Child skill production: Accounting for parental and market-based time and goods investments

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Motivation

- Growing evidence suggests that parental investments in children are critical to intergenerational mobility & inequality
- These investments come in many forms:
  - parental time
  - home goods & services (e.g. books, computers, lessons)
  - market-based child care services

2 Outstanding Questions

We explore 2 issues theoretically & empirically:

- How does parental human capital affect different investments in children & child development?
  - through wages: time input prices & family income
  - child skill productivity differences
  - preferences for children's skills

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- How does parental human capital affect different investments in children & child development?
  - through wages: time input prices & family income
  - child skill productivity differences
  - preferences for children's skills
- How do different tax/subsidy policies affect different types of investments & child development?
  - e.g., income taxes, EITC, subsidies for sports & arts programs, child care subsidies
  - substitutability of inputs is critical

Related Literature & Our Focus

- Most of the literature on child development & estimation of skill production functions focuses on the dynamics of investments
  - studies generally reduce investment to a single endogenous input (e.g. Cunha & Heckman 2007, Cunha, Heckman & Schennach 2010, Agostinelli & Wiswall 2020, Caucutt & Lochner 2020)
  - or impose strong assumptions about substitutability between inputs (e.g. Del Boca, Flinn & Wiswall 2014, Griffen 2019, Lee & Seshadri 2019, Mullins 2022, Attanasio et al. 2020)
  - a few recent exceptions free up some assumptions about substitutability (Abbott 2022, Moschini 2023, Molnar 2023)

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  - or impose strong assumptions about substitutability between inputs (e.g. Del Boca, Flinn & Wiswall 2014, Griffen 2019, Lee & Seshadri 2019, Mullins 2022, Attanasio et al. 2020)
  - a few recent exceptions free up some assumptions about substitutability (Abbott 2022, Moschini 2023, Molnar 2023)
- We focus mainly on *intratemporal* allocation decisions about the types of investments families make each period
  - allow substitutability to differ across 3 broad types of inputs
    - parental time, home goods/services & market child care
  - allow parental skills to impact the relative productivity of inputs

Main Contributions

- We document strong growth in 3 broad types of investment with maternal education
- Using a dynamic household model of child development, we
  - characterize effects of input prices & parental education on input choices
  - show when the family decision problem can be separated into *intratemporal* & *intertemporal* decisions
- We develop & implement a relative demand estimation strategy for the within-period technology of skill production
  - estimate flexible substitution & relative productivity of different inputs
  - estimate effects of parental education on relative input productivity
  - account for unobserved heterogeneity in parental skills
  - address measurement error in inputs & parental wages
- Exploit relative demand restrictions to simplify estimation of dynamics of skill prod.
  - incorporate panel data on (noisy) skill measures
  - test whether beliefs about skill technology are accurate

Key Findings

- Estimate input elasticities of substitution of 0.2–0.5 for
  - parental time vs. home goods/services
  - home inputs (time & goods/services) vs. child care services
- This moderately strong complementarity implies co-movement of all inputs to price changes
  - wage increases can lead to increases in parental time investments
  - adjustments in other inputs have important implications for the public costs of free child care & other investment subsidies
- No evidence that maternal education makes child investment inputs more productive
  - more educated parents invest more in all inputs, because they have higher incomes & stronger preference for child skills (or higher perceived returns to investment)

Cross-Sectional Investment Patterns

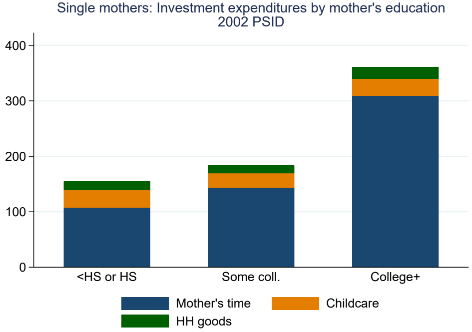
Investments in 2002 PSID-CDS

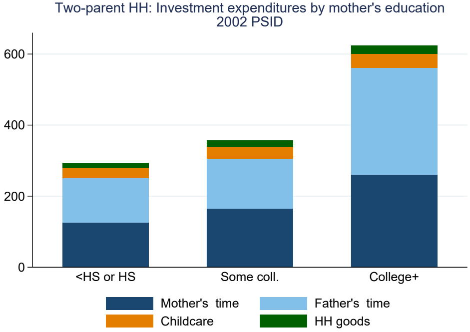
Consider weekly expenditures for families with 1–2 children, both ages 0–12

- Nearly all children were ages 5–12 in 2002 CDS
- “HH goods” investments: school supplies; books & toys; services like tutoring, lessons, community groups & sports
- Parental time: time actively engaging with children in developmental & social activities
  - based on time diaries
  - stricter definition than Del Boca, Flinn, and Wiswall (2014)

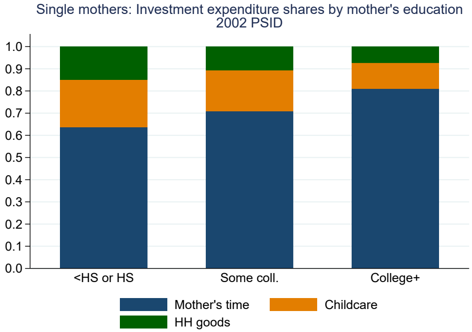
Investment Expenditures by Mother's Education

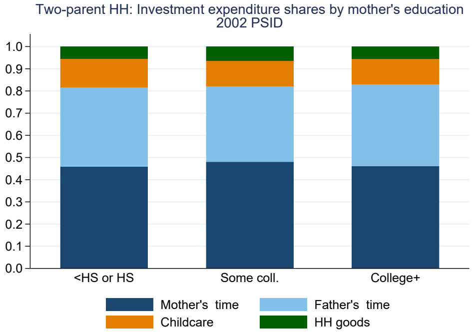
- Expenditures dominated by time investments
- Expenditures strongly increase with maternal education
  - increase in time expenditures partly reflects higher wages ▸ Parental Time
- Main difference by marital status is father's time





- Expenditure shares are similar across mother's education, especially for two-parent households
  - → More educated mothers spend more on all forms of investment





Household Model of Child Development

Model Basics

- Two-parent households differ by
  - child's ability: θ
  - child's initial skill: Ψ_1
  - human capital of mother & father: H_m & H_f
  - non-labor income: y_t
- Every period, households choose
  - consumption: c_t
  - future assets: A_{t+1}
  - mother's and father's leisure: $l_{m,t}$ & $l_{f,t}$
  - investments in children
    - home investments: goods, g_t, and parental time, $\tau_{m,t}$ & $\tau_{f,t}$
    - market-based child care services: $Y_{c,t}$
  - parental time working: $h_{j,t} = 1 - l_{j,t} - \tau_{j,t}$ for $j = m, f$

Prices

- Input price vector: $\Pi_t \equiv (W_{m,t}, W_{f,t}, p_t, P_{c,t})$
  - parental wages: $W_{j,t} = w_{j,t} H_j$
  - price of home investment goods: p_t
  - price of market child care: $P_{c,t}$
- Interest rate for borrowing/saving: r
  - borrowing limit: $A_{t+1} \geq A_{t,min}$

Preferences

- Per-period household preferences:

$$u(c_t) + \nu_m(l_{m,t}) + \nu_f(l_{f,t})$$

- Time discount rate: $\beta > 0$
- Parents invest in their children for T periods with period $T+1$ household continuation value:

$$\tilde{V}(H_m, H_f, A_{T+1}, \Psi_{T+1})$$

  where Ψ_{T+1} reflects child's final skill level

Child Skill Production

- Child skills evolve according to:

$$\Psi_{t+1} = \mathcal{H}_t \left(f_t \left(\tau_{m,t}, \tau_{f,t}, g_t, Y_{c,t}; H_m, H_f \right), \theta, \Psi_t \right)$$

- Key Assumptions:
  - weak intertemporal substitutability of inputs through “total investment” $f_t(\cdot)$
  - $f_t(\cdot)$ is homogenous of degree 1

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- Key Assumptions:
  - weak intertemporal substitutability of inputs through “total investment” $f_t(\cdot)$
  - $f_t(\cdot)$ is homogenous of degree 1
- We primarily use a nested CES:

$$f_t = \left[\left(a_{m,t}(H_m)\tau_{m,t}^{\rho} + a_{f,t}(H_f)\tau_{f,t}^{\rho} + a_{g,t}(H_m, H_f)g_t^{\rho} \right)^{\frac{\gamma}{\rho}} + a_{Y\,c,t}Y_{c,t}^{\gamma} \right]^{\frac{1}{\gamma}}$$

where $\rho < 1, \gamma < 1$

  - accommodates flexible substitution patterns: $\varepsilon_{\tau,g} = \frac{1}{1-\rho}$ & $\varepsilon_{H,Y} = \frac{1}{1-\gamma}$
  - parental human capital can affect (relative) productivity of inputs

Household's Problem

Household's problem for $t = 1, ..., T$:

$$V_t(\theta, H_m, H_f, A_t, y_t, \Pi_t, \Psi_t)$$
$$= \max_{l_{m,t}, \tau_{m,t}, l_{f,t}, \tau_{f,t}, g_t, Y_{c,t}, A_{t+1}} u(c_t) + v_m(l_{m,t}) + v_f(l_{f,t}) + \beta V_{t+1}(\theta, H_m, H_f, A_{t+1}, y_{t+1}, \Pi_{t+1}, \Psi_{t+1})$$

subject to

$$c_t + \underbrace{p_t g_t + W_{m,t}\tau_{m,t} + W_{f,t}\tau_{f,t} + P_{c,t}Y_{c,t}}_{E_t} + A_{t+1} = (1+r)A_t + y_t + W_{m,t}(1 - l_{m,t}) + W_{f,t}(1 - l_{f,t})$$

$$\Psi_{t+1} = \mathcal{H}_t\Big(\underbrace{f_t\left(\tau_{m,t}, \tau_{f,t}, g_t, Y_{c,t}; H_m, H_f\right)}_{X_t}, \theta, \Psi_t\Big)$$

$$V_{T+1}(\theta, H_m, H_f, A_{T+1}, y_{T+1}, \Pi_{T+1}, \Psi_{T+1}) = \tilde{V}(H_m, H_f, A_{T+1}, \Psi_{T+1})$$

$$A_{t+1} \geq A_{min,t}, \qquad l_{j,t} + \tau_{j,t} \leq 1 \qquad \text{and} \qquad l_{j,t}, \tau_{j,t}, g_t, Y_{c,t} \geq 0, \quad \text{for } j = m, f$$

Household's Problem

- When parents work, the household problem can be separated into
  - **Intratemporal problem:** within-period input allocation decision given a "total" investment amount, X_t
    - $\rightarrow$ implies a composite price of total investment: $\bar{p}_t$
  - **Intertemporal problem:** dynamic decision about savings, leisure & total investment each period given all $\bar{p}_t$
- Like the 2-stage budgeting approach commonly used in labor supply literature (Gorman 1959, Heckman 1974, Altonji 1986, Blundell & Walker 1986)
- Our main innovations are centered on the *intratemporal* problem
  - we specify *intertemporal* problem as in Del Boca, Flinn, and Wiswall (2014), but consider both borrowing constrained & unconstrained households

Intratemporal Problem

Expenditure minimization:

$$E_t = \min_{g_t, \tau_{m,t}, \tau_{f,t}, Y_{c,t}} p_t g_t + P_{c,t} Y_{c,t} + W_{m,t} \tau_{m,t} + W_{f,t} \tau_{f,t}$$

subject to

$$X_t = f_t(\tau_{m,t}, \tau_{f,t}, g_t, Y_{c,t}; H_m, H_f)$$

$$\tau_{j,t} < 1 \qquad \& \qquad \tau_{j,t}, g_t, Y_{c,t} \geq 0, \quad \text{for } j = m, f$$

- Optimal inputs are proportional to each other & total investment, X_t
  - input ratios depend only on relative input prices & within-period technology $f_t(\cdot)$

Intratemporal Problem

When $f_t(\cdot)$ is nested CES,

$$\frac{\tau_{j,t}}{g_t} = \Phi_{j,t}\left(\frac{W_{j,t}}{p_t}\right) = \left[\frac{a_{g,t}}{a_{j,t}}\frac{W_{j,t}}{p_t}\right]^{\frac{1}{\rho-1}}, \qquad \text{for } j = m, f$$

$$\frac{Y_{c,t}}{g_t} = \Phi_{c,t}\left(\frac{W_{m,t}}{p_t}, \frac{W_{f,t}}{p_t}, \frac{P_{c,t}}{p_t}\right) = \left[\frac{a_{g,t}}{a_{Y c,t}}\frac{P_{c,t}}{p_t}\right]^{\frac{1}{\gamma-1}} \left(a_{m,t}\Phi^{\rho}_{m,t} + a_{f,t}\Phi^{\rho}_{f,t} + a_{g,t}\right)^{\frac{\gamma-\rho}{\rho(\gamma-1)}}$$

- Note: $a_{g,t},\ a_{m,t},\ a_{f,t}$ are functions of parental human capital
- Nesting of home inputs $(g, \tau_m, \tau_f) \Rightarrow \frac{\tau_{j,t}}{g_t}$ does not depend on $\frac{P_{c,t}}{p_t}$
  - simplifies estimation of ρ

Composite Price of Investment

Investment expenditure can be written as

$$E_t \equiv \bar{p}_t(\Pi_t; H_m, H_f) X_t \qquad = p_t g_t + P_{c,t} Y_{c,t} + W_{m,t}\tau_{m,t} + W_{f,t}\tau_{f,t}$$

where the composite/unit price of total investment, $X_{i,t}$, is

$$\bar{p}_t(\Pi_t; H_m, H_f) = \frac{p_t + P_{c,t}\Phi_{c,t} + W_{m,t}\Phi_{m,t} + W_{f,t}\Phi_{f,t}}{\left[\left(a_{m,t}\Phi^{\rho}_{mt} + a_{f,t}\Phi^{\rho}_{f,t} + a_{g,t}\right)^{\frac{\gamma}{\rho}} + a_{Y c,t}\Phi^{\gamma}_{c,t}\right]^{\frac{1}{\gamma}}}$$

- depends on technology, input prices & parental human capital (H_m, H_f)
- cost of investment varies across families!

Intertemporal Problem

Can write the **Intertemporal Problem** in terms of X_t and $\bar{p}_t(\Pi_t, H_m, H_f)$:

$$
\begin{aligned}
& V_t(\theta, H_m, H_f, A_t, y_t, \Pi_t, \Psi_t) \\
& \quad = \max_{l_{m,t}, l_{f,t}, X_t, A_{t+1}} u(c_t) + v(l_{m,t}) + v(l_{f,t}) + \beta V_{t+1}(\theta, H_m, H_f, A_{t+1}, y_{t+1}, \Pi_{t+1}, \Psi_{t+1})
\end{aligned}
$$

subject to

$$c_t + \bar{p}_t(\Pi_t, H_m, H_f) X_t + A_{t+1} = (1+r)A_t + y_t + W_{m,t}(1 - l_{m,t}) + W_{f,t}(1 - l_{f,t})$$

$$
\begin{aligned}
\Psi_{t+1} &= \mathcal{H}_t\left(X_t, \theta, \Psi_t\right) \\
A_{t+1} &\geq A_{min,t} \\
V_{T+1}(\theta, H_m, H_f, A_{T+1}, y_{T+1}, \Pi_{T+1}, \Psi_{T+1}) &= \tilde{V}(H_m, H_f, A_{T+1}, \Psi_{T+1})
\end{aligned}
$$

$$0 \leq l_{m,t}, l_{f,t} \leq 1 \qquad \& \qquad X_t \geq 0$$

- Within-period production technology only enters through $\bar{p}_t$

Intertemporal Problem

Two assumptions from DFW (2014) yield an easy-to-work-with FOC for X_t:

1. $\Psi_{t+1} = \theta X_t^{\delta_1} \Psi_t^{\delta_2}$
2. $\tilde{V}(H_m, H_f, A_{T+1}, \Psi_{T+1}) = \tilde{U}(H_m, H_f, A_{T+1}) + \alpha \ln(\Psi_{T+1})$

FOC for X_t:

$$\underbrace{\bar{p}_t X_t}_{E_t} = \frac{K_t}{u'(c_t)} \qquad \text{where } K_t \equiv \alpha \beta^{T-t+1} \delta_2^{T-t} \delta_1 > 0$$

$\Rightarrow$ Investment expenditures E_t co-move with c_t

Standard FOC for $l_{j,t}$: $\quad v'(l_{j,t}) = u'(c_t) W_{j,t}, \qquad j = m, f$

Dynamics for X_t

Combining FOC for X_t with Euler equation yields dynamics for *total investment*:

$$X_{t+1} \geq \left(\frac{\bar{p}_t}{\bar{p}_{t+1}}\right)\left(\frac{1+r}{\delta_2}\right) X_t$$

- satisfied with equality for unconstrained households
- stronger investment growth for constrained households
  - X_t depends on $\bar{p}_t$ & full household income $(W_{m,t} + W_{f,t} + y_t)$
- paper discusses impacts of changes in input prices & parental skills on X_t & input allocations

Estimation Approaches

Estimation: Within-Period Production $f_t(\cdot)$

- We use revealed preferences & relative demand to estimate the substitutability & relative productivity of different inputs within periods
- Key requirements:
  - parents work positive hours (wage reflects the price of time)
  - intertemporal separability of inputs through $f_t(\cdot)$
  - $f_t(\cdot)$ is homogeneous of degree 1
  - no preferences over specific inputs
  - implicitly assume families are knowledgeable about $f_t(\cdot)$; otherwise, identifies beliefs about skill production
- Key advantages (relative to "direct" estimation approach):
  - requires no additional assumptions about dynamics of skill production, $\mathcal{H}_t(\cdot)$, or input-neutral child ability θ
  - only requires cross-sectional data on inputs & prices, not panel data on skills
  - easy to deal with measurement error in inputs – no need for multiple measures of each input

Notation and Empirical Specification for $f(\cdot)$

$$f(\tau_{m,i,t}, \tau_{f,i,t}, g_{i,t}, Y_{i,t} | Z_{i,t}) =$$
$$\left[\left(a_m(Z_{i,t}, \eta_{m,i}) \tau_{m,i,t}^{\rho} + a_f(Z_{i,t}, \eta_{f,i}) \tau_{f,i,t}^{\rho} + a_g(Z_{i,t}) g_{i,t}^{\rho} \right)^{\frac{\gamma}{\rho}} + a_Y(Z_{i,t}) Y_{c,i,t}^{\gamma} \right]^{\frac{1}{\gamma}}$$

- $Z_{i,t}$ reflects observed household characteristics
  - parents: marital status, education, age, race
  - child: age
  - household: number of children
- $\eta_{j,i}$ reflects unobserved productivity of parental time for $j = m, f$
- Assume $a_j(Z, \eta_j) = \exp(Z\phi_j + \eta_j)$ for $j = m, f$; $a_g(Z) = \exp(Z\phi_g)$; and $a_Y(Z) = \exp(Z\phi_Y)$

Measurement Error

Measurement error in investment inputs & parental wages:

$$\ln(x^o_{i,t}) = \ln(x_{i,t}) + \xi_{x,i,t}, \quad \text{for } x \in \{\tau_m, \tau_f, g, Y_c, W_m, W_f\}$$

Measurement errors are:

- mean zero
- independent of true variables (inputs, prices, observed & unobserved characteristics) and other measurement errors

Relative Demand: Parental Time vs. HH Goods Inputs

Model implies that relative demand for parental time vs. household goods (for working parents) is given by

$$\ln\left(\frac{\tau_{j,i,t}}{g_{i,t}}\right) = \varepsilon_{\tau,g}\ln\left(\frac{a_j(Z_{i,t},\eta_{j,i})}{a_g(Z_{i,t})}\right) - \varepsilon_{\tau,g}\ln\left(\frac{W_{j,i,t}}{p_{i,t}}\right), \quad j = m, f$$

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Incorporating measurement error, *relative observed expenditures* are

$$\ln(R_{j,i,t}) = Z'_{i,t}\tilde{\phi}_{jg} + (1-\varepsilon_{\tau,g})\ln\tilde{W}^o_{j,i,t} + \tilde{\eta}_{j,i} + \tilde{\xi}_{j,i,t}, \quad j = m, f$$

where $R_{j,i,t} \equiv \frac{W^o_{j,i,t}\tau^o_{j,i,t}}{p_{i,t}g^o_{i,t}}$ & $\tilde{W}^o_{j,i,t} \equiv W^o_{j,i,t}/p_{i,t}$

- $\tilde{\phi}_{jg} \equiv \varepsilon_{\tau,g}(\phi_j - \phi_g)$ & $\tilde{\eta}_{j,i} \equiv \varepsilon_{\tau,g}\eta_{j,i}$
- $\tilde{\xi}_{j,i,t} \equiv \xi_{\tau_j,i,t} - \xi_{g,i,t} + \varepsilon_{\tau,g}\xi_{W_j,i,t}$

Econometric Challenges

$$\ln(R_{j,i,t}) = Z'_{i,t}\tilde{\phi}_{jg} + (1-\varepsilon_{\tau,g})\ln \tilde{W}^o_{j,i,t} + \tilde{\eta}_{j,i} + \tilde{\xi}_{j,i,t}$$

3 econometric challenges:

1. unobserved parenting skill $\eta_{j,i}$ likely correlated with wages
2. measurement error in wages is correlated with observed wages
3. unobserved heterogeneity in $\eta_{j,i}$ implies selection into work

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First 2 challenges can be addressed with instrumental variables:

- we use measure of predicted log wages from 2000 Census based on gender, race, education, experience, state, occupation, and industry

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To address selection:

- condition on parents with high predicted probability of work (bias $\rightarrow$ 0 as probability $\rightarrow$ 1)
- include log wage fixed effects from panel data (estimates of $\eta_{j,i}$) in our set of observed characteristics $Z_{i,t}$
- also consider a Heckman two-step estimator for married mothers

Relative Demand: Child Care vs. HH Goods Inputs

Relative demand for child care vs. household goods for *single mothers* is given by

$$\ln(R_{Y_c,i,t}) = Z'_{i,t}\phi_{Y,g} + \left[\frac{\varepsilon_{Y,H} - \varepsilon_{\tau,g}}{1-\varepsilon_{\tau,g}}\right] \ln\left(1 + R_{m,i,t}e^{-\xi_{W_m\tau_m/g,i,t}}\right)$$
$$+(1-\varepsilon_{Y,H})\ln\tilde{P}_{c,i,t} + \xi_{Y_c/g,i,t}$$

where $R_{Y_c,i,t} \equiv \frac{P_{c,i,t}Y^o_{c,i,t}}{p_{i,t}g^o_{i,t}}$ & $R_{m,i,t} \equiv \frac{W^o_{m,i,t}\tau^o_{m,i,t}}{p_{i,t}g^o_{i,t}}$

- $\phi_{Y,g} \equiv \varepsilon_{\tau,g}\left(\phi_Y - \frac{\varepsilon_{\tau,g}}{\varepsilon_{Y,H}}\frac{1-\varepsilon_{Y,H}}{1-\varepsilon_{\tau,g}}\phi_g\right)$
- $\xi_{\tau_m W_m/g,i,t} \equiv \xi_{\tau_m,i,t} + \xi_{W_m,i,t} - \xi_{g,i,t}$ & $\xi_{Y_c/g,i,t} \equiv \xi_{Y_c,i,t} - \xi_{g,i,t}$

Note: Estimating equation is similar for *two-parent households*

Relative Demand: Child Care vs. HH Goods Inputs

$$\begin{aligned}\ln(R_{Y_c,i,t}) &= Z'_{i,t}\phi_{Y,g} + \left[\frac{\varepsilon_{Y,H} - \varepsilon_{\tau,g}}{1-\varepsilon_{\tau,g}}\right] \ln\left(1 + R_{m,i,t}e^{-\xi_{W_m\tau_m/g,i,t}}\right) \\ &\quad + (1-\varepsilon_{Y,H})\ln \tilde{P}_{c,i,t} + \xi_{Y_c/g,i,t}\end{aligned}$$

We consider 4 cases:

1. $\varepsilon_{Y,H} = \varepsilon_{\tau,g}$: nonlinear term drops out
2. No measurement error: estimate using OLS
3. Measurement error in inputs: substitute in predicted values $\ln\widehat{(R_{m,i,t})}$ from relative demand estimation for mother's time vs. HH goods & estimate via OLS
4. Measurement error in inputs & wages: use a 2nd order Taylor approximation & estimate via GMM or OLS

Estimation: Full Production Function

We specify dynamics of skill production as in DFW (2014):

$$\Psi_{i,t+1} = \theta_{i,t} f(\tau_{m,i,t}, \tau_{f,i,t}, g_{i,t}, Y_{c,i,t})^{\delta_1} \Psi_{i,t}^{\delta_2}$$

- Use this to derive intertemporal moment conditions on inputs & skill measures
- Combine with relative demand moments just discussed
- Estimate via optimally weighted GMM

Estimation: Full Production Function

Defining $\tilde{\Psi}_{i,t} \equiv \ln(\Psi_{i,t})$, intertemporal moments based on observing children with 5-year gaps:

$$\tilde{\Psi}_{i,t+5} = \sum_{s=0}^{4} \delta_2^{4-s} \left[\delta_1 \ln(X_{i,t+s}) + \ln(\theta_{i,t+s})\right] + \delta_2^5 \tilde{\Psi}_{i,t}$$

where $X_{i,t} = f_t(\tau_{m,i,t}, \tau_{f,i,t}, g_{i,t}, Y_{c,i,t})$ using nested CES

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where $X_{i,t} = f_t(\tau_{m,i,t}, \tau_{f,i,t}, g_{i,t}, Y_{c,i,t})$ using nested CES

Issues to deal with:

1. Some inputs missing in 1997 & no inputs are observed in between 1997, 2002 & 2007
2. Skills & inputs are measured with error

Estimation: Full Production Function

- Intratemporal optimality implies $\tau_{m,i,t} = \Phi_{m,X}(\Pi_{i,t})X_{i,t}$, where $\Phi_{m,X}(\cdot)$ depends on within-period technology $f_t(\cdot)$
- For unconstrained case, optimal dynamics of investment allow us to solve for X_{t+s} as a function of X_t & $\bar{p}_{t+s}/\bar{p}_t$

Together, these imply the following skill dynamics for unconstrained families:

$$\tilde{\Psi}_{i,t+5} = \delta_1 \sum_{t=s}^{4} \delta_2^{4-s} \ln\left(\frac{\bar{p}_{i,t}\tau_{m,i,t}}{\bar{p}_{i,t+s}\Phi_{m,X}(\Pi_{i,t})}\right) + Z_{i,t}\hat{\phi}_\theta + \delta_2^5 \tilde{\Psi}_{i,t} + \tilde{\xi}_{\theta,i,t+5}$$

- for constrained families, an additional term accounting for growth in full income must also be included

Estimation: Full Production Function

Other estimation issues:

- Use two measures of cognitive skills (Letter Word and Applied Problems scores) to address measurement error in skills
- Use measure of $\tau^{o}_{m,t+5}$ to instrument for $\tau^{o}_{m,t}$ to address measurement error in mother's time investment
- Allow for unobserved heterogeneity (in θ and a's) by using group fixed effects estimator based on classification routine of Bonhomme and Manresa (2015)
    - use full panel of mother's wages in PSID
- Can relax assumption that relative demand is driven by same technology parameters as those determining actual skill dynamics
    - $\Phi_{m,X}(\cdot)$ becomes a function of both "perceived" & "true" within-period technology parameters
    - we implement several tests for equality of these parameters

Data

Data Overview

PSID

- 1968-2007: hours, wages, HH structure, race, education
- Estimate $\eta_{j,i}$ from parental log wage regressions

PSID-CDS

- 1997: test scores, parental time, child care expenditure
- 2002 & 2007: test scores, parental time, child care expenditure, home goods/services expenditure

Prices (by state & year)

- *Child Care Aware of America*: state averages 2006–2018, $P_{c,t}$
  - combine with CPS data to predict earlier years
- *Regional Price Parities* from BEA for goods and services, p_t

Empirical Results

Empirical Implementation

- Sample: children ages 0–12 from families with 1–2 children in that age range (1997, 2002, 2007)
  - mostly use children ages 5–12 in 2002 & 10–12 in 2007
- To alleviate concerns about selection into work, we restrict analysis to mothers with predicted probability of at least 0.75 (at least 0.90 for fathers)
  - predictions based on educational attainment, age, race, household structure, children's ages
  - results are similar for other probability thresholds

Within-Period Production Function Estimation: Preliminaries

- All within-period specifications control for: child's age, parental education (some coll, coll$+$) or log wage FE, mother's race, number of young children in HH, and number of children in HH
- Estimate relative demand for child care vs. home inputs using only families with positive child care expenditure
  - little evidence that reporting zero expenditure is related to $P_{c,i,t}$
  - probability of positive child care increasing in mother's education and decreasing in older children & older relatives living in house
  - consistent with families receiving free child care from extended family or older children

Summary of Within-Period Production Function Estimates

- elasticities of substitution ranging from 0.2 to 0.5 imply moderately strong complementarity
- effects of characteristics on input productivity:
  - no consistent effects of parental education on relative productivity of their time or HH goods inputs
  - most specifications suggest that the relative productivity of HH goods inputs is increasing in child's age
  - no effect of state-year child care staff/child ratios
- estimates are largely insensitive to how we account for unobserved heterogeneity
  - 2SLS estimates using predicted log wages from 2000 Census
  - including parental log wage FE
- abstracting from measurement error in wages has little effect on estimated elasticities
- cannot reject similar relative demand
  - by child's ages 5–8 vs. 9–12
  - by child's prior achievement $\rightarrow$ intertemporal substitutability
  - by father's wage $\rightarrow$ homotheticity of $f_t(\cdot)$

Full Production Function Estimation: Preliminaries

Estimation of the full production function, $\mathcal{H}(\cdot)$ & $f(\cdot)$

- Exploit moments related to:
  - same input ratios as before for 2002 & 2007 plus Y_c/τ_m from 1997
  - achievement dynamics from 1997 to 2002 & 2002 to 2007
- Estimate separate specifications for skill dynamics assuming:
  - non-binding borrowing constraints
  - no borrowing/saving (also assume log preferences for consumption & leisure)
- Allow share parameters (a_k) to vary by marital status, but assume the same elasticity parameters (γ, ρ) and skill dynamics (δ_1, δ_2)
- Allow for unobserved heterogeneity in θ and share parameters using group fixed effects estimator
  - classification based on mother's lifetime wages $\rightarrow$ 3 unobserved types

Full Production Function Estimates (GMM Using All Moments)

	No Borrowing/Saving	Unconstrained
$\varepsilon_{\tau,g}$	0.20	0.20
	(0.05)	(0.05)
$\varepsilon_{Y,H}$	0.49	0.49
	(0.08)	(0.08)
δ_1	0.12	0.08
	(0.04)	(0.04)
δ_2	0.93	0.93
	(0.01)	(0.01)

$$\Psi_{i,t+1} = \theta_i X_{i,t}^{\delta_1} \Psi_{i,t}^{\delta_2}$$

- Moderately strong complementarity, stronger between home inputs
- 10pp increase in investment leads to a roughly 0.01 SD increase in skill
- High self-productivity of skill, δ_2
- Modest effects of mother's education & unobserved skill on productivity of her time
- Cannot reject that relative demand is consistent with "true" $f_t(\cdot)$

Counterfactual Analysis

Counterfactual Analysis

We use our GMM estimates for the case of no borrowing/saving to study

1. investment differences by maternal education
2. effects of input price changes
3. cost of free childcare

- Calibrate preference parameters (α, ψ_m, ψ_f) to match time use patterns from 2002 PSID separately by maternal education (college vs. non-college)

▸ Calibration Targets ▸ Calibrated Parameters

Investment & Expenditure Gaps by Parental Education

Among **single mothers**, college-educated spend 50% more on investments than non-college-educated Table

- some of the gap comes from paying higher prices (wages)
- also a nearly 33% gap in total investment X
  - equalizing technology differences by maternal human capital only closes the investment gap by about 10%
- Investment gaps by mother's education are largely driven by income effects and differences in preferences (or beliefs about the productivity of investment), not productivity differences

Simulating the Effects of Price Changes

- Many policies influence investment decisions by changing input prices
  - e.g., child care subsidies, tax & welfare policies
- We simulate the effects of 30% reductions in input prices when children are ages 5–12
- Study the effects on
  - investments at age 5
  - child achievement at age 13
- Contrast with implications from a Cobb-Douglas production function with identical expenditure shares

30% Reduction in Prices: Single Mothers

	Nested CES				Cobb-Douglas			
	Wages	Wages (Constant income)	Goods	Child Care	Wages	Wages (Constant income)	Goods	Child Care
A. Change in Investment at Age 5 (%)								
Total Expenditure (E)	-30.00	0.00	0.00	0.00	-30.00	0.00	0.00	0.00
Investment Quantity:								
Mother's Time (τ_m)	-5.70	34.71	1.23	3.82	0.00	42.85	0.00	0.00
Goods (g)	-11.94	25.80	8.60	3.67	-30.00	0.00	42.86	0.00
Child Care (Y_c)	-20.16	14.06	0.68	23.54	-30.00	0.00	0.00	42.86
Total (X)	-9.59	29.15	1.37	7.58	-9.18	29.75	1.60	8.37
B. Effects on Age 13 Achievement								
100 $\times$ Log Achievement at age 13	-8.25	18.68	1.83	5.20	-7.65	19.28	2.07	5.57
Value (% Cons. over Ages 5–12)	-4.99	12.44	1.15	3.28	-4.63	12.87	1.31	3.52

- Due to complementarity, all input quantities move together
- Cobb-Douglas implies stronger own-price & zero cross-price effects

30% Reduction in Prices: Single Mothers

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- Effects of income reduction with lower wages dominate price-reduction effects, even for mother's time investment
  - modest reductions in achievement are broadly consistent with effects of EITC on achievement (Dahl & Lochner 2012, Agostinelli & Sorrenti 2018)

30% Reduction in Prices: Other Thoughts

- Price effects of temporary wage change dominate income effects for unconstrained HH
- Policy effects are generally smaller but qualitatively similar for two-parent HH
  table
- Input substitutability is important for effects of large (but not small) price changes on total investment & achievement table

Costs of Free Child Care

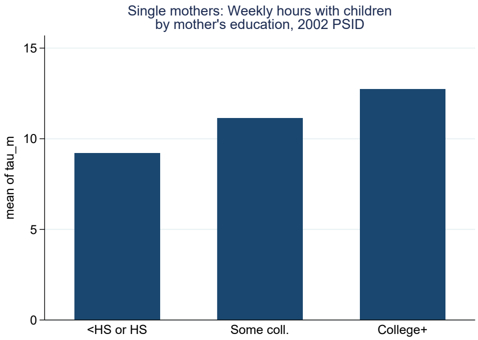
How much would it cost to provide free child care to non-college mothers to eliminate total investment gaps (ages 5–12) by mother's education?

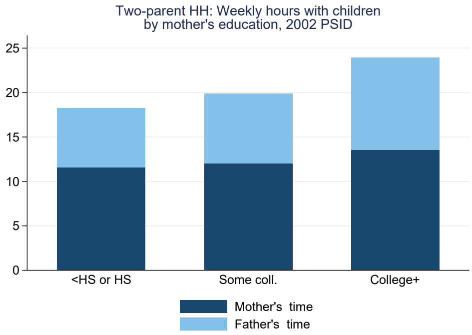
- would cost only \$100/week for single non-college mothers
- families respond to savings from free care by increasing other inputs
  - absent these responses, cost would be prohibitive
  - Cobb-Douglas specification implies 8% higher costs due to smaller responses in other inputs
- Reinforcing investment responses lower public expenditures but are costly for families
  - manageable for single mothers who save a lot from reduced child care expenses
  - non-college two-parent HH save less from free child care $\rightarrow$ unwilling to increase other investments enough to eliminate the education gap even with substantial amounts of free child care
  - by contrast, Cobb-Douglas specification suggests it would "only" cost about \$300/week in public child care to eliminate the gap for two-parent HH

Conclusions

- Families investments take many forms: time, goods/services & child care
- We develop a relative demand estimation approach to identify input substitutability & relative productivity
  - only requires data on input prices & quantities (not skills)
  - avoids assumptions about dynamics of skill production & child ability
  - easily addresses unobserved heterogeneity & measurement error in inputs
  - we also exploit relative demand to greatly simplify estimation of skill dynamics
    - can even test whether beliefs are accurate
- Substitutability of different investments is important
  - our estimates suggest moderately strong complementarity of inputs
  - implies that inputs co-move in response to taxes/subsidies
  - income effects of wage increases dominate price effects for constrained families, leading to stronger investment & skill accumulation
- No consistent effect of parental education on productivity of investments
  - positive parental education gradient is driven by overall demand – resources & tastes (or perceptions) – not factor augmentation

Time Investment by Mother's Education (PSID)





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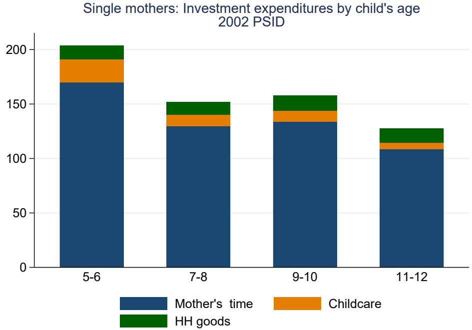
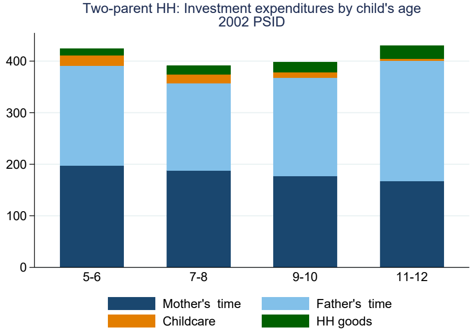
Investment Expenditures by Child's Age (PSID)





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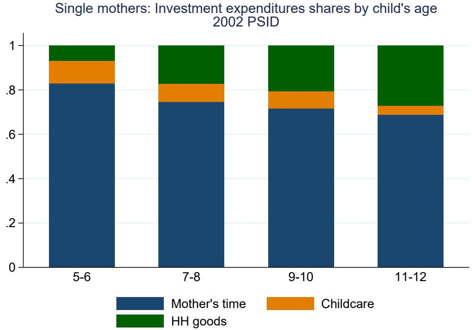
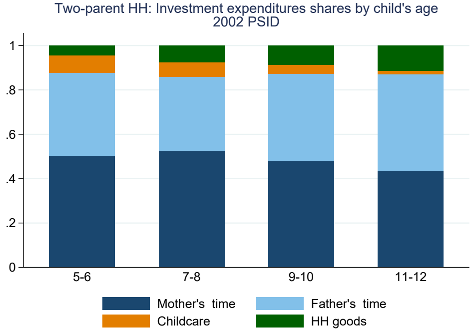
Investment Expenditure Shares by Child's Age (PSID)





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Linking Empirical & Theoretical Specifications

- Let parental human capital be $H_{j,i,t} = exp(Z_{i,t}\Gamma_j + \tilde{\eta}_{j,i})$, so

$$\ln(W_{j,t}) = \ln(w_{j,t}) + Z_t\Gamma_j + \tilde{\eta}_j$$

- Assuming that $\varphi_j(H_j) = H_j^{\bar{\varphi}_j}$ implies that $a_j(Z, \eta_j) = \exp(Z\phi_j + \eta_j)$ where $\phi_j = \Gamma_j\bar{\varphi}_j\rho$ and $\eta_j = \tilde{\eta}_j\bar{\varphi}_j\rho$
- For $\rho < 0$ $(0 < \varepsilon_{\tau,g} < 1)$, the marginal effects of characteristics that improve parental wages ($\Gamma_j > 0$) will imply $\phi_j < 0$ when parental skills raise the marginal value of parental time inputs (i.e., $\varphi_j'(H) > 0$)
- Because parental HC is factor augmenting, an increase in parental HC raises the total effective time input, which may cause parents to spend relatively less time investing

No Measurement Error in Wages, Time or Goods Inputs

If $\xi_{W_m \tau_m / g,i,t} = 0$, then estimating equation simplifies to:

$$\ln(R_{Y_c,i,t}) = Z'_{i,t}\tilde{\phi}_g + \left[\frac{\gamma-\rho}{\rho(\gamma-1)}\right] \ln\left(1 + R_{m,i,t}\right) + \left(\frac{\gamma}{\gamma-1}\right) \ln \tilde{P}_{c,i,t} + \xi_{Y_c/g,i,t}$$

- Can estimate using OLS

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Measurement Error in Inputs

If $\xi_{W_m,i,t} = 0$, then estimating equation simplifies to:

$$\ln(R_{Y_c,i,t}) = Z'_{i,t}\tilde{\phi}_g + \left[\frac{\gamma-\rho}{\rho(\gamma-1)}\right]\ln\left(1+e^{\ln(\tilde{\Phi}_{m,i,t})}\right) + \left(\frac{\gamma}{\gamma-1}\right)\ln\tilde{P}_{c,i,t} + \xi_{Y_c/g,i,t}$$

where $\tilde{\Phi}_{m,i,t} \equiv \frac{W_{m,i,t}\tau_{m,i,t}}{p_{i,t}g_{i,t}}$

- Substitute predicted values $\ln\widehat{(R_{m,i,t})}$ (from relative demand estimation for mother's time vs. HH goods) in for $\ln(\tilde{\Phi}_{m,i,t})$ above and estimate via OLS

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Measurement Error in Inputs & Wages

$$E\left[\ln(R_{Y_c,i})\middle|Z_i, R_{m,i}, \tilde{P}_{c,i}, g_i^o\right] = Z_i'\tilde{\phi}_g + \left[\frac{\gamma-\rho}{\rho(\gamma-1)}\right] E\left[\ln\left(1 + R_{m,i}e^{-\xi_{W_m\tau_m/g,i}}\right)\middle|R_{m,i}\right] + \left(\frac{\gamma}{\gamma-1}\right)\ln\tilde{P}_{c,i} - E[\xi_{g,i}|g_i^o]$$

- Distributional assumptions on measurement errors enable a GMM approach (requires integrating over expectation term in red)

Taking a second order Taylor approximation for term in red and assuming normality in $(g_i, \xi_{g,i})$ yields:

$$E\left[\ln(R_{Y_c,i})\middle|Z_i, R_{m,i}, \tilde{P}_{c,i}, g_i^o\right] \approx Z_i'\tilde{\phi}_g + \left(\frac{\gamma-\rho}{\rho(\gamma-1)}\right)\ln(1+R_{m,i}) + \sigma^2_{W_m\tau_m/g}\left(\frac{\gamma-\rho}{\rho(\gamma-1)}\right)\left(\frac{R_{m,i}}{2(1+R_{m,i})^2}\right) + \left(\frac{\gamma}{\gamma-1}\right)\ln(\tilde{P}_{c,i}) - \sigma^2_{\xi_g}\left(\frac{\ln(g_i^o)-E[\ln(g_i^o)]}{Var(\ln(g_i^o))}\right)$$

- Can estimate via GMM or OLS

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Estimation: Full Production Function

We use the following:

- Intratemporal optimality implies $\tau_{m,i,t} = \Phi_{m,X}(\Pi_{i,t})X_{i,t}$, where $\Phi_{m,X}(\cdot)$ depends on within-period technology $f_t(\cdot)$
- Optimal dynamics of investment allow us to solve for X_{t+s} as a function of
  - X_t & $\bar{p}_{t+s}/\bar{p}_t$ in the unconstrained case
  - $W_{m,t+s} + W_{f,t+s} + y_{t+s}$ & $\bar{p}_{t+s}$ in the constrained case

to obtain the following skill dynamics based on observed data:

$$\begin{aligned}\tilde{\Psi}_{i,t+5} \quad = \quad & \delta_1 \sum_{t=s}^{4} \delta_2^{4-s} \left[\ln \left(\frac{\bar{p}_{i,t}\tau_{m,i,t}}{\bar{p}_{i,t+s}\Phi_{m,X}(\Pi_{i,t})} \right) + \kappa \ln \left(\frac{W_{m,i,t+s} + W_{f,i,t+s} + y_{i,t+s}}{W_{m,i,t} + W_{f,i,t} + y_{i,t}} \right) \right] \\ & + Z_{i,t}\hat{\phi}_\theta + \delta_2^5 \tilde{\Psi}_{i,t} + \tilde{\xi}_{\theta,i,t+5}\end{aligned}$$

- $\kappa = 0$ reflects unconstrained case; $\kappa = 1$ reflects no borrowing/saving case
- assumes log utility over consumption & leisure in no borrowing/saving case
- age is only time-varying factor affecting $\theta_{i,t}$

Estimating Log Wage Fixed Effects, $\eta_{j,i}$

Estimating log wage fixed effects, $\eta_{j,i}$, for mothers & fathers, we

- use gender-specific regressions of log wages on experience, experience-squared, year & state indicators
- drop all years with children ages ≤ 12 in HH
- require at least 5 observations over 1968–2007
  - median of 10 obs. per person

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PSID-CDS Data

- CDS followed children ages 0–12 in 1997, re-surveying them in 2002 & 2007
  - we focus on children ages 0–12 in any given year
- Cognitive measures: *Letter-Word (LW)* & *Applied Problems (AP)* scores from Woodcock-Johnson tests at ages 3+
- Time investment: time parents spend actively engaging in social & developmental activities with child
  - 1 random weekday & 1 random weekend day
- Child care expenditures based on following:
  - child-specific weekly expenditures from current arrangement
  - total weekly HH expenditures on child care divided by number of children ages 0–12
- HH goods/services inputs (2002 & 2007): spending on school supplies; toys; sporting activities; tutoring; lessons (dance, music, other hobbies); and community group activities

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Price Data

- Price of child care services, P_{ct}
  - *Child Care Aware of America* provides average annual prices for full-time family-based care centers for 4-year-old children by state & year
  - using data from 2006–2018, we regress state-year costs on state FE, linear time trend, and average state-year hourly wages for child care workers from CPS ($R^2 = 0.86$), then predict state-year values back to 1997
- HH goods/services input prices, p_t
  - *Regional Price Parities by State (RPP)* from BEA measures differences in prices by state & year for 2008–2017
    - use goods & services (excluding rent/shelter) components
    - combine *RPP* with regional *CPI* (separately for goods & services excluding rent) to project back from 2008 values
  - weighted average of prices for goods (70%) and services (30%) — based on rough breakdown of HH goods & services investment spending in CEX & PSID-CDS

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Summary statistics for full sample: 2002 and 2007

	N	mean	sd	min	max
$\ln(\tilde{W}_m)$	1110	2.44	0.66	-3.07	3.99
$\ln(\tilde{W}_f)$	835	2.93	0.60	1.25	4.90
$\ln(\tilde{P}_{c,i})$	1512	1.10	0.32	0.27	1.89
Child's age	1512	9.53	2.10	5.00	12.00
Mother HS grad	1510	0.33	0.47	0.00	1.00
Mother some coll.	1510	0.32	0.47	0.00	1.00
Mother coll+	1510	0.27	0.44	0.00	1.00
Mother's age	1512	37.56	6.43	21.00	55.00
Father HS grad	951	0.36	0.48	0.00	1.00
Father some coll.	951	0.22	0.42	0.00	1.00
Father coll+	951	0.33	0.47	0.00	1.00
Father's age	937	40.50	7.04	20.00	65.00
Mother white	1499	0.58	0.49	0.00	1.00
Num children age 0-5	1512	0.19	0.42	0.00	2.00
Num of children	1512	2.02	0.73	1.00	6.00
Year=2007	1512	0.22	0.41	0.00	1.00

OLS & 2SLS estimates for mother time/goods relative demand

	OLS	OLS	OLS	OLS	2SLS (pred wage)	2SLS (state, year)
$\ln(\tilde{W}_{m,t})$	0.645*	0.646*	0.609*	0.758*	0.553*	0.749*
	(0.071)	(0.071)	(0.078)	(0.092)	(0.196)	(0.216)
Married	-0.075	-0.074	-0.121	0.022	-0.071	-0.069
	(0.095)	(0.095)	(0.104)	(0.108)	(0.096)	(0.095)
Child's age	-0.141*	-0.141*	-0.147*	-0.147*	-0.140*	-0.139*
	(0.022)	(0.022)	(0.025)	(0.024)	(0.022)	(0.022)
Mother HS grad	0.099					
	(0.350)					
Mother some coll.	0.106	0.011	-0.043		0.026	-0.018
	(0.351)	(0.102)	(0.117)		(0.113)	(0.117)
Mother coll+	-0.061	-0.157	-0.245		-0.119	-0.218
	(0.357)	(0.112)	(0.131)		(0.155)	(0.164)
Mother's age	-0.008	-0.008	-0.002		-0.007	-0.009
	(0.008)	(0.008)	(0.008)		(0.008)	(0.008)
Mother white	-0.244*	-0.243*	-0.095	-0.328*	-0.233*	-0.249*
	(0.090)	(0.089)	(0.107)	(0.102)	(0.091)	(0.090)
Num. of children ages 0-5	0.156	0.158	0.081	0.163	0.168	0.155
	(0.126)	(0.125)	(0.144)	(0.169)	(0.126)	(0.125)
Num. of children	0.089	0.089	0.090	0.027	0.082	0.097
	(0.062)	(0.062)	(0.068)	(0.066)	(0.063)	(0.063)
Mother's cognitive score			-0.005			
			(0.003)			
Mother's log wage FE				-0.346*		
				(0.114)		
Constant	2.126*	2.213*	2.602*	1.745*	2.398*	1.999*
	(0.469)	(0.355)	(0.449)	(0.366)	(0.520)	(0.553)
R-squared	0.190	0.190	0.167	0.193		
Sample size	727	727	603	562	720	727

OLS estimates for parental time vs. goods relative demand, by parent type

	(1) All Mothers	(2) Single Mothers	(3) Married Mothers	(4) Married Fathers
$\ln(\tilde{W}_{j,t})$	0.646*	0.711*	0.628*	0.678*
	(0.071)	(0.155)	(0.079)	(0.090)
Married	-0.074			
	(0.095)			
Child's age	-0.141*	-0.162*	-0.132*	-0.107*
	(0.022)	(0.043)	(0.026)	(0.027)
Parent some coll.	0.011	0.198	-0.124	-0.130
	(0.102)	(0.173)	(0.128)	(0.131)
Parent coll+	-0.157	0.009	-0.269*	0.071
	(0.112)	(0.222)	(0.132)	(0.127)
Parent's age	-0.008	-0.014	-0.005	-0.010
	(0.008)	(0.014)	(0.009)	(0.008)
Mother white	-0.243*	-0.413*	-0.170	-0.053
	(0.089)	(0.167)	(0.107)	(0.123)
Num. of children age 0-5	0.158	-0.139	0.291*	0.148
	(0.125)	(0.239)	(0.147)	(0.134)
Num. of children	0.089	0.081	0.107	0.168*
	(0.062)	(0.109)	(0.076)	(0.080)
Constant	2.213*	2.471*	1.982*	1.282*
	(0.355)	(0.691)	(0.429)	(0.434)
R-squared	0.190	0.197	0.194	0.154
Sample size	727	236	491	582

Calibration Targets

Table: Weekly Hours of Time Investment and Work

	Mother's Education	
	Non-College	College
A. Single Mothers		
Mother's Time Investment	10.04	12.42
Mother's Hours Worked	42.26	38.22
B. Two-Parent Households		
Mother's Time Investment	9.56	12.13
Mother's Hours Worked	38.43	38.58
Father's Hours Worked	43.85	44.03

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Calibrated Preference Parameters (No Borrowing/Saving)

	Mother's Education	
	Non-College	College
A. Single Mothers		
α	3.93	4.90
ψ_m	1.27	1.46
B. Two-Parent Households		
α	2.26	3.11
ψ_m	0.50	0.54
ψ_f	0.66	0.57

Back

Gaps by Parental Education

	Baseline	Equalizing: Preferences	Preferences and Wages	All but Technology	Wages	Technology	Wages and Technology
A. Single Mothers							
Total Investment							
Expenditure (E)	50.56	34.09	3.32	0.00	15.98	50.56	15.98
Price ($\bar{p}$)	14.23	14.23	-6.08	-1.67	-6.08	19.02	-4.01
Quantity (X)	32.31	17.86	9.17	2.11	22.54	28.42	20.35
Mother's Time Investment (τ_m)	23.75	10.24	5.57	0.12	18.51	22.79	18.21
B. Two-Parent Households							
Total Investment							
Expenditure (E)	102.68	49.28	-2.01	0.00	33.04	102.68	33.04
Price ($\bar{p}$)	46.88	46.88	2.32	0.71	2.32	48.77	1.58
Quantity (X)	37.82	1.52	-3.75	-0.56	30.67	36.33	31.01
Mother's Time Investment (τ_m)	26.97	-6.49	-6.98	-4.58	26.29	31.47	31.99

Back

Effects of Small vs. Large Price Changes

Table: Elasticity of Total Investment Quantity with Respect to Input Prices

Price Change	Nested CES				Cobb-Douglas				% Difference between Co Douglas and Nested CE		
	Wages	Wages (Constant income)	Goods	Child Care	Wages	Wages (Constant income)	Goods	Child Care	Wages	Wages (Constant income)	Goods
A. Single Mothers											
10% Change	0.28	-0.80	-0.04	-0.23	0.28	-0.80	-0.05	-0.24	0.37	-0.14	6.20
30% Change	0.32	-0.97	-0.05	-0.25	0.31	-0.99	-0.05	-0.28	-4.35	2.04	16.56
50% Change	0.38	-1.24	-0.05	-0.29	0.34	-1.32	-0.06	-0.34	-9.78	5.95	31.80
B. Two-Parent Households											
10% Change	0.16	-0.93	-0.03	-0.13	0.16	-0.94	-0.03	-0.13	-2.65	0.51	4.39
30% Change	0.19	-1.16	-0.03	-0.14	0.17	-1.18	-0.03	-0.15	-8.07	1.88	14.80
50% Change	0.23	-1.54	-0.03	-0.15	0.20	-1.60	-0.04	-0.18	-14.57	4.39	29.96

Back

30% Reduction in Prices: Two-Parent Households

	Nested CES				Cobb-Douglas			
	Wages	Wages (Constant income)	Goods	Child Care	Wages	Wages (Constant income)	Goods	Child Care
A. Change in Investment at Age 5 (%)								
Total Expenditure (E)	-30.00	0.00	0.00	0.00	-30.00	0.00	0.00	0.00
Investment Quantity:								
Mother's Time (τ_m)	-3.33	38.10	0.75	2.10	0.00	42.86	0.00	0.00
Father's Time (τ_f)	-3.22	38.26	0.73	2.01	0.00	42.86	0.00	0.00
Goods (g)	-9.74	28.94	8.07	1.96	-30.00	0.00	42.86	0.00
Child Care (Y_c)	-18.52	16.40	0.44	21.63	-30.00	0.00	0.00	42.85
Total (X)	-5.68	34.75	0.88	4.14	-5.22	35.40	1.01	4.46
B. Effects on Age 13 Achievement								
100×Log Achievement at age 13	-4.71	22.22	1.12	2.78	-4.29	22.64	1.29	3.01
Value (% Cons. over Ages 5–12)	-1.78	9.03	0.43	1.07	-1.63	9.23	0.50	1.16

- Achievement effects for two-parent HH are smaller but qualitatively similar

back