed, the indifference scale method is convenient and highly tractable; moreover, it allows identification of economies of scale for private consumptions, which raises specific difficulties in the MMWI context, at least if one exclusively considers multi-person families. Conversely, the MMWI technology introduces an additional dimension, namely that individuals may value differently the same amount of public good – a fact that should be taken into account when assessing intrahousehold inequality when a significant fraction of household expenditures relates to publicly consumed commodities.

2.4 Identification

While the conceptual tools just presented help clarify some of the issues involved, we now turn to their empirical content. In this section, we briefly summarize some of the main results.[7]

We start with the 'pure' identification problem. Assume that the entire demand function of a household can be observed; what can be recovered from this (and this only)? A first result, due to Chiappori and Ekeland (2009b), is that under mild regularity conditions, one exclusion restriction per agent is sufficient to fully identify the collective, indirect utilities. The exclusion restriction, here, requires that for each agent there exists a least one commodity that is not consumed by this

[7] For a detailed presentation, the reader is referred to Chiappori and Ekeland (2009a) and Browning, Chiappori, and Weiss (2014).

agent. In practice, leisure will be the exclusive commodity in the empirical section below. This result generalizes an earlier contribution by Blundell, Chiappori, and Meghir (2005) who established a similar result in a model with three commodities (two leisures and a public good).

A crucial remark is that what is identified (up to an increasing transformation) is the indirect collective utility of each member. From a welfare perspective, this is the only relevant concept, since it fully characterizes the utility reached by each agent. However, its implications for the previous discussion must be carefully considered. The case where all goods are public is the easiest: Chiappori and Ekeland (2009b) show that when all commodities are publicly consumed, recovering a person's indirect collective utility is equivalent to recovering their direct utility. It follows that all the concepts previously defined (in particular the MMWI) are exactly identified under either the exclusion condition or the assignable and distribution factor case.

Private goods, however, raise specific difficulties. Remember that, in the absence of public goods, the various concepts (conditional sharing rule, generalized sharing rule, money metric welfare index) coincide with the sharing rule, and the collective indirect utility takes the form:

$$V^a(p, y) = v^a(p, \rho^a(p, y))$$

where, as above, v^a is a's indirect utility and ρ is the sharing rule. Under assumptions stated above, the function V^a is identified. The sharing rule, however, is not; identification only obtains up to an additive function of the prices of the non exclusive private goods. The corresponding indeterminacy is not welfare relevant, since the different solutions correspond to the same collective indirect utilities for each agent. In that case, and somewhat paradoxically, one can identify the intrahousehold distribution of *welfare* (although only up to the usual restrictions: one can only identify individual utilities in an ordinal sense), but not the intrahousehold distribution of *income*.

It is important to remark, however, that this non identification result is only local. In particular, it disregards additional, global restrictions such as non negativity contraints. Adding non negativity restrictions (reflecting the fact that if household income goes to zero then all consumptions should go to zero as well) typically pins down the sharing rule in general. This result should be related to recent work on the estimation of the sharing rules based on a revealed preference approach (see for instance Cherchye, De Rock, Lewbel *et al.*, 2015). Since the revealed preference approach is global by nature, it can generate bounds on the sharing rule, which can actually be quite narrow.[8] In the 'differentiable' case, such as the one below, the specific functional forms generally used implicitly imposes non negativity restrictions on individual consumptions (which enter through their log), which leads to full identification.[9]

3 Empirical model

We now use the MMWI to characterize the distribution of welfare in Japanese households. We draw data from Japan Panel Survey of Consumers (JPSC), which we explain in detail in Section 4. JPSC makes it necessary to accommodate certain features of observed household decisions. First, a substantial share of 48.1 percent of wives do not work. Second, husbands contribute little to house work. 55 percent of husbands do not do housework at all on a typical weekday and the median hours that husbands spend on house work is 0.5 hours per day. Given these observations, we assume all housework is performed by the woman and we condition on the observed amount (Browning and Meghir, 1991).

The setting. We consider a two-person household, consisting of person 1 (primary earner) and person 2 (secondary earner or household manager) who jointly make decisions about consumption

[8]In all cases, the global restrictions are generated at one end of the distribution of expenditures, so their use for identifying the sharing rule outside this range should be submitted to the usual caution.

[9]Alternatively, one may assume that individual preferences remain (partly) unchanged after marriage, and use information about the demand of single individuals - a line followed by Bargain, Beblo, Beninger *et al.* (2006), Vermeulen, Bargain, Beblo *et al.* (2006) and Lise and Seitz (2011) for labor supply, and by the Indifference Scale literature initiated by Browning, Chiappori, and Lewbel (2013) for consumption. Additional constraints on intrahousehold allocations can also be derived from the equilibrium conditions on the 'marriage market'. These approaches refer either to frictionless, matching models (Choo and Siow, 2006; Chiappori, Iyigun, and Weiss, 2009; Chiappori, Salanié, and Weiss, 2017; Chiappori, Costa-Dias, and Meghir, 2018) or to a search framework (Jacquemet and Robin, 2013; Goussé, Jacquemet, and Robin, 2017). In all these cases, complete identification of the sharing rule obtains.

as well as labor force participation. Their individual preferences are Cobb-Douglas:

$$U^a = \alpha^a \ln L^a + \beta^a \ln C^a + \gamma_1^a \ln K_1 + \gamma_2^a \ln K_2 \quad \text{for } a = 1,2 \text{ and } \alpha^a + \beta^a + \gamma_1^a + \gamma_2^a = 1 \quad (5)$$

Here, L^a denotes person a's leisure, C^a their consumption of some Hicksian composite good that is privately consumed, and K_1 and K_2 are two household public goods, one represents child related expenditures (K_1) and the other a general public good defined in the data section (K_2). Note that these utilities are (strongly) separable; in particular, individual demands for private goods, as functions of the sharing rule, do not depend on the level of public consumption. All coefficients in the utility function are assumed random allowing for substantial heterogeneity. This is discussed further in the estimation section.

The time constraint on person $i = 1,2$ is $T = L^i + H^i + h^i$, where H^i is hours worked and h^i is hours spent on household chores respectively. The collective household optimization problem under efficiency is

$$\max \quad V = (1-\mu)U^1 + \mu U^2 \quad (6)$$

subject to the budget constraint

$$C^1 + C^2 + w_1 L^1 + w_2 L^2 + P_1 K_1 + P_2 K_2 = \hat{y}^1 + \hat{y}^2 + Y \equiv X \quad (7)$$

where $\hat{y}^1 = w_1 T$ and $\hat{y}^2 = w_2(T-h)$ are the maximum potential income of the primary earner and the household manager respectively, which in the context of Japan is almost always the woman and we take that to be the case from now on. Moreover, we assume the male (member 1), always works.

The solution depends on whether the woman works or not, which is determined by

$$\text{Works} \Leftrightarrow \mu \leq \frac{\hat{y}^2}{\alpha^2 X} \quad (8)$$

For the case where the woman does work, the resulting demands for the private goods (consumption and leisure) are

$$w_a L^a = \zeta_\mu^a \alpha^a X; \qquad C^a = \zeta_\mu^a \beta^a X, \quad a = 1, 2 \tag{9}$$

where $\zeta_\mu^1 = 1 - \mu$ for person 1 and $\zeta_\mu^2 = \mu$ for person 2. The household demand for the public goods is

$$P_j K_j = (\zeta_\mu^1 \gamma_j^1 + \zeta_\mu^2 \gamma_j^2) X \quad \text{for public goods } j = 1, 2 \tag{10}$$

which shows how the individual preference for the public good (γ_j^a) is weighed by their relative bargaining power (ζ_μ^a).

In the case where she does not work, the structure of the demands is the same. However total income X is replaced by $\tilde{X} \equiv X - \hat{y}_2$ and the relative bargaining power of the man ζ_μ^1 is replaced by $\tilde{\zeta}_\mu^1 \equiv (1-\mu)/((1-\mu) + \mu(1-\alpha^2))$, while that of the women by $\tilde{\zeta}_\mu^2 \equiv \mu/((1-\mu) + \mu(1-\alpha^2))$.

Money Metric Welfare Index. We define the Money Metric Welfare Index (MMWI) to be the minimal expenditure needed as a single to achieve the level of utility implied by the efficient allocations as a married couple:

$$\begin{aligned} &\text{Working when single:} \qquad MMWI_p^a = E_p^a(U_m^a, P_1, P_2, w_a) \quad a = 1, 2 \\ &\text{Not working when single:} \quad MMWI_{np}^a = E_{np}^a(U_m^a, P_1, P_2) \quad a = 1, 2 \end{aligned} \tag{11}$$

where U_m^a is the indirect utility of person a when living with their partner and the intrahousehold allocations are efficient as described above.[10]

Preference heterogeneity. We assume that the preference parameters ($\alpha^a, \beta^a, \gamma_1^a, \gamma_2^a, a = 1, 2$) depend on a vector of characteristics ($\boldsymbol{x}_i$), which includes their educational attainment (an indicator variable taking one if one has some college degree or above), the age of the youngest child, and the number of children as well as on a random term. Hence we have that:

$$\alpha^a = \boldsymbol{x}_i' \boldsymbol{\alpha}^a + \epsilon_{i\alpha}^a, \quad \beta^a = \boldsymbol{x}_i' \boldsymbol{\beta}^a + \epsilon_{i\beta}^a, \quad \gamma_1^a = \boldsymbol{x}_i' \boldsymbol{\gamma_1}^a + \epsilon_{i\gamma 1}^a, \quad \gamma_2^a = \boldsymbol{x}_i' \boldsymbol{\gamma_2}^a + \epsilon_{i\gamma 2}^a \tag{12}$$

[10]We provide the exact formulae for the indirect utility functions and the expenditure functions corresponding to our specification in the appendix.

The additive errors, ϵ^a_α, ϵ^a_β, $\epsilon^a_{\gamma 1}$ and $\epsilon^a_{\gamma 2}$ ($a \in 1, 2$), are specified to be normally distributed, but must add up to zero for the husband and the wife respectively. Hence there are just three variances that are free in each case. We denote these by ς^a_k, where k indicates the corresponding parameter.[11]

Pareto weight. We assume that the wife's Pareto weight μ depends on a vector of prices and distribution factors (z_h), which include the couple's log wage gap, childcare availability, an indicator variable taking one for the presence of a pre-school child, and the interaction between the childcare availability and the presence of a pre-school child for each household h:

$$\mu = \frac{1}{1 + exp(-z'_h \boldsymbol{\mu})} \tag{13}$$

4 Data

Our main data come from the Japanese Panel Survey of Consumers (hereafter JPSC), the longest-running nationwide panel survey of individuals in Japan.
We restrict our sample to heterosexual,[12] legally married couples with at least one child under the age of 15[13] between 1998 and 2020.[14] We restrict our sample to households where a husband works with positive earnings (12% dropped). We further exclude households that did not complete reporting monthly expenses, income, and time use (8% dropped). Finally, we omit households that did not provide a comprehensive history of the wife's employment since she completed school (32% dropped). The resulting sample consists of 1433 households each observed for an average of 6.4 years and 9131 observations in total.[15] Key summary statistics are shown in Table 1.

For our empirical analysis, JPSC provides necessary information on household expenditure, household expenditure on children (which corresponds to public good 1), household expenditure on

[11]Normality is an approximation since it allows the remote possibility of coefficients occasionally turning negative.

[12]During the period of our study, same-sex marriage was not yet legalized in Japan and thus was not captured in our data. Our focus on heterosexual couples does not diminish the scholarly importance of studying same-sex marriages in Japan. We leave it for future studies.

[13]In Japan, out-of-wedlock births still account for a very small portion of all births. In 2020, only 2.4 percent of all births were out of marriage.

[14]Although the JPSC starts in 1993, some key variables exist only after 1998.

[15]See Table A.2 in the Online Appendix for the number of households dropped to meet each criteria.

Table 1: Descriptive statistics

	Mean	Std.dev.
Demographic characteristics		
Wife's actual market experience	8.38	6.18
Wife some college	0.23	0.42
Wife 4yr+ university	0.16	0.37
Husband 4yr+ university	0.37	0.48
No. of children	2.01	0.81
Dummy = 1 if the youngest child age 0-6	0.58	0.49
Dummy = 1 if the youngest child age 7-12 (primary school)	0.31	0.46
Dummy = 1 if the youngest child age 13-15 (middle school)	0.11	0.32
Childcare availability per child	0.41	0.15
Household income		
Dummy = 1 if wife participates	0.54	0.50
Dummy = 1 if husband participates	1.00	0.00
Wife's real hourly rate of pay	1163.29	585.60
Husband's real hourly rate of pay	1860.75	891.68
Household real labor income per day	14795.62	6344.34
Wife's real labor income per day	5064.04	3397.55
Husband's real labor income per day	12202.79	5376.18
Real nonlabor income per day	-6605.03	5669.07
Household expenditure		
Daily total household expenditure (real, JPY)	8190.58	3571.19
Daily expenditure on nonchild public goods (real, JPY)	5239.57	2567.09
Daily expenditure on child public goods (real, JPY)	1355.88	1502.84
Daily expenditure on private goods (real, JPY)	1595.13	1308.28
Daily expenditure share of private goods	0.20	0.13
Daily expenditure share of nonchild public goods	0.64	0.17
Daily expenditure share of child public goods	0.16	0.12
Daily expenditure share of wife's private goods	0.06	0.06
Daily expenditure share of husband's private goods	0.14	0.10
Wife's relative private expenditure per day	0.30	0.23
Time use		
Wife's hrs worked per day	3.63	3.66
Wife's hrs leisure per day	2.00	2.03
Wife's hrs housework per day	7.60	4.27
Husband's hrs worked per day	10.23	2.23
Husband's hrs leisure per day	1.83	1.63
Husband's hrs housework per day	0.80	1.19
Dummy = 1 for husband's zero housework	0.44	0.50

Exchange rate: 1 yen = 0.011 US$ in 2010 common use (which corresponds to public good 2),[16] time use, income, homeownership, housing characteristics, prefecture of residence and individual characteristics such as highest educational attainment.

5 Estimation

Our estimation method proceeds in steps. We first deal with missing wages for non-working women. We then use simulated method of moments to estimate the parameters characterizing preferences and the Pareto weight.

5.1 Wife's wage process

We take wages as exogenous, in the sense that unobserved components of wages are assumed independent of preference heterogeneity for both men and women. While all men in our sample are working and hence their wages are observed, this is not true for the high proportion of women who do not work and whose wages are missing and have to be integrated out. We thus estimate the distribution of women's wages outside of the main model using a wage equation of the form

$$\ln w_{it} = \boldsymbol{z}'_{w,it}\boldsymbol{\gamma}_w + \epsilon_{it}, \qquad \epsilon_{it} \sim N(0, \sigma^2) \tag{14}$$

where $\boldsymbol{z}_{w,it}$ represents individual characteristics, including her actual labor market experience since completing her schooling, its square, and education. We estimate equation (14) correcting for selection based on the Heckman (1979) estimator and assuming joint normality. The participation equation, used to correct for selection additionally includes dummies for the number of children (1-3 children), dummies for the age of the youngest child (0-6 years old, 7-12 years old, and 13-15

[16] The Japanese Panel Survey of Consumers asks about monthly household expenses for common resources and personal use by the wife, husband, children, and other household members. These breakdowns are mutually exclusive and should add up to the total household monthly expenses. The survey question does not aim to identify which commodities each household member consumes. Instead, it leaves the interpretation of what constitutes a common resource or personal use to the respondents. This survey question was originally motivated by sociological studies on the control and allocation of money within families, particularly Jan Pahl's (1989) "Money and Marriage" published by MacMillan. In our study, we use expenses for children as public good 1 and expenses for common resources as public good 2.

years old), child care availability and its interaction with the age of the youngest child. Childcare availability is defined as the ratio of daycare slots to the population aged 0-4, and is reported by prefecture level and rural urban status (i.e. two measures per prefecture). Childcare availability is measured as the ratio of public childcare capacity to the population aged 0-4. Across prefectures, childcare supply grew during the studied period, but the timing varied, which gives us the variation we need, beyond aggregate and prefecture effects. Both the wage equation and the participation equation include time, prefecture and rural/urban dummies. Table 2 shows the results from estimating this wage equation. As our results show, greater access to childcare increases the participation of mothers in market work. This together with the demographic composition of the household are strong instruments for selection correction.

5.2 Estimation

We estimate 34 preference parameters, the coefficients defining the Pareto weight, and the variances of the random coefficients in preferences. The full set of parameters is given by

$$\theta = \left(\boldsymbol{\alpha}^a, \boldsymbol{\beta}^a, \boldsymbol{\gamma}_1^a, \boldsymbol{\gamma}_2^a, \boldsymbol{\mu}, \varsigma_\alpha^a, \varsigma_{\gamma 1}^a, \varsigma_{\gamma 2}^a\right) \qquad a = 1, 2$$

by using the simulated method of moments (McFadden, 1989; Pakes and Pollard, 1989). Specifically, we maximize the following criterion function

$$L(\theta) = -\sqrt{n}(g_n(\theta))' W_n(\theta) g_n(\theta) \tag{15}$$

where $g_n(\theta)$ is a vector whose elements are defined by $g_{n,j}(\theta) = \dfrac{m_j^D - m_j^S(\theta)}{m_j^D}$. In the above, m_n^D are the moments estimated from the data and $m_s^S(\theta)$ the simulated moments from the model, produced by S simulations. The errors that are drawn for these s simulations of the model are drawn only once and held the same throughout the optimization problem. The subscript n emphasizes the dependence of the data moments on the sample size.

The moments include the mean and variance of leisure, private consumption and public goods interacted with the four couple education pairs,[17] the three categories of the number of children (one child, two children, and three or more children), the three categories of the age of the youngest child (age 0-6, 7-12, and 13-15), and the work status of wife. We use observed wages for working women and we draw wages from the pre-estimated wage model for nonworking women, assuming the error term is normally distributed. We set the weighting matrix $W_n(\theta)$ to be an identity matrix instead of an optimal weighting matrix. This is to address small-sample biases in GMM covariance estimation that Altonji and Segal (1996) point out.

To find the optimum we employ Markov Chain Monte Carlo methods (MCMC) to implement a Laplace-type estimator (LTE) (Chernozhukov and Hong, 2003).[18] The standard errors are computed based on the usual sandwich formula for the variance-covariance matrix of $\boldsymbol{\theta}$ which in our case is:

$$Cov(\boldsymbol{\theta}) = (G'G)^{-1} G'\Omega G(G'G)^{-1} \tag{16}$$

where G is the derivative matrix of the model moments with respect to the parameters ($\boldsymbol{\theta}$), and Ω is the variance-covariance matrix of the moments.[19]

6 Results

Wage and participation equations. Table 2 presents the parameter estimates for the participation equation and wage equation described in Section 5.1. The estimates indicate that wives are more likely to participate as their experience and education go up, in particular having four-year university degree or above (*Wife 4yr+ university*). Their participation non-linearly increases in the number of children. Most importantly, the availability of childcare increases female participation, particularly for those with pre-school children. However, it also seems to increase female labor supply more

[17]The four pairs are (husband's educational attainment, wife's educational attainment) = ((high school or below, high school or below), (high school or below, some college or above), (some college, some college or above), (some college, some college or above)).

[18]Details are given in Appendix A.2.

[19]We estimate Ω by taking an outer product of a vector of percentage differences between data moments and simulated moments evaluated at the MCMC estimates.

Table 2: Labor Force Participation and the Wage Equation

	Estimate	(Std.err.)	Estimate	(Std.err.)
Participation equation				
Intercept	-2.639	(0.099)	-2.980	(0.169)
Wife's actual market experience (years)	0.258	(0.013)	0.271	(0.013)
Wife's actual experience squared	-0.004	(0.001)	-0.005	(0.001)
Wife some college	0.030	(0.039)	0.060	(0.041)
Wife 4yr+ university	0.211	(0.049)	0.216	(0.053)
Dummy = 1 if the youngest child age 7-12 (primary school)	0.312	(0.094)	0.296	(0.099)
Dummy = 1 if the youngest child age 13-15 (middle school)	0.465	(0.115)	0.414	(0.122)
Childcare availability	1.147	(0.174)	1.608	(0.464)
Availability x primary school dummy	-0.085	(0.228)	0.048	(0.244)
Availability x middle school dummy	-0.282	(0.278)	-0.006	(0.301)
No. of children = 2	0.079	(0.035)	0.070	(0.037)
No. of children $\geq$3	0.024	(0.044)	-0.012	(0.047)
Log wage equation				
Intercept	6.030	(0.040)	5.969	(0.057)
Wife's actual market experience (years)	0.064	(0.005)	0.059	(0.005)
Wife's actual experience squared	-0.001	(0.000)	-0.001	(0.000)
Wife some college	0.104	(0.015)	0.094	(0.015)
Wife 4yr+ university	0.348	(0.018)	0.298	(0.019)
Selection				
sigma	0.405	(0.006)	0.385	(0.006)
rho	0.742	(0.020)	0.720	(0.023)
Prefecture fixed effects	NO		YES	
Year fixed effects	NO		YES	

This table presents the estimated coefficients for women's participation equation and log wage equation. The first pair of columns does not include prefecture and year fixed effects whereas the second pair includes them both in the participation and wage equations. Asymptotic standard errors in parentheses.

generally, likely because the child care availability proxies the area's generosity of family-friendly policies, such as after-school child care.

We use the estimated wage equation to predict those who do not work and therefore their wages are not observed. In Appendix A.7, Figure A.1 shows the distribution of the log wage gap in couples. The mean wage gap is -0.74 log points (std. dev. = 0.52). This wage gap feeds into wife's Pareto weight equation in the main estimation.

Preferences. Table 3 presents the estimated parameters for the utility functions of husband and wife. Our focus is on the relative preference for the two public goods, general and expenditure

Table 3: Main parameter estimates

	Estimate	(Std.err.)
α^1 *Husband's leisure*		
Constant	0.784	(0.029)
No. of children	0.018	(0.005)
Age of the youngest child	-0.002	(0.002)
Child care availability	-0.026	(0.017)
University	-0.011	(0.003)
Std.dev. of random preference ς^1_α	0.008	(0.005)
γ^1_1 *Husband's preference for child expenditures*		
Constant	0.004	(0.071)
No. of children	-0.013	(0.010)
Age of the youngest child	0.005	(0.006)
Std.dev. of random preference $\varsigma^1_{\gamma 1}$	0.005	(0.011)
γ^1_2 *Husband's preference for other public goods*		
Constant	0.145	(0.042)
No. of children	-0.012	(0.003)
Age of the youngest child	0.000	(0.004)
Std.dev. of random preference $\varsigma^1_{\gamma 2}$	0.004	(0.029)
α^2 *Wife's leisure*		
Constant	0.335	(0.011)
No. of children	-0.068	(0.004)
Age of the youngest child	0.008	(0.001)
Child care availability	-0.042	(0.014)
University	-0.004	(0.000)
Std.dev. of random preference ς^2_α	0.023	(0.008)
γ^2_1 *Wife's preference for child expenditures*		
Constant	0.041	(0.078)
No. of children	0.066	(0.013)
Age of the youngest child	-0.004	(0.006)
Std.dev. of random preference $\varsigma^2_{\gamma 1}$	0.004	(0.020)
γ^2_2 *Wife's preference for other public goods*		
Constant	0.570	(0.068)
No. of children	0.008	(0.011)
Age of the youngest child	-0.002	(0.006)
Std.dev. of random preference $\varsigma^2_{\gamma 2}$	0.008	(0.009)

This table presents estimates for preference parameters. Standard errors are computed using the sandwich formula as in equation (16) and shown in parentheses.

on children. Based on the results in Table 3, the wife values public goods substantially more than her husband on average. And perhaps of particular interest is the fact that on average he puts almost no weight on expenditures relating to children relative to his wife. Of course, there is some heterogeneity around these results, as reflected in the estimates of the standard deviation of the random coefficients, although the only large and significant heterogeneity is in the wife's preference for leisure. Her preference for child expenditures rises fast with the number of children in the household, although this could also reflect a selection effect into larger families by women who value expenditures for children. In any case the result is striking in the importance that women place on public expenditure relative to men, whether this is the overall public good that includes shared consumption such as housing and utility costs or expenditures on children: the ratio of her weight on children relative to her consumption (γ_1/β) is 2.88 and the ratio between non-child public good and her consumption (γ_2/β) is 11.87. While the husband also places more weight on the non-child public good than on his own consumption (ratio = 1.60), he places less weight on children relative to his consumption (ratio = 0.25).[20,21] The difference in preferences for public goods between spouses becomes even more pronounced as the number of children increases. Table A.5 shows that as the number of children increase the wife's preferences shift to place additional weight on the public goods and away from her consumption. The husband's preferences are not significantly affected and, if anything, he slightly shifts weight away from public goods to his own leisure and consumption.[22]

Pareto weight. Table 4 presents parameter estimates of the wife's Pareto weight. When the couple pay gap closes by 10 percentage points, the Pareto weight increases by about 7 percentage points. It confirms that relative earnings capacity matters for intrahousehold allocations. It also suggests that women's economic empowerment translates into the intrahousehold bargaining power. If we

[20] As equation shows, the household expenditure share of children is weighted sum of the wife's and the husband's preferences where the weight equals the wife's Pareto weight: $P_jK_j/X = \zeta_\mu^1\gamma_1 + \zeta_\mu^2\gamma_2$ where $\zeta_\mu^1 = 1-\mu$ and $\zeta_\mu^2 = \mu$.

[21] Previous literature also finds that mothers tend to weigh more children than fathers do. See for example Lundberg, Pollak, and Wales (1997).

[22] The stark gender gap in preferences over children may partly reflect Japan's specific circumstances regarding child custody, which tend to favor mothers. In Japan, married parents typically share custody and responsibility for their children, unless a court order states otherwise. However, in cases of divorce, parents must agree on who will have sole custody and responsibility for their children. Japan's legal system doesn't allow for shared custody. When custody disputes are brought to family courts, it is now common for the mother to be granted full custody. According to Vital Statistics, between 2000 and 2020, 83 percent of newly divorced mothers received custody for all of their children.

Table 4: Main parameter estimates (Pareto weight)

	Estimate	(Std.err.)
m^0 constant	0.765	(0.026)
m^{12} couple pay gap	0.724	(0.012)
m^{cc} child care availability	-0.371	(0.109)
m^y unearned income (1,000 JPY)	0.037	(0.003)

This table presents the estimated coefficients for Pareto weights. Standard errors are computed using the sandwich formula as in equation (16) and shown in parentheses.

completely close the couple wage gap, it would increase the wife's Pareto weight by 28 percent.

Consistent with what we observe in existing studies in family economics, higher household unearned income also increases the wife's bargaining power. Lastly, the child care availability reduces the wife's bargaining power and also reduces her preference for leisure. These two forces act to increases her labor market participation and on net more child care availability increases her welfare.

Evaluated at the mean household observables, the average Pareto weight for wife is 0.46. It, however, varies significantly across households. At the top 5th percentile, wife's Pareto weight amounts to 0.61. At the bottom 5th percentile, it goes down to 0.30.

Lindahl prices. Now we are ready to compute personal Lindahl prices for the public goods. Two curves in Figure 1 show the wife's Lindahl prices P_j^2 ($j = 1$ for child expenditures and $j = 2$ for other public goods) as a function of her Pareto weight μ while holding her preference parameters at the data mean. The black vertical line shows μ evaluated at the data mean. The prices of the public good are normalized to one. Hence husband's Lindahl price P_j^1 is $1 - P_j^2$. Evaluated at the data mean, the wife's Lindahl price for public good 1 is 0.81 whereas the husband's is 0.19. Similarly, the average wife's Lindahl price for public good 2 is 0.79 whereas the husband's is 0.21. The wife's higher Lindhahl prices reflect her higher preference weights on children and non-child public good.

Conditional on preferences, the wife's Lindahl price monotonically increases with her Pareto

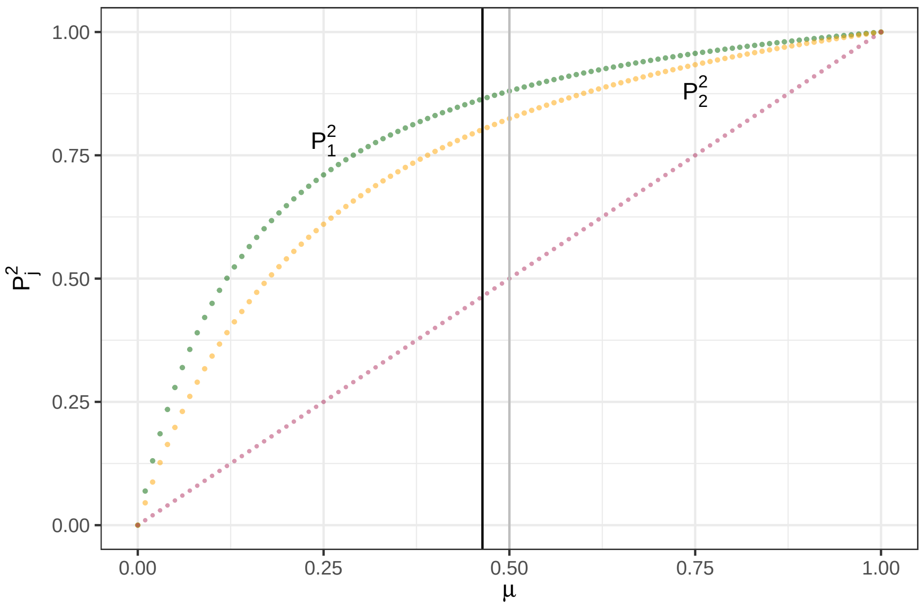

Figure 1: Wife's Lindahl price as a function of her Pareto weight μ. Her personal Lindahl price is given by $P_j^2 = \mu\gamma_j^2/((1-\mu)\gamma_j^1 + \mu\gamma_j^2)$. P_1^2 refers to the wife's Lindahl price for child expenditures and P_2^2 to her Lindahl price for other public goods. When computing Lindahl price of each public good, preferences are evaluated at the data mean ($\gamma_1^1 = 0.02$, $\gamma_1^2 = 0.12$, $\gamma_2^1 = 0.14$, $\gamma_2^2 = 0.56$). In case the wife and the husband have identical preferences ($\gamma_j^1 = \gamma_j^2$), her Lindahl price equals her Pareto weight μ, which is shown as a 45 degree line. Two vertical lines show two different Pareto weights. The black line equals to the case $\mu = 0.46$, which is the average Pareto weight in our data. The gray line equals to the case $\mu = 0.5$, which represents equal power between the wife and the husband.

weight. As her Pareto weight increases, her private consumption also increases. This creates an upward pressure on the marginal rate of substitution between her private consumption and the public good, leading to a higher willingness to pay for public goods.

If wife and husband had equal power within a household then her prices for public goods would have been even higher. Setting $\mu = 0.5$, her price for expenditures on children would have been 0.83 and her price for expenditures on the non-child public good would have been 0.82. The discrepancy arises precisely because, on average, the wife has weaker power than her husband, i.e., $\mu = 0.46 < 0.5$. When the wife is at the weaker side of the intrahousehold power equation, she gets her personal price discounted and therefore benefits more from the household shared goods.

Alternatively if both wife and husband assigned equal preference weight to public goods, then the wife's personal Lindahl price would be equal to her Pareto weight μ, which is shown by a 45 degree line in Figure 1. It is interesting to note that household demand for public goods is unaffected by μ if the wife's and husband's preferences are symmetric. Even so, the Pareto weight, or distribution of power, matters for personal Lindahl prices, and therefore it has an important welfare implication. Within a household, welfare is redistributed through public goods.

Money Metric Welfare Index (MMWI). Finally, we turn to discuss MMWI. Figure 2 shows a monetary evaluation of the benefits generated by public consumption as a function of μ, i.e., $(MMWI_1(\mu) + MMWI_2(\mu))/X$. The green dots show the case when preferences are set at the data mean. The fact that the maximum is reached for $\mu < .5$ reflects the wife's larger preference for public goods. We also consider two other scenarios: the maroon dots represent the case when husband's preferences were set to be wife's data mean. Alternatively, the navy dots represent the case when wife's preferences were set to be husband's data mean. Since husband's and wife's preferences are symmetric, the maximum is attained for μ. However, the maximum is higher when we set preferences at wife's because she place more weights on public goods.

As discussed in the theory section, the share of total resources provides an imperfect description of individual welfare, because it ignores variations in individual prices of public consumption. The notion of MMWI is introduced precisely to address this problem. Here, we characterize intrahousehold welfare inequality by wife's relative MMWI (i.e. the ratio of her MMWI to the sum of wife's and husband's MMWIs). The average MMWI ratio is 0.44, which is smaller than case of equality in the couple. There is, however, substantial variability in the MMWI ratio across households, driven by the observables.

Taking into account public consumption reveals that intrahousehold inequality is not as severe as private consumption would suggest. Figure 3 juxtaposes the MMWI-based inequality measure (solid line) and the inequality measure that is based on private consumption (dotted line). Specifically, we compute consumption-based inequality by the ratio of wife's predicted private consumption to the sum of wife's and husband's private consumption that is predicted by the model. The distribution shifts to the right from the private-consumption-based measure to the MMWI-based

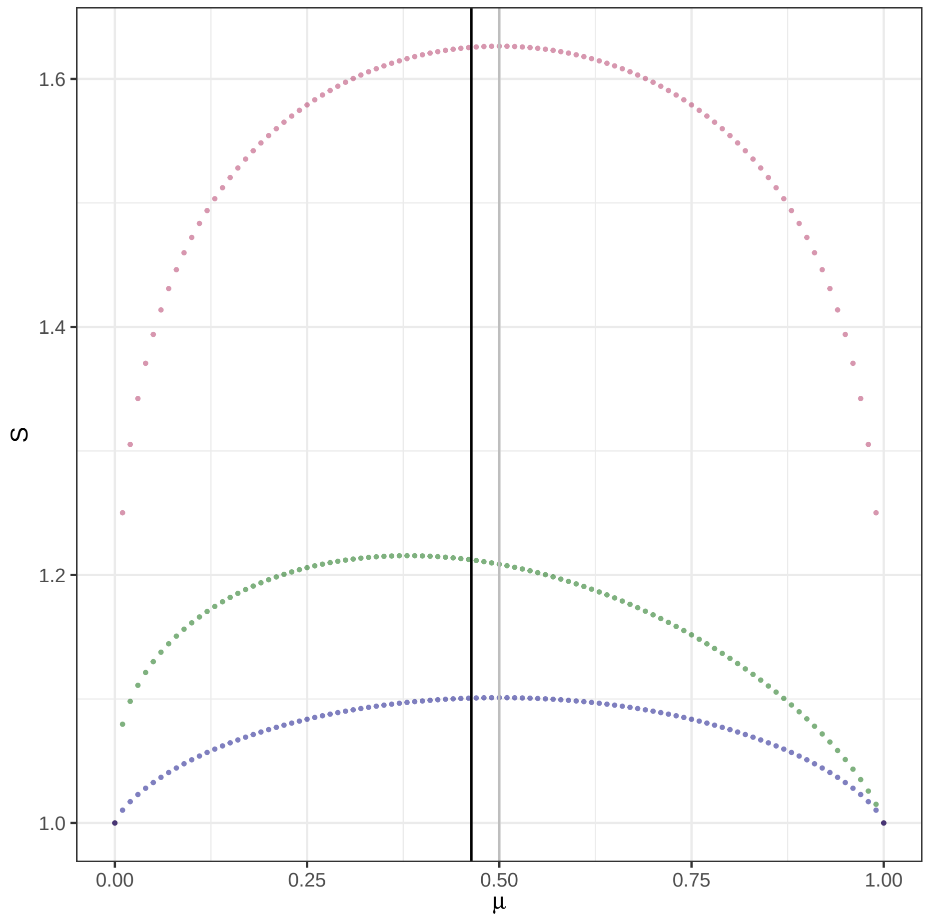

Figure 2: A monetary evaluation of the benefits generated by public consumption as a function of μ. The green dots shows the case when preferences are evaluated at the data mean. Alternatively, the maroon dots show the case when husband's preference were the same as wife's. The navy dots show the case when wife's preference were set to be the same as husband's. Two vertical lines show two different Pareto weights. The black line equals to the case $\mu = 0.46$, which is the average Pareto weight in our data. The gray line equals to the case $\mu = 0.5$, which represents equal power between the wife and the husband.

measure. The mean shifts from 0.36 to 0.44. At this point we should recall that we have conditioned on the observed housework time for the wife and we have not valued this in any way. Housework represents in part a transfer from the wife to the husband, and the MMWI would change as it varies.

The gap between the consumption based inequality and the one based on MMWI arises because the wife values public consumption more, and because her personal Lindahl prices get discounted due to her weaker bargaining power. If we consider a household as a micro-society consisting of individuals with conflicting preferences, distinct comparative advantages, and unequal

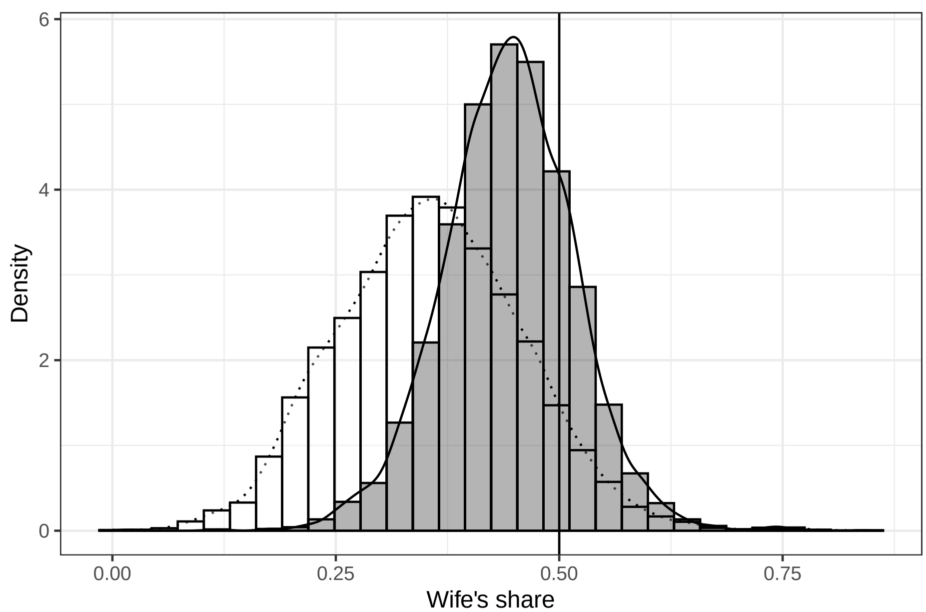

Figure 3: Distributions of the wife's MMWI ratio (solid line and black bar) and her consumption ratio (dotted line and white bar). The MMWI ratio is calculated as a share of the wife's MMWI out of the sum of the wife's and husband's money metric welfare indices. The consumption share is calculated as a share of the wife's predicted consumption out of the sum of the wife's and husband's predicted private consumption. The solid vertical line indicates the point where the wife's share equals the husband's.

power, public goods provide a crucial mechanism to redistribute welfare from haves to have nots. Our findings highlight the fact that ignoring public consumption would lead us to overstate intra-household welfare inequality, at least conditional on observed housework.

7 Conclusion

How is welfare distributed within a household? Intrahousehold welfare distribution based solely on private consumption reveals significant inequity between couples within a household. However, this approach fails to consider couples' potentially asymmetric preferences for private consumption or, more importantly, the welfare derived from public goods, which accounts for the majority of household expenses. The Money Metric Welfare Index (MMWI) takes both factors into account. In the context of Japanese households we show that allowing for public goods reduces the amount of measured inequality, relative to the measure that relies on private consumption only. This is because women tend to value public goods much more than men. As a result, they benefit more from public expenditures; while they also 'pay' more for the public goods (in terms of Lindahl prices), the net gain remains positive. In addition, the Lindahl prices are also influenced by Pareto weights, or the intrahousehold distribution of power. In cases where a wife has less power than her husband, as is the case in Japan, her Lindahl price becomes 'cheaper', resulting in her benefiting even more from household public goods than she otherwise would. Therefore, particularly in situations where there is unequal intrahousehold distribution of power, public goods play a crucial role in understanding intrahousehold welfare distribution, providing a different perspective from solely considering personal consumption. This insight has important policy implications, for example, in determining the appropriate level of compensation for a wife in the event of marital dissolution. More broadly, since public goods represent such a high percentage of household expenditure and since these also relate to intergenerational transmission because one of the public goods relates to investment in children, the study of the role of public goods is central to our understanding of household behavior.

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A Online Appendix

A.1 An example with Cobb-Douglas Preferences

The previous concepts can be illustrated on a very simple example, which will be directly generalized in the empirical section. Assume two individuals 1 and 2, four commodities - one private consumption good q, one public consumption good Q and two leisures L^1 and L^2 - and Cobb-Douglas preferences:

$$U^n\left(L^n, q^n, Q\right) = \alpha^n \ln L^n + \gamma^n \ln Q + \left(1 - \alpha^n - \gamma^n\right) \ln q^n, \quad n = 1, 2$$

The couple's aggregate budget constraint is given by

$$w^1 L^1 + w^2 L^2 + q^1 + q^2 + PQ = \hat{y}^1 + \hat{y}^2 = Y$$

where w^n denotes n's wage, P is the price of the public good (the price of the private good being normalized to 1), and $\hat{y}^n = w^n T$ denotes n's maximum labor income (T being the total time available for leisure and work)[23]; for simplicity, we disregard non labor income. Finally, let μ be the Pareto weight of individual 2 (then the weight of individual 1 is normalized to be $1-\mu$ to keep the sum constant and equal to 1).

Assume, for the time being, that the wife has a positive market labor supply.[24] The individuals' private consumptions and demand for leisure are given by:

$$q^1 = (1-\mu)\left(1-\alpha^1-\gamma^1\right) Y, \;\; q^2 = \mu\left(1-\alpha^2-\gamma^2\right) Y \tag{17}$$

$$L^1 = \alpha^1 (1-\mu) \frac{Y}{w^1}, \;\; L^2 = \alpha^2 \mu \frac{Y}{w^2} < T \tag{18}$$

[23]Domestic work is not considered in this example, although it will be taken into account in the empirical estimation.
[24]The case of a non participating wife is considered in the online Appendix.

while the demand for public good is:

$$Q = \left((1 - \mu) \gamma^1 + \mu \gamma^2 \right) \frac{Y}{P}$$

giving individual utilities

$$\begin{aligned} U_M^1 &= \ln Y - \ln w^1 - \ln P + \ln (1 - \mu) + K^1 \\ U_M^2 &= \ln Y - \ln w^2 - \ln P + \ln (1 - \mu) + \left(1 - \gamma^2\right) \ln \frac{\mu}{1 - \mu} + K^2 \end{aligned} \tag{19}$$

where

$$\begin{aligned} K^1 &= \alpha^1 \ln \alpha^1 + \gamma^1 \ln \left(\frac{(1 - \mu) \gamma^1 + \mu \gamma^2}{1 - \mu} \right) + \left(1 - \alpha^1 - \gamma^1\right) \ln \left(1 - \alpha^1 - \gamma^1\right) \\ K^2 &= \alpha^2 \ln \alpha^2 + \gamma^2 \ln \left(\frac{(1 - \mu) \gamma^1 + \mu \gamma^2}{1 - \mu} \right) + \left(1 - \alpha^2 - \gamma^2\right) \ln \left(1 - \alpha^2 - \gamma^2\right) \end{aligned}$$

It follows that:

- The ratio of private consumptions is

$$r_p = \frac{q^2}{q^1 + q^2} = \frac{\mu (1 - \alpha^2 - \gamma^2)}{(1 - \mu) (1 - \alpha^1 - \gamma^1) + \mu (1 - \alpha^2 - \gamma^2)}$$

- The conditional sharing rules are:

$$\begin{aligned} \tilde{\rho}^1 &= q^1 + w^1 L^1 = (1 - \mu) \left(1 - \gamma^1\right) Y \\ \tilde{\rho}^2 &= q^2 + w^2 L^2 = \mu \left(1 - \gamma^2\right) Y \end{aligned}$$

hence the ratio

$$r_{CSR} = \frac{\tilde{\rho}^2}{\tilde{\rho}^1 + \tilde{\rho}^2} = \frac{\mu (1 - \gamma^2)}{(1 - \mu) (1 - \gamma^1) + \mu (1 - \gamma^2)}$$

- The Lindahl prices for the public good are

$$P^1 = \frac{(1-\mu)\,\gamma^1}{(1-\mu)\,\gamma^1 + \mu\gamma^2} P,\;\; P^2 = \frac{\mu\gamma^2}{(1-\mu)\,\gamma^1 + \mu\gamma^2} P$$

giving the Generalized Sharing Rules:

$$\begin{aligned}\rho^{*1} &= q^1 + w^1 L^1 + P^1 Q = (1-\mu)\,Y \\ \rho^{*2} &= q^2 + w^2 L^2 + P^2 Q = \mu Y\end{aligned}$$

so that the ratio of 2's to total GSR is

$$r_{GSR} = \frac{\rho^{*2}}{\rho^{*1} + \rho^{*2}} = \mu$$

- Finally, the MMWI requires estimating, for each individual, the utility they would reach for some given income if they had to purchase the public good at market price. Individual n if endowed with an income $\bar{y}^n$, would then choose

$$w^n L_S^n = \alpha^n \bar{y}^n,\;\; P Q_S^n = \gamma^n \bar{y}^n,\;\; q_S^n = (1-\alpha^n-\gamma^n)\,\bar{y}^n,$$

therefore reach the utility level

$$U_S^n = \ln \bar{y}^n - \ln w^n - \ln P + \alpha^n \ln \alpha^n + \gamma^n \ln \gamma^n + (1-\alpha^n-\gamma^n) \ln (1-\alpha^n-\gamma^n)$$

Attaining in this context the utility levels given by (19) would require incomes given by:

$$U_S^1 = U_M^1 \Rightarrow \ln \bar{y}^1 = \ln Y + \ln (1-\mu) + \gamma^1 \ln \left(\frac{(1-\mu)\,\gamma^1 + \mu\gamma^2}{(1-\mu)\,\gamma^1} \right)$$

or

$$\bar{y}^1 = (1-\mu)\,Y \left(\frac{(1-\mu)\,\gamma^1 + \mu\gamma^2}{(1-\mu)\,\gamma^1} \right)^{\gamma^1}$$

and similarly

$$\bar{y}^2 = \mu Y \left(\frac{(1-\mu)\,\gamma^1 + \mu\gamma^2}{\mu\gamma^2} \right)^{\gamma^2}$$

giving an index

$$I = \frac{\bar{y}^2}{\bar{y}^1 + \bar{y}^2} = \frac{\mu \left(\frac{(1-\mu)\gamma^1 + \mu\gamma^2}{\mu\gamma^2} \right)^{\gamma^2}}{(1-\mu) \left(\frac{(1-\mu)\gamma^1 + \mu\gamma^2}{(1-\mu)\gamma^1} \right)^{\gamma^1} + \mu \left(\frac{(1-\mu)\gamma^1 + \mu\gamma^2}{\mu\gamma^2} \right)^{\gamma^2}}$$

This ratio of 2's to total MMWI is our preferred measure of welfare allocation within the household. Note that, unlike r_{GSR}, it depends not only on the Pareto weight μ, but also on individual preferences for the public good.

The case of identical preferences

To better understand the underlying mechanisms, it is useful to start with a very particular case, namely when individuals have identical preferences; in our context, this implies that $\alpha^1 = \alpha^2$ and $\gamma^1 = \gamma^2$. Then we have that

$$r_p = r_{CSR} = r_{GSR} = \mu \tag{20}$$

whereas

$$I = \frac{\bar{y}^2}{\bar{y}^1 + \bar{y}^2} = \frac{\mu^{1-\gamma}}{(1-\mu)^{1-\gamma} + \mu^{1-\gamma}}$$

which coincides with the previous measures only when $\gamma = 0$.

Thus, taking public consumption into account reduces the measure of intrahousehold inequality; the larger the share of expenditures devoted to public goods, the more important the dampening effect. Note that even the Generalized Sharing Rule fails to detect this impact; only the MMWI ratio provides an effective measure of actual intrahousehold inequality. Technically, access to (implicitly) 'cheaper' public goods benefits both spouses. Even with identical preferences, the corresponding gain is added to both welfare measures, thus diluting the inequality. In addition, an individual's Lindahl price of the public good increases with the individual's Pareto weight; unequal distribution is therefore partly compensated by an even 'cheaper' access to public consumption for the disadvantaged party.[25]

An important point is that individual MMWIs add up to more than total household income, precisely because public consumptions generate an economic gain. It is therefore interesting to consider the ratio

$$S = \frac{\bar{y}^1 + \bar{y}^2}{Y}$$

which provides a monetary evaluation of the benefits generated by public consumption. Here:

$$S = (1-\mu)\left(\frac{(1-\mu)\,\gamma^1 + \mu\gamma^2}{(1-\mu)\,\gamma^1}\right)^{\gamma^1} + \mu\left(\frac{(1-\mu)\,\gamma^1 + \mu\gamma^2}{(\mu)\,\gamma^2}\right)^{\gamma^2}$$

Interestingly, one can easily see that the benefit is maximum for $\mu < 1$ if and only if $\gamma^2 > \gamma^1$.

Finally, this simple example does not take domestic work into account. In the empirical estimation, given the characteristics of most Japanese households (in which domestic work is almost entirely female), we consider it as a fixed imposition on the wife's leisure that (also) benefits the husband; de facto, there is thus a transfer of resources from the wife to the husband that should also be taken into account.

A.2 Optimazation Algorithm

In particular, we employ the adaptive Metropolis algorithm (Haario, Saksman, and Tamminen 2001) with uniform priors $\pi(\theta) = c$ on the parameter space Θ. As Chernozhukov and Hong (2003) shows, the following transformation of our criterion function $\Omega_n(\theta)$ is a quasi-posterior function of θ:

$$p_n(\theta) = \frac{e^{\Omega_n(\theta)}\pi(\theta)}{\int_{\theta\in\Theta} e^{\Omega_n(\theta)}\pi(\theta)d\theta} \tag{21}$$

[25] All these ideas generalize to the case where some of the private goods carry individual specific prices, such as the (opportunity cost of) leisure.

We evaluate this function at the current parameter guess θ^t and at an alternative draw $\tilde{\theta}$ from a multivariate normal distribution. The parameter guess is then updated according to:

$$\theta^{t+1} = \begin{cases} \tilde{\theta} & \text{with probability} \qquad \rho(\theta^t, \tilde{\theta}) \\ \theta^t & \text{with probability} \qquad 1 - \rho(\theta^t, \tilde{\theta}) \end{cases} \tag{22}$$

where[26]

$$\rho(x, y) = min\left(\frac{p_n(y)q(y|x)}{p_n(x)q(x|y)}, 1\right) = \min\left(\frac{e^{\Omega_n(y)}}{e^{\Omega_n(x)}}, 1\right) \tag{23}$$

The resulting estimator follows as the quasi-posterior mean

$$\hat{\theta} = \int_{\theta \in \Theta} \theta p_n(\theta) d\theta \tag{24}$$

which we compute by computing the arithmetic mean over all T MCMC samples of the converged Markov chain, i.e., $\hat{\theta} = \Sigma_{t=1}^{T} \theta^t / T$.

A.3 Derivations

Collective indirect utilities Collective indirect utilities can be expressed as functions of μ and prices.

1. She works:

$$\begin{aligned} V^1 = {} & \alpha^1 \ln\left(\frac{\alpha^1}{1+\mu}\frac{X}{w_1}\right) \\ & + (1 - \alpha^1 - \gamma_1^1 - \gamma_2^1) \ln\left(\frac{1 - \alpha^1 - \gamma_1^1 - \gamma_2^1}{1+\mu} X\right) \\ & + \gamma_1^1 \ln\left(\frac{\gamma_1^1 + \mu\gamma_1^2}{1+\mu}\frac{X}{P_1}\right) + \gamma_2^1 \ln\left(\frac{\gamma_2^1 + \mu\gamma_2^2}{1+\mu}\frac{X}{P_2}\right) \end{aligned} \tag{25}$$

[26]When the transitional kernel q is symmetric, i.e., $q(y|x) = q(x|y)$, then Hastings ratio $(p_n(y)q(y|x))/(p_n(x)q(x|y))$ reduces to $p_n(y)/p_n(x)$.

$$
\begin{aligned}
V^2 = {} & \alpha^2 \ln\left(\frac{\mu\alpha^2}{1+\mu}\frac{X}{w_2}\right) \\
& + (1-\alpha^2-\gamma_1^2-\gamma_2^2)\ln\left(\frac{\mu(1-\alpha^2-\gamma_1^2-\gamma_2^2)}{1+\mu}X\right) \\
& + \gamma_1^2 \ln\left(\frac{\gamma_1^1+\mu\gamma_1^2}{1+\mu}\frac{X}{P_1}\right) + \gamma_2^2 \ln\left(\frac{\gamma_2^1+\mu\gamma_2^2}{1+\mu}\frac{X}{P_2}\right)
\end{aligned} \tag{26}
$$

2. She does not work:

$$
\begin{aligned}
V_{NP}^1 = {} & \alpha^1 \ln\left(\frac{\alpha^1}{1+\mu-\alpha^2\mu}\frac{X-\hat{y}^2}{w_1}\right) \\
& + (1-\alpha^1-\gamma_1^1-\gamma_2^1)\ln\left(\frac{(1-\alpha^1-\gamma_1^1-\gamma_2^1)}{1+\mu-\alpha^2\mu}\left(X-\hat{y}^2\right)\right) \\
& + \gamma_1^1 \ln\left(\frac{\gamma_1^1+\mu\gamma_1^2}{1+\mu-\alpha^2\mu}\frac{X-\hat{y}^2}{P_1}\right) + \gamma_2^1 \ln\left(\frac{\gamma_2^1+\mu\gamma_2^2}{1+\mu-\alpha^2\mu}\frac{X-\hat{y}^2}{P_2}\right)
\end{aligned} \tag{27}
$$

$$
\begin{aligned}
V_{NP}^2 = {} & \alpha^2 \ln\left(\bar{L}\right) \\
& + (1-\alpha^2-\gamma_1^2-\gamma_2^2)\ln\left(\frac{\mu\left(1-\alpha^2-\gamma_1^2-\gamma_2^2\right)}{1+\mu-\alpha^2\mu}\left(X-\hat{y}^2\right)\right) \\
& + \gamma_1^2 \ln\left(\frac{\gamma_1^1+\mu\gamma_1^2}{1+\mu-\alpha^2\mu}\frac{X-\hat{y}^2}{P_1}\right) + \gamma_2^2 \ln\left(\frac{\gamma_2^1+\mu\gamma_2^2}{1+\mu-\alpha^2\mu}\frac{X-\hat{y}^2}{P_2}\right)
\end{aligned} \tag{28}
$$

Money Metric Welfare Index Finally, consider how much money an individual needs to achieve some utility level U by themselves. For men, the cost-minimization problem is as follows:

$$
\begin{aligned}
& \min_{L^1,C^1,K_1,K_2} w^1L^1 + C^1 + P_1K_1 + P_2K_2 \\
& \qquad s.t. \quad \alpha^1 \ln L^1 + \left(1-\alpha^1-\gamma_1^1-\gamma_2^1\right)\ln C^1 + \gamma_1^1 \ln K_1 + \gamma_2^1 \ln K_2 = U
\end{aligned} \tag{29}
$$

which gives:

$$
E = \exp\left(U + \alpha^1 \ln w_1 + \gamma_1^1 \ln P_1 + \gamma_2^1 \ln P_2 - B^1\right) \tag{30}
$$

where

$$B^1 = \alpha^1 \ln \alpha^1 + \left(1 - \alpha^1 - \gamma_1^1 - \gamma_2^1\right) \ln\left(1 - \alpha^1 - \gamma_1^1 - \gamma_2^1\right) + \gamma_1^1 \ln \gamma_1^1 + \gamma_2^1 \ln \gamma_2^1$$

Similarly, for working women:

$$E = \exp\left(U + \alpha^2 \ln w_2 + \gamma_1^2 \ln P_1 + \gamma_2^2 \ln P_2 - B^2\right) \tag{31}$$

where

$$B^2 = \alpha^2 \ln \alpha^2 + \left(1 - \alpha^2 - \gamma_1^2 - \gamma_2^2\right) \ln\left(1 - \alpha^2 - \gamma_1^2 - \gamma_2^2\right) + \gamma_1^2 \ln \gamma_1^2 + \gamma_2^2 \ln \gamma_2^2$$

For non working women, the program is:

$$\begin{aligned} &\min_{C^2, K_1, K_2} C^2 + P_1 K_1 + P_2 K_2 \qquad (32) \\ &\qquad s.t. \quad \alpha^2 \ln \bar{L} + \left(1 - \alpha^2 - \gamma_1^2 - \gamma_2^2\right) \ln C^2 + \gamma_1^2 \ln K_1 + \gamma_2^2 \ln K_2 = U \end{aligned}$$

which gives

$$E = \left(1 - \alpha^2\right) \exp\left(\frac{U}{1 - \alpha^2} + \frac{\gamma_1^2}{1 - \alpha^2} \ln P_1 + \frac{\gamma_2^2}{1 - \alpha^2} \ln P_2 - B'^2\right) \tag{33}$$

where

$$B'^2 = \frac{\alpha^2 \ln \bar{L} + (1 - \alpha^2 - \gamma_1^2 - \gamma_2^2) \ln (1 - \alpha^2 - \gamma_1^2 - \gamma_2^2) + \gamma_1^2 \ln \gamma_1^2 + \gamma_2^2 \ln \gamma_2^2}{1 - \alpha^2}$$

Then the money metric index corresponding to some given efficient allocation (as defined by some specific μ) can be computed by replacing U with the indirect utilities.

A.4 Women's position in Japan and OECD countries

Japan is one of the most developed nations economically, but one of the most under-developed in terms of gender equality in political and economic opportunities. While Japan ranks among the top three countries in terms of the GDP per capita, it lags far behind its peer nations in gender equality. In 2022, it ranked 121 out of 153 countries in the Global Gender Gap Report 2022 (World Economic Forum, 2022). Especially, among four arenas that the World Economic Forum analyses (health, education, workplaces and politics), workplaces and politics are skewed in favor of men in Japan.

At workplace, Japanese women face a thicker glass ceiling than women in the peer nations. The top panel of Table A.1 shows the gender gap in the labor market outcomes. At the median, Japanese women earn 78 cents while Japanese men earn one dollar. The gap, 22 percent relative to men's median earnings, is twice as large as the OECD median (11 percent). The proportion of female managers is only 13 percent, which puts Japan as the worst among all OECD countries. While more women are employed than before, nearly 40 percent of them work part time. From the intrahousehold bargaining perspective, weak economic opportunities put women at a weaker position in the household bargaining.

In the household sphere, the bulk of unpaid work and childcare responsibilities still fall on women's shoulders. The bottom panel of Table A.1 focuses on women and men who are partnered and have at least one child at the age of 0-14. Japanese women work and engage in unpaid care work as likely as other OECD nations. What sets Japan apart from other OECD countries is the disengagement of men from domestic duties. Japanese men devote significantly less time to care work: for example, men with one child spend only 2.5 percent of their time on care work out of all the time they spend on primary activities (paid work, unpaid work, care work, personal care, leisure, and unspecified activities). This is only half the amount that men in other OECD countries spend. As a result, the distribution of care work within households in Japan is skewed toward women.

Table A.1: Women's position in Japan and OECD countries

Variable	Definition	Japan	OECD
Gender wage gap	Gender difference in the median earnings relative to men's median earnings.	22.11	11.31
Share managers (women)	Proportion of managers that are women.	13.20	35.40
Share part time (women)	Women's part-time employment as a proportion of women's employment.	39.02	20.71
Maternal employment rate	Employment rates (%) for women 15-64 yrs old with children (aged 0-14)	70.61	72.96
Care work time (men, 1 child)	% of time dedicated to care work (men with 1 child)	2.50	5.27
Care work time (men, 2 children or more)	% of time dedicated to care work (men with >1 children)	4.10	6.67
Care work time (women, 1 child)	% of time dedicated to care work (women with 1 child)	11.70	11.39
Care work time (women, 2 children or more)	% of time dedicated to care work (women with >1 children)	16.60	16.08

[1] *Source*: OECD Family database (https://www.oecd.org/els/family/database.htm). The column *OECD* shows the median of all OECD countries available in the database.

A.5 Sample construction

Table A.2: Sample construction

Criteria	No. of households	No. of household-year observations
Original married sample	2669	30468
At least one child aged 15 or below	2297	21021
Husband works	2198	18461
Complete expenditure and income data	2165	17072
Wife's market history	1498	11689

A.6 Key data moments

Table A.3: Data moments by demographic group

	One child			*Two children*			*Three or more children*		
	age 0-6	age 7-12	age 12-15	age 0-6	age 7-12	age 12-15	age 0-6	age 7-12	age 12-15
Wife does not participate									
Husband's consumption	1007.4	1233.6	1290.7	1024.8	1191.1	1261.6	963.2	1138.7	1137.2
Wife's consumption	408.8	574.7	358.0	371.5	412.3	390.2	288.1	300.3	290.0
Husband's leisure	1.5	1.7	1.9	1.5	1.8	1.9	1.6	1.7	1.7
Wife's leisure	2.1	4.1	5.1	1.8	3.3	3.9	1.3	2.4	3.0
Child expenses	600.0	789.4	1320.7	935.0	1269.9	1987.3	1224.3	1259.8	1823.4
Non child expenses	4774.0	5207.3	5521.7	4850.1	5227.5	5894.4	4549.5	4913.3	5338.4
Wife's housework time	11.4	7.2	7.2	12.1	8.9	7.8	13.2	11.1	9.9
Wife participates									
Husband's consumption	1096.6	1268.6	1346.2	953.7	1108.0	1204.1	1108.5	1075.7	1159.8
Wife's consumption	677.5	682.7	702.2	517.1	493.7	492.9	510.2	451.6	501.9
Husband's leisure	1.7	2.1	2.2	1.6	2.0	2.2	1.6	2.0	2.1
Wife's leisure	1.2	2.0	2.2	1.0	1.6	1.8	0.8	1.4	1.6
Child expenses	814.8	841.7	1574.6	1251.6	1257.3	1853.9	1480.5	1404.1	1856.8
Non child expenses	4581.1	5084.3	5950.9	4799.2	5005.1	5651.3	4413.4	5128.9	5508.2
Wife's housework time	5.1	4.1	3.9	6.6	5.1	4.5	7.4	6.1	5.2
Wife's participation rate	0.38	0.64	0.69	0.37	0.60	0.71	0.34	0.50	0.61

A.7 Couple wage gap

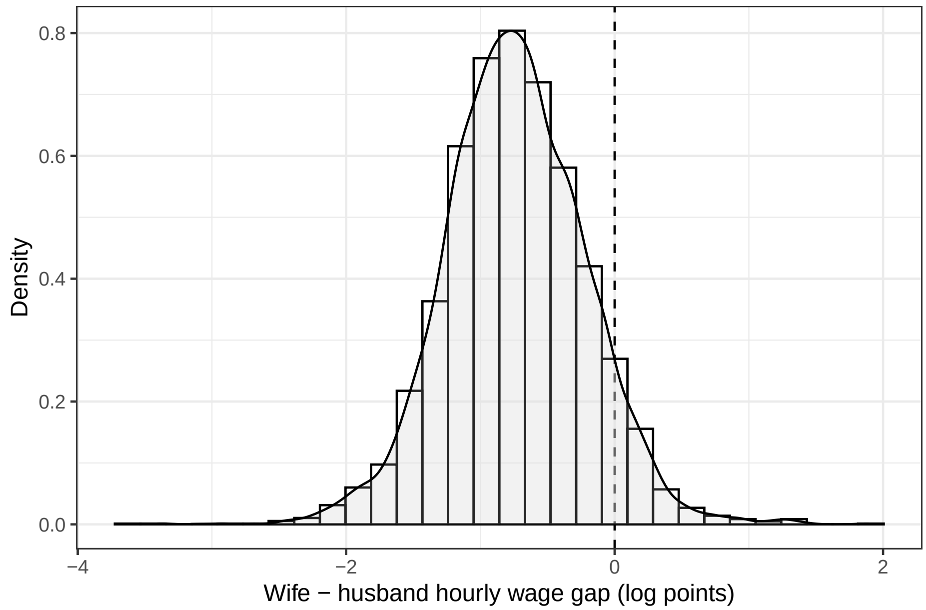

Figure A.1: Distribution of wife-husband log wage gap. For non-working wives, we predict their wages based on the wage regression 14. For working wives, we use their observed wages. The dotted vertical line indicates the point where wife and husband earn equally. The mean couple wage gap is -0.74 (std.dev. = 0.52).

A.8 Preference estimates at the data mean

Table A.4: Preference parameters evaluated at the data mean

	Husband	Wife
α leisure	0.786	0.251
β private consumption	0.075	0.048
γ_1 preference for child expenditures	0.019	0.137
γ_2 preference for other public goods	0.120	0.565

Note: This table presents wife's and husband's preference parameters evaluated at the data mean. Underlying parameter estimates are presented in Table 3.

A.9 Preference estimates by household observables

Table A.5: Preference estimates by the number of children

	One child	Two children	Three children
Husband			
α leisure	0.768	0.786	0.804
β private consumption	0.068	0.075	0.082
γ_1 preference for child expenditures	0.031	0.019	0.006
γ_2 preference for other public goods	0.132	0.120	0.108
Wife			
α leisure	0.319	0.251	0.183
β private consumption	0.053	0.048	0.042
γ_1 preference for child expenditures	0.070	0.137	0.203
γ_2 preference for other public goods	0.557	0.565	0.573

Note: This table presents preference parameters by the number of children while holding other household observables constant at the data mean. Underlying parameter estimates are presented in Table 3.