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# Estimating intra-household sharing from time-use data

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# Estimating Intra-Household Sharing from Time-Use Data

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**Abstract**

Households do not share resources equally between their members, so estimating intra-household inequality is crucial to understanding overall inequality. However, estimating the sharing rule is difficult because expenditure data is almost always at the household level. A growing literature proposes methods to estimate sharing from individual-level demand data for a single private good, the 'assignable good'. Building on recent developments in this literature, this paper extends the underlying structural household model and identification results in several directions. Using private leisure as the assignable good, I show it is possible to estimate sharing by linear regression from time-use data alone. I illustrate this approach for UK working couples. Women command fewer resources than men on average, and characteristics such as wages affect sharing in a manner consistent with bargaining theory.

*Keywords:* Collective model, Sharing rule, Household bargaining, Bargaining power, Consumption inequality, Time-use

## 1 Introduction

It is empirically challenging to measure individual-level inequality because expenditure data is typically collected at the household level. The simplest solution to this problem, which is

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still widely used, for instance by the World Bank, is to assume there is no intra-household inequality, so that household-level expenditure, divided by number of household members, directly provides estimates of individual-level consumption (the per capita approach). A growing body of evidence suggests that the equal sharing assumption is unrealistic and that it is crucial to account for intra-household, as well as inter-household, inequality (e.g. Lechene et al. (2022)). Not doing so leads to inaccurate, generally downward-biased, estimates of the aggregate level of inequality. It also fails to accurately capture key dimensions of inequality, such as gender and age, and their intersection (e.g. see Calvi (2020)).

Recent literature has made important advances in the estimation of individual-level consumption from household-level expenditure.[1] While earlier papers such as Chiappori (1992), Browning, Bourguignon et al. (1994) and Blundell, Chiappori, Magnac et al. (2007) were able to identify the sharing rule up to a constant, later work has shown the theoretical possibility of identifying the sharing rule, including its location, under quite a general model (e.g. Blundell, Chiappori and Meghir (2005) and Chiappori and Ekeland (2009)). A variety of different approaches have been proposed, which can broadly be categorised into three strands. The first draws on revealed preference techniques to estimate bounds on household sharing, e.g. Cherchye, De Rock, Lewbel et al. (2015). The other two strands point-identify the sharing rule from estimates of demand functions combined with strong identifying assumptions. One of these strands identifies the sharing rule from expenditure systems for multiple goods, e.g. Browning, Chiappori and Lewbel (2013) (henceforth BCL), Lewbel and Pendakur (2008) and Bargain and Donni (2012). The other strand identifies the sharing rule from data on individual-level demand for a single private good, known as the 'assignable good', e.g. Lise and Seitz (2011), Dunbar et al. (2013) (henceforth DLP), Bargain, Donni and Hentati (2022), and Lechene et al. (2022) (henceforth LPW).

I contribute to the latter strand of the literature, which has the advantage of yielding point estimates for resource shares with transparent identification and a methodology which is easily applied in a wide variety of contexts. These are important advantages for a literature that aims to develop methods with the potential of replacing the 'per capita' approach as the standard method to estimate individual-level inequality, to be employed both by academics and by non-academic institutions, and understood and trusted by policy-makers to guide decisions on targeting and evaluating policy.

Most papers in this strand of the literature use clothing as the assignable good, as some ex-

[1] Some papers estimate sharing directly from data on individual-level expenditure e.g. Cherchye, De Rock and Vermeulen (2012). However, this type of data is very unusual and, even when it is available, we may be concerned about its reliability. Often this kind of data is based on self-reported recall of how broad expenditure categories were split between members, raising questions about its accuracy.

penditure surveys contain clothing expenditure divided into men's, women's and children's clothing. They also generally model only material goods. I extend the underlying structural model of the household to detailed time-use.[2] This extension opens the possibility of using private leisure as the assignable good for identification of the sharing rule. Private leisure is time spent on leisure activities with nobody in the household co-present. Where time-use data is available, using private leisure as the assignable good, instead of clothing, is likely to increase accuracy of estimates. Importantly, private leisure is more credibly private than clothing, which is crucial to identification. Moreover, clothing expenditure is infrequent and hence has a high proportion of zeros in the data. Identification relies on demand functions which treat observations as interior solutions, so using an assignable good with a high proportion of zeros in the data adds substantial inaccuracy to the estimation approach, as corner solutions are treated as interior solutions. By contrast, leisure has virtually no zeros in the data, and has a larger budget share than clothing, promoting accuracy of estimates (see section 3.1 for a more detailed discussion). Additionally, as argued by Becker (1965), time-use is a key determinant of well-being. The extension to time-use enables applications of resource share estimates to measuring 'full' individual-level inequality, inclusive of time-use as well as material consumption.

I also extend the structural model in a second direction by incorporating public goods as well as private goods. Most of this strand of the literature treats all goods as non-public and models household economies of scale through the shareable goods framework (e.g. see BCL, DLP and LPW). In this framework, larger households face lower prices for goods. This way of capturing economies of scale is very specific and is not equivalent to modelling public goods.[3] However, public goods are a key driver of household economies of scale, and households spend a very large proportion of their budgets on public goods, e.g. housing. Properly modelling public goods is therefore crucial to accurate estimation of individual-level inequality.

Most identification results in this literature, including this paper, apply to broad families of preferences. In order to identify the way that households share all resources just from demand data for one assignable good, this strand of the literature requires strong identifying

[2]This extension makes some material differences to the model, for instance the household budget is full income, not realised earnings.

[3]Most importantly, shareable goods do not incorporate the restriction of equal consumption by different household members which is at the core of public consumption (see Browning, Chiappori and Lewbel (2013)). Household optimisation under fully shareable goods and public goods will in general yield different solutions. A strength of the shareable goods framework is that it does not impose the degree of economies of scale of different goods, and in some cases this can be recovered empirically (in BCL, but not in DLP or LPW). This paper focuses only on the pure private-public framework (which can incorporate partially public goods by appropriately defining goods e.g. splitting car fuel into car fuel used for holidays and car fuel used for work trips) but it readily extends to a model with both shareable and public goods.

assumptions. This is also true for this paper, but I propose a somewhat milder variation of the assumptions generally made in the literature. For implementation, I illustrate a simple parametric example which has important advantages in terms of tractability, applicability to realistic data and the feasibility of estimating not just the sharing rule, but also individual-level consumption and money-metric measures of welfare.

While some of the earlier identification results and estimation approaches in this literature were challenging to understand and complex to replicate, the literature has moved towards more transparent identification and simple estimation. The recent contribution by LPW shows it is possible to estimate resource shares by linear regression, with a very clear and elegant identification proof. That contribution substantially furthers the goal of developing methods to estimate intra-household inequality which can be feasibly understood, trusted, and used outside of academia. Therefore, this paper retains the linear estimation approach developed in LPW, to advance the aim of this literature having more of a concrete impact on policy.

Another important step to developing impactful, widely applicable methodologies, is extending the contexts in which these approaches can be applied. I make two contributions in this regard. Firstly, I show that if private leisure is used as the assignable good then it is possible to estimate the sharing rule from time-use data alone. This opens applications to contexts where we may lack high-quality expenditure data, or where expenditure data does not contain any assignable expenditures. Secondly, my identification result applies to contexts with or without price variation, while most identification results in this strand of the literature assumed constant prices. This extension is crucial to modelling time-use, since the price of time varies at the individual level. Allowing price variation also enables applications where prices of material goods vary substantially in the sample.

We can think of the identification result in this paper as an extension of the linear identification approach in LPW to a setting with public goods, time-use, price variation and broader families of preferences. In this sense, my result bridges between LPW and papers in the strand of the literature which identifies sharing from demand data on multiple goods. The latter, unlike LPW, generally model time-use, public goods, and price variation. However, compared to LPW, they have much more complex and hard-to-implement identification proofs and estimation strategies.

I apply my novel methodology to UK data on heterosexual working couples without co-habiting children. To my knowledge, this is the first paper which estimates resource shares from time-use data alone. I estimate that women generally command a lower proportion of resources than men,[4] 45% on average, but this varies substantially between households. Characteristics affect resource shares in a manner consistent with bargaining theory, e.g. women with higher wages, or matched with less educated men, command a higher resource share. While the sharing rule is interesting in itself, we are often interested in applying it to a separate question. I apply the sharing rule, estimated from time-use data, to a separate but comparable expenditure dataset, allowing me to estimate individual-level consumption. I find that, on average, men's consumption is 7.86% higher than women's. This is a substantial gender gap, especially when considering the context. The UK is one of the most gender equal countries in the world[5] and previous literature finds that women command higher resource shares in working couples than in households with children or where the woman doesn't work (e.g. Bargain, Donni and Hentati (2022)). Therefore we can think of this as an approximate lower bound for the kind of intra-household inequality we are likely to encounter in other applications, in other countries and with broader household compositions. This reinforces previous findings in the literature that equal sharing does not hold, and that it is important to estimate intra-household inequality instead of using a per capita approach.

I compare my baseline results to estimates obtained by using two alternative assignable goods. First, I consider non-market-work instead of private leisure, as in Lise and Seitz (2011).[6] Where detailed time-use data is not available, data on usual hours worked can be considered a second-best substitute. However, because this choice of assignable good includes private leisure but also joint leisure and time spent on domestic work, it is not credibly private. Consistently with this, resource shares estimated from non-market-work are biased towards equality relative to estimates from private leisure. This effect would likely be much stronger for households with cohabiting children, where women spend disproportionately more time than men on childcare. In addition, inequality within the household is underestimated due to the higher measurement error associated with recall questions on usual hours worked relative to detailed time-use data.

I repeat my analysis with clothing as the assignable good, and find that this approach

[4] In this paper I use 'resource share' and 'share of resources' as a short-hand for 'conditional resource share' or 'share of household private expenditure'.

[5] United Nations Development Programme. (2022). Human development report 2021-22. http://report.hdr.undp.org.

[6] My paper differs builds on Lise and Seitz (2011) in a few dimensions. Firstly, I add detailed time-use to the model, use a more credible assignable good (private leisure) and estimate the sharing rule from high-quality time-use diary data, instead of expenditure data with recall question on hours worked. Secondly, I drop the 'symmetry assumption' in Lise and Seitz (2011), that men and women with the same hourly wage share household full income equally, in favour of a more widely adopted, transparent and realistic identifying assumption restricting preference heterogeneity. Finally, I adopt the more transparent identification and simple estimation approach pioneered by LPW.

yields results that are hard to reconcile with bargaining theory (and also overestimate female resource shares). This appears driven by strong externalities which are inconsistent with treating clothing as private for UK couples. Therefore, it appears that using private leisure as the assignable good is a useful innovation from an empirical viewpoint, as well as theoretically, which in contexts such as the UK may contribute in a substantial way to estimate accuracy.

# 2 The model

In this section I set out a general static collective model of the household with both private and public goods and both material goods and time-use. The collective model of the household is both more general, and provides a better empirical fit, than alternatives in the literature (e.g. see Browning, Chiappori and Weiss (2014)). This structural model underpins the methodology proposed in this paper.

## 2.1 Framework

An individual $i$ belongs to a household $h$. Each individual has a person type $t$ (e.g. man, woman, child).[7] Households are in the same category $g$ (e.g. singles, heterosexual couples...) if they have the same composition, i.e. the same number of household members of each type: $N^t_{h,g} = N^t_g, \forall h \in g$.[8] A household may have multiple members of the same type. Individuals have a vector of characteristics $\pi_{i,h}$ (e.g. age, educational attainment) and their households have a vector of characteristics $\zeta_h$ (e.g. the gender ratio in the region).

The household purchases two types of goods on the market:[9] private goods and public goods.[10] $c_{i,h}$ is the vector of market purchased private goods consumed by individual $i$. $c$ is the sum of these vectors over all household members. A specific good, e.g. food, is indexed by

[7]Depending on the application of interest, types can be defined more granularly to model more heterogeneity. Children may be modelled as decision-makers or as public goods, depending on their age.

[8]To keep the notation leaner, I index individual variables only by $i$ and $h$ (even though these have an associated $t$ and $g$), and household variables only by $h$ (not $g$). I index type-specific parameters that vary between household categories with both $t$ and $g$, and household category-specific parameters only with a $g$. Where summing over multiple individuals, I use the letter $s$ instead of $i$, and the notation $st$ for the type of person $s$.

[9]See appendix D for a discussion of why time-use goods are not modelled as marketable.

[10]If desired, this model in this paper can readily be extended to shareable goods as well as public goods. In this case, actual consumption of each member is obtained by multiplying the vector of market purchases for that member by an economies of scale matrix $A$. The $A$ matrix depends only on household size. Analysis remains the same, with the addition of this matrix (potentially leading to more complex forms for demands, especially where cross-good economies of scale are allowed).

$j \epsilon \Omega^c$. The private good $j$ has price $p^j$, and the vector of prices is $p$. $X$ is the vector of market purchased public goods. Since these are public goods, all household members consume the full amount purchased by the household. A specific good, e.g. housing, is indexed by $j \epsilon \Omega^X$.[11] The public good $j$ has price $r^j$, and the vector of prices is $r$.[12] I do not index $p^j$ and $r^j$ by h because in many applications they will be constant across the sample, but where they do vary between households everything carries through with the small addition of $h$ subscripts to the material good prices.

Time is continuous,[13] and each individual has time-endowment normalised to 1, which can be spent in different activities. For exposition, I distinguish between four types of time-use: private leisure, joint leisure,[14] market work and domestic work. Private leisure $\ell_{i,h}$ includes leisure activities enjoyed by an individual without the co-presence of other household members. It is therefore a private good, which enters the utility function directly. Joint leisure $jt_{i,h}$ and domestic work $d_{i,h}$ may instead be thought of as inputs to public goods $D = f^D(\boldsymbol{d})$ and $JT = f^{JT}(\boldsymbol{jt})$ enjoyed by the household (or by specific sub-sets of members in the case of joint leisure involving only some household members). These production functions can accommodate heterogeneous productivity by type. Market work $m_{i,h}$ does not enter the utility function.[15] In order to use private leisure as the assignable good, the key is to distinguish between it and other types of time-use (a more detailed categorisation is consistent with the model, but is not necessary). In particular, it is important to distinguish between private and public leisure[16] as well as between leisure and non-leisure activities such as domestic work.

We write $Q_h$ for the vector of public goods including both material and time-use public goods $(X_h, JT_h, D_h)$ and $R_h$ for the associated price vector. $R_h$ is indexed by h because it includes the wages $w_{i,h}$ of each of its members. Each individual commands an exogenous

[11] I do not restrict the types of goods. They can be normal or inferior, goods or bads...

[12] The more commonly used notation in this literature is q and p for the quantity and price of private goods, and Q and P for those of public goods. I deviate from it for two reasons. Firstly, to highlight the fact that c and X are only material private and public goods. This allows me to use Q to refer to the set of all public goods. Secondly, for the very practical reason that it is easy to confuse lower and upper case instances of the same letter, particularly for the letter p. For this reason, I use R, rather than P for the price vector associated to Q.

[13] By examining UK time-use data used for the application in this paper, this modelling assumption appears realistic.

[14] The distinction between private and joint leisure here is a generalisation of the distinction made in Fong and Zhang (2001) between individual and spousal leisure.

[15] This model can be extended to accounting for individuals taking some pleasure in their work (and/or domestic work) by modifying the time budget constraint so that an hour spent working reduces leisure time by less than an hour, capturing the fact that part of the time spent working is enjoyed. See Browning, Chiappori and Weiss (2014) for a discussion.

[16] Browning, Donni et al. (2020) finds that these are far from perfect substitutes.

wage $w_{i,h}$ for a unit of market work.[17] We can think of an individual's unobserved skills as determining their hourly pay. Individuals then choose, within the set of jobs available to them given their pre-determined skills, whether to work in a longer-hour, higher-overall-pay job, or a shorter-hour, lower-overall-pay job. For instance, someone with high numerical literacy will likely command a high hourly wage, and might choose between the longer hours and higher overall salary of investment banking, and the shorter hours and lower (though still substantial) overall pay of industry forecasting. In the model, $m_i$ are actual hours worked, not contracted hours.

Given the time endowment has been normalised to 1, we model each individual as having a labour income endowment equal to $w_{i,h}$. In addition, each member can be endowed with non-labour income $y_{i,h}^{NL}$ (or alternatively the household as a whole can have non-labour income $y_h^{NL}$). An individual's overall endowment is $y_{i,h} = y_{i,h}^{NL} + w_{i,h}$. The household's endowment (or full income) is $y_h = \sum_{i \epsilon h} y_{i,h}$. I refer to this as the household budget.

Each individual $i$ of type $t$ living in a household category $g$ has utility function $u_{t,g}$. This allows preference heterogeneity across types and household compositions. For instance, a woman living alone can have different preferences from a woman living with a partner, and also different from a man living with a partner. Person types and household categories can be chosen to be arbitrarily granular, allowing additional heterogeneity in preferences. However, for implementation, it will be practical to restrict the number of person types and household categories. Types should be chosen to capture the key likely dimensions of heterogeneity in the context of interest, but without reducing the sample size for each category too drastically.

Estimation of the sharing rule must be conducted separately for households of different categories. This is because the form of the bargaining solution of each household depends on the number and types of members, so that the sharing rule for different household categories must be estimated separately.

## 2.2 The household's optimisation problem

In a household, the constituent individuals bargain over how to divide resources. Depending on the bargaining process, and on the outside options of the individuals, the different individuals will have different bargaining power and the resulting division of resources will be different.

[17] The collective model is in general incompatible with hourly pay being endogenous. Since hourly pay generally affects bargaining power in the household, the household problem becomes inefficient if it is a choice variable. We can reconcile a dynamic wage progress, which depends on time spent working, with modelling exogenous wages, by considering individuals to be myopic about the impact of their present time-use on future periods.

The collective model of the household does not restrict bargaining to any specific solution, and only requires that this process be efficient.[18] Relative bargaining power will in general depend both on (i) market variables such as prices (including wages), and (ii) distribution factors, which enter the household's optimisation problem only indirectly through the distribution of bargaining power, e.g. age and education of members (elements of the vectors of individual and household characteristics $\pi_{i,h}, i \in h$ and $\zeta_h$). I refer to the vector of variables that affect bargaining power as $z_h$. Note that a specific member's bargaining power will depend not only on their own characteristics, but also on the characteristics of all other household members, hence the household-level subscript.

A key result from the existing literature (see Browning, Chiappori and Weiss (2014)) is that the problem solved by any collective household, regardless of the underlying bargaining process and outside options, can be represented as an optimisation problem where the maximand is the weighted sum of the members' utility functions. Each member's utility function is weighed by their Pareto weight $\mu_{i,h}(z_h)$ normalised so that $\sum_{i\epsilon h} \mu_{i,h}(z_h) = 1$. The higher an individual's Pareto weight, the more weight the collective household gives their utility in determining its choices. The household's optimisation problem is therefore to maximise $\sum_{i\epsilon h} (\mu_{i,h}(z_h) u_{t,g}(c_{i,h}, X_h, l_{i,h}, JT_h, D_h))$.

## 2.3 A problem in two stages

We can re-cast this as a two-stage problem.[19] This representation is very helpful for identifying individual-level resources. In the first stage, the household chooses expenditure on public goods $R_h Q_h = \sum_{j\epsilon\Omega^X} r^j X_h^j + \sum_{i\epsilon h} w_{i,h}(d_{i,h} + jt_{i,h})$, and how to divide the remaining household budget into individual budgets $\rho_{i,h} = (y_h - R_h Q_h)\eta_{i,h}$ for members. The sharing rule[20] determines the share of household budget net of public good expenditure assigned to each member (the individual's resource share $\eta_{i,h}$), with the shares normalised to sum to one $\sum_{i\in h} \eta_{i,h} = 1$. In the second stage, members decide how to allocate their individual budgets $\rho_{i,h}$ to private expenditure.

**First stage: public goods and individual budgets**

$\max_{\rho,X,d,jt} \sum_{i\epsilon h} (\mu_{i,h}(z_h) v_{t,g}(\rho_{i,h}, X_h, D_h, JT_h))$ s.t. the following constraints:

[18] As opposed to models with specific bargaining solutions e.g. McElroy and Horney (1981), for which there is little consensus in the literature

[19] Separability is often assumed when employing the two-stage representation of a collective model with public goods (e.g. Lise and Seitz (2011)), but it is not a necessary assumption. Without separability, second-stage demands generally depend on public good consumption.

[20] In a model with private and public goods, what is called the sharing rule here is sometimes referred to as the conditional sharing rule. This is because it only affects the share of private expenditure of members, conditional on the household's choice of public good expenditure.

- Budget constraint: $\sum_{i\epsilon h} w_{i,h}\left(d_{i,h}+jt_{i,h}\right)+\sum_{j\epsilon\Omega^X} r^j X_h^j+\sum_{i\in h}\rho_{i,h}=y_h$
- Time feasibility constraint: $d_{i,h}+jt_{i,h}\leq 1$
- Non-negativity constraints: $\rho_{i,h}, d_{i,h}, X_h, jt_{i,h}\geq 0$
- Domestic and joint leisure production functions: $D=f^D(\boldsymbol{d})$, $JT=f^{JT}(\boldsymbol{jt})$

**Second stage: individual optimisation over private good consumption**

$\max_{c_{i,h},l_{i,h},m_{i,h}} u_{t,g}(c_{i,h}, l_{i,h}, X_h, D_h, JT_h)$ s.t. the following constraints:

- Budget constraint: $\sum_{j\epsilon\Omega^c} p^j c_{i,h}^j+w_{i,h}l_{i,h}=\rho_{i,h}$
- Time feasibility constraint: $l_{i,h}+m_{i,h}=1-(d_{i,h}+jt_{i,h})$
- Non-negativity constraints: $c_{i,h}, l_{i,h}, m_{i,h}\geq 0$

In general, second-stage demand depends on the prices of all private goods, including leisure (but not on the prices of public goods). Unless preferences are separable in private and public goods, it depends on public good consumption as determined in the first stage. Finally, it depends on the individual's second-stage budget, which is their resource share multiplied by the household budget net of public good expenditure (determined in the first stage). Individual i's (of type t) second-stage demand for private good $c^a$ takes the form: $c_{t,g}^a\left(p, w_{i,h}, Q_h, (y_h-R_hQ_h)\,\eta_{i,h}\right)$.

# 3 Identifying the sharing rule

Crucially for identification, individual i's second-stage demand for any private good, including the assignable good, depends only on i's own resource share. Hence, if we observe one individual-level demand for an assignable good for each household member, and make an identifying assumption restricting preference heterogeneity, we can in general identify resource shares. The advantage of this approach is that identification requires individual-level demand for one good only, rather than the whole unobservable individual-level expenditure system. The structural model simplifies the mapping from household-level expenditures to individual-level expenditures, so that we only need to estimate a very limited number of parameters (the resource share parameters).

## 3.1 Assignable good: advantages of private leisure

The assignable good $c^a_{i,h}$ may be clothing, as in the previous literature, private leisure, as in this paper, or some other assignable good.[21] Where time-use data is available, using private leisure as the assignable good has several important advantages relative to clothing.

### 3.1.1 Availability and accuracy of assignable good data

Using clothing expenditure requires availability of expenditure data on clothing broken down into categories that coincide with the types one wishes to use in the analysis (generally men's, women's and children's clothing). Where it is available, the accuracy of clothing expenditure data may vary substantially by context, e.g. whether it was derived from expenditure diaries or recall questions. Where expenditure by men, women and children is not available, some papers e.g. Calvi (2020) estimate these expenditure categories from expenditure on specific types of clothing e.g. assigning pyjamas to men. However, this approach is likely to substantially add to measurement error, especially in countries where clothing items are less clearly gendered. Narrowing down to specific clothing types such as skirts and ties is not a viable alternative as these purchases are very infrequent and command a very small budget share.

Similarly, time-use data may be more or less accurate depending on whether it is based on recall questions or on a detailed time-diary. High-quality time-use data is available for many countries, and generally includes information both on very detailed activities and who was co-present during the activity (e.g. the UK Time Use Data used in the application in this paper). Using the detailed activity information it is possible to construct accurate measures of time spent on leisure activities, and using the co-presence information it is possible to exclude leisure time which was joint rather than private. Where this is possible, private leisure is likely measured with high accuracy. Measurement error will be more substantial for measures of leisure obtained from recall questions on hours of leisure in a typical week.

[21] In practice, estimation is likely to proceed from individual-level demand for the same assignable good for everyone in the household. For this reason, the notation in the identification proof implies that the same assignable good $c^a$ is observed for all members. However, it is worth noting that identification can also proceed from different assignable goods for different types of people, as long as the same assignable good is used for each person of the same type in the same household category. In this case, the identification result is unchanged, apart from indexing the assignable good for different members differently depending on their type.

### 3.1.2 Credibility of the assignable good being private

Identification relies on the assignable good being private (for more detail see C). Where this is not the case, identification does not hold. In general, using an assignable good with a substantial public element (externalities) will lead to biased resource share estimates, and likely systematic bias by type of individual.

Unfortunately, clothing is highly shareable, even between different types of people. Especially in poorer households, this is likely to introduce a substantial public element of clothing. Moreover, even when each person wears their own clothing, clothing has substantial externalities. One may enjoy one's partner being well-dressed just as much as one enjoys being well-dressed oneself, or more. Also, the way one's partner dresses in company can be felt as an important source of pride or embarrassment for oneself. Furthermore, clothing (which is generally defined as including accessories) is frequently exchanged as a gift in couples, with dynamics that can undermine a standard representation as a private good e.g. getting promoted to a higher earning job may be an occasion to purchase a gift to a supportive partner who has been taking on more domestic work. As discussed in section 5.4, for UK working couples, patterns in clothing consumption are hard to reconcile with clothing being private. Externalities appear so strong as to outweigh own enjoyment of one's clothing. In this context, clothing cannot be reliably used as the assignable good. Where high-quality time-use data is available, private leisure is more credibly a private good than clothing, as joint leisure and domestic work have been separated out from it.

### 3.1.3 Frequency of purchases and magnitude of budget share

Clothing is an infrequent purchase, and the proportion of zeros is very high in many datasets used in the literature (most expenditure surveys are taken over short periods of time, e.g. two weeks). In the UK expenditure data used in the application in this paper, 27% of households have zero recorded expenditure on male or female clothing.[22] This is problematic because identification relies on demand functions which treat observations as interior solutions. Therefore, using an assignable good with a high proportion of zeros adds substantial inaccuracy to the estimation approach, as corner solutions are treated as interior solutions. Moreover, where zeros are not driven by infrequency but are actual zeros, due to low resource shares, applying a methodology that treats all assignable good expenditure as if it were at an interior solution may lead to biased estimates of the sharing rule. By contrast, households spend a large proportion of their resources on private leisure, with virtually no zeros in the

[22]In some cases, surveys may ask recall questions on clothing expenditure to alleviate this problem, but the accuracy of responses to such questions is more doubtful.

data, so using it as the assignable good allows for greater accuracy of estimates.

#### 3.1.4 Further considerations

**Clothing durability**. Clothing is highly durable and often passed down through generations (e.g. older to younger cousins), especially for small children. This means that clothing consumption can be substantially different from clothing purchases. This may lead to additional inaccuracies arising when using clothing as the assignable good. It might potentially also contribute to biasing estimates if e.g. clothes hand-me-downs are more frequently enjoyed by certain sub-groups, such as small children or young women.

**Clothing expenditure is not recorded at the individual level**. Clothing expenditure is almost never measured at the individual level in expenditure data, and is only available in broad break-downs such as men's, women's and children's clothing. In contexts with larger households and multiple members of each type, common especially in developing countries, this reduces the granularity of estimates to type-level, rather than individual-level, estimates of resources.

**Wage endogeneity.** While I model wages as exogenous, in reality wages are likely to be endogenous to some degree. However, this cannot be accounted for within the collective framework, as it would lead to inefficient bargaining. Therefore, if this endogeneity is substantial, that is an issue for resource share estimation. While this issue is more salient when using private leisure as the assignable good, it would be problematic regardless of the choice of assignable good.

**Restricting the sample to working couples**. In order to retain the simple estimation approach pioneered in LPW, if using private leisure as the assignable good, estimation must be restricted only to households where all members participate in labour markets. This is because, for individuals who don't work, private leisure falls out of first-stage decisions, and hence we cannot write an interior second-stage demand function for private leisure. This is explained in more detail in appendix C.[23] This limits the possibility of estimating individual-level inequality for the whole population, and is why the application of this paper focuses only on UK working couples without cohabiting children. Analysis of this sub-sample of the population is still of great interest, and in particular can shed light on gender inequalities and some of their drivers. It is important to note that, for the UK, selecting only couples

[23] It is theoretically possible to extend identification to contexts with non-participation, although it requires adopting a more complex identification approach, and may require additional data or assumptions (e.g. Blundell, Chiappori, Magnac et al. (2007) estimate the sharing rule - although only up to a constant - using a revealed preference approach). In order to maintain the simplicity and point-identification of LPW's framework, this paper does not address this issue, and focuses only on working couples without cohabiting children. An extension to non-participation is left to future work.

where both members work is unlikely to introduce any bias in the estimates (contrary to excluding households with zero clothing expenditure). The reason is that non-participation in couples without cohabiting children is similar between genders in the UK and is generally due to reasonably exogenous drivers such as long-term illness or temporary unemployment. I discuss this in greater detail in appendix C.

### 3.2 Parametric assumptions and estimating equations

The identification result holds for several families of preferences. Once we have chosen a parametric form for implementation we can derive the functional form of second-stage demands. We may directly use assignable good demands as estimating equations, or else choose some function f of demands: $f\left(c^a_{t,g}\left(p, w_{i,h}, Q_h, \left(y_h - R_h Q_h\right)\eta_{i,h}\right)\right)$. This could be for instance: demand for the assignable good, expenditure on the assignable good, the budget share spent on the assignable good, or expenditure on the assignable good relative to the expenditure on some sub-set of goods.[24] We may choose the most appropriate function depending on the functional form choices made for implementation, or depending on the data available. In general, we are likely to observe individual-level expenditures rather than individual-level demands, in expenditure data, so I illustrate estimation from expenditure functions in my UK application.

### 3.3 Linear approximation of resource shares

Resource shares will generally depend on household budget, prices (including wages of all household members) and Pareto weights of household members: $\eta_{i,h}\left(y_h, p, r, w_h, \mu_h\right)$. The form of an individual's resource share will depend on their type and household category so we can write $\eta_{t,g}\left(y_h, p, r, w_h, \mu_h\right)$.[25] Recall that Pareto weights are an unknown function of a vector of variables $z_h$ that determine relative bargaining power in the household. Since $z_h$ in general includes $y_h, p, r, w_h$ we can write $\eta_{t,g}\left(z_h\right)$. The collective model only restricts bargaining to be efficient, so to estimate resource shares we must approximate the Pareto weights, or directly approximate the resource shares. For different household categories $g$, resource shares (and Pareto weights) will be different functions of household characteristics

[24] Different assignable goods or functions of demand could be used for different types of people, but the same must be used for people of the same type in the same household category. The generalised notation is $f_{t,g}\left(c^{a,t,g}_{t,g}\left(p, w_{i,h}, Q_h, \left(y_h - R_h Q_h\right)\eta_{i,h}\right)\right)$ and the same result applies with minimal changes.

[25] Where there are multiple individuals of the same type in a household, the notation must be amended to acknowledge the fact that an individual's share depends both on their type and on their own individual characteristics / Pareto weight. Everything goes through in the same way, so for simplicity of notation I avoid this extension of notation.

$z_h$. For this reason, the sharing rule $\eta_g(z_h)$, which assigns a resource share $\eta_{i,h}$ to each member of households $h$ of category $g$, is estimated separately for households of different categories.[26]

To linearly estimate resource shares, we linearly approximate them as:

$\eta_{i,h} = \eta^0_{t,g} + \sum_z \eta^z_{t,g} \left(z_h - \bar{z}\right)$

- $\eta^0_{t,g}$ is the average resource share of a type (living in a specific household category). This is the resource share evaluated at the average characteristics in the sample. By definition, $\sum_{i\epsilon h} \eta^0_{t,g} = 1$. In the context of heterosexual couples, the average resource share of men and women sum to one.

- $\eta^z_{t,g}$ captures the impact on sharing of a household's characteristic $z_h$ deviating from the sample average $(z_h - \bar{z})$. For instance, a higher-than-average wage for the woman might increase the woman's resource share, so that she would have a higher-than-average-for-women resource share. Since resource shares must sum to one within the household, this implies her partner must have a correspondingly lower-than-average-form-men resource share: $\sum_{i\epsilon h} \eta^z_t = 0$. We can interpret $\eta^z_{t,g}$ as the marginal impact of characteristic $z_h$ on the resource share.

A linear approximation does not guarantee estimates of resource shares which fall within the unit interval. However, by construction, only estimates in this range are theory-consistent. This provides a useful test of model fit. Reassuringly, in my application to UK data, my baseline resource share estimates from time-use data are all within the unit interval.

Depending on the parametric choice made for implementation, it may be possible to linearly approximate the Pareto weights, rather than the resource shares, and substitute them into structural resource share equations (e.g. with Cobb-Douglas preferences).[27] This has the advantage that empirically finding that Pareto weights vary with market factors would be evidence in favour of the collective model and against the unitary model.[28] This is the case in my application to UK data, where Pareto weights significantly vary with members' wages.

[26] The approximation is more effective if person types and household categories are defined sufficiently granularly that the bargaining process, as a function of characteristics, would not differ too substantially within each household category sub-sample.

[27] In the identification result, I illustrate the case of directly approximating the resource shares, but the theorem and proof can be straightforwardly extended for the case of approximating the Pareto weights instead of the resource shares.

[28] The unitary model is inconsistent with market variables affecting relative bargaining power, while it can be reconciled with other variables affecting bargaining. In the unitary model, decision-making is represented through a single household utility function. The utility function cannot depend directly on prices or the budget, but it can depend on e.g. age and education of household members. For a more detailed discussion see Browning, Chiappori and Weiss (2014).

By contrast, when we directly approximate the resource shares, finding that the sharing rule depends on market factors does not guarantee that the Pareto weights do. Hence this finding is consistent with both the unitary and collective model of the household.

### 3.4 Identification result

Having substituted in the linear approximation of the resource share into the preferred function of assignable good demand, we obtain:

$f\left(c^a_{i,h}\right) = f\left(c^a_{t,g}\left(p, w_{i,h}, Q_h, \left(y_h - R_h Q_h\right)\left(\eta^0_{t,g} + \sum_z \eta^z_{t,g}\left(z_h - \bar{z}\right)\right)\right)\right)$

**Theorem 3.1.** *The linear approximation of the sharing rule $\eta_g$ is fully identified for households of category $g$ if (i) for all $N^t_g$ household members we observe a function $f\left(c^a_{i,h}\right)$ of demand for assignable good $c^a_{i,h}$, and (ii) the following Assumptions hold.*

**Assumption 3.1.1.** *$f\left(c^a_{i,h}\right)$ can be decomposed into a component $f^0_{t,g}\eta_{i,h}$ which is linear in the resource share approximation:*

$$f\left(c^a_{i,h}\right) = \sum_{\varphi} f^{\varphi}_{t,g}\left(p, w_{i,h}, Q_h, \left(y_h - R_h Q_h\right)\left(\eta^0_{t,g} + \sum_z \eta^z_{t,g}\left(z_h - \bar{z}\right)\right)\right) + f^0_{t,g}\left(p, w_{i,h}, Q_h, \left(y_h - R_h Q_h\right)\right)\left(\eta^0_{t,g} + \sum_z \eta^z_{t,g}\left(z_h - \bar{z}\right)\right) \quad (1)$$

This assumption imposes that the assignable good demand functions (or some function of demand) have a component which is linear in the individual's second-stage budget. Roy's identity can be used to easily check whether a specific indirect utility function is consistent or not with this requirement. Some functional forms that are consistent with this assumption are: (i) the Linear Expenditure System (Cobb-Douglas or Stone-Geary, see Stone (1954) and Geary (1950)), (ii) Price-Independent Generalised Linear (PIGL) (indirect utility functions that are a function of $\rho^k_{i,h}, k > 0$, see Muellbauer (1976)) and (iii) Price-Independent Generalised Logarithmic (PIGLOG) (indirect utility functions that are a function of $\ln \rho_{i,h}$ e.g. the Almost Ideal Demand System, see Muellbauer (1976) and Deaton and Muellbauer (1980)).[29] This assumption relaxes similar assumptions made in the related literature, allowing for broader families of preferences (e.g. DLP assumes budget share functions are linear

[29] Weaker parametric assumptions are also consistent with this assumption. For instance, preferences may be represented as Cobb-Douglas over the assignable good, an aggregate private and an aggregate public good (instead of Cobb-Douglas over each granular good). Alternatively, demand for the assignable good may be modelled as being linear in the budget without restricting the functional form of demands for other goods, and only requiring them to be jointly consistent with rational preferences.

in functions of expenditure, and LPW assumes that Engel curves be of the Almost Ideal Demand System, both of which are examples of functions of demand that have a component which is linear in the resource share).

**Assumption 3.1.2.** *Unknown preference parameters*[30] $\alpha_{t,g}$ *and unknown resource share parameters* $\eta_{t,g}$ *enter* $f\left(c^a_{i,h}\right)$ *linearly, so that we can write:*

$$f\left(c^a_{i,h}\right) = a_{t,g} + \sum_{\varphi} b^{\varphi}_{t,g} g^{\varphi}\left(p, w_{i,h}, Q_h, \left(y_h - R_h Q_h\right), \left(y_h - R_h Q_h\right)\left(z_h - \bar{z}\right)\right)$$
$$+ \gamma_{t,g} g^0\left(p, w_{i,h}, Q_h, \left(y_h - R_h Q_h\right)\right)\left(\eta^0_{t,g} + \sum_z \eta^z_{t,g}\left(z_h - \bar{z}\right)\right) \quad (2)$$

*Where:*[31] *(i)* $a_{t,g}$*,* $b^{\varphi}_{t,g}$ *and* $\gamma_{t,g}$ *are unknown constants which may be (potentially composite) preference parameters, or composite parameters composed of preference parameters and unknown resource share parameters, and (ii)* $g^{\varphi}$ *and* $g^0$ *are observed in the data.*[32]

The assumption that unknowns enter linearly is required so that the estimating equations can be estimated by linear regression. Where the assumption does not hold, it is possible to relax it by using a linear approximation of $f\left(c^a_{i,h}\right)$, as exemplified for the Almost Ideal Demand System in appendix A (and similarly to LPW).

**Assumption 3.1.3.** *No component of* $g^0\left(p, w_{i,h}, Q_h, \left(y_h - R_h Q_h\right)\right)\left(\eta^0_{t,g} + \sum_z \eta^z_{t,g}\left(z_h - \bar{z}\right)\right)$ *is co-linear with any other component of* $f\left(c^a_{i,h}\right)$

Writing $f\left(c^a_{i,h}\right)$ as:

$$f\left(c^a_{i,h}\right) = a_{t,g} + \sum_{\varphi} b^{\varphi}_{t,g} g^{\varphi}\left(p, w_{i,h}, Q_h, \left(y_h - R_h Q_h\right), \left(y_h - R_h Q_h\right)\left(z_h - \bar{z}\right)\right)$$
$$+ \gamma_{t,g}\eta^0_{t,g} g^0\left(p, w_{i,h}, Q_h, \left(y_h - R_h Q_h\right)\right) + \sum_z \eta^z_{t,g}\gamma_{t,g} g^0\left(p, w_{i,h}, Q_h, \left(y_h - R_h Q_h\right)\right)\left(z_h - \bar{z}\right) \quad (3)$$

[30] If modelling shareable goods, then read 'preference or shareability parameters' for 'preference parameters'

[31] Depending on the chosen functional form specification, some of these components may be absent from the estimating equations (e.g. there may be no constant term $a_{t,g}$).

[32] Note that, as long as we assume the same parametric form for preferences of all individuals (of all types, and in all household categories) these fully observable objects have the same functional form for everyone. Identification also works if we assume different parametric forms for preferences of different types of individuals, or individuals in different household categories, and in that case we write $g^{\varphi}_{t,g}$ and $g^0_{t,g}$. Here I focus on the case where the same parametric form of preferences is assumed for all individuals as this is generally the case for implementation, and it leads to much more natural and realistic interpretations of the identifying assumptions.

we can see that the no-colinearity assumption is required so that coefficients $\gamma_{t,g}\eta^0_{t,g}$ and $\gamma_{t,g}\eta^z_{t,g}$ can be identified separately from other coefficients in $f\left(c^a_{i,h}\right)$. This is necessary because we will identify $\eta^0_{t,g}$ and $\eta^z_{t,g}$ from $\gamma_{t,g}\eta^0_{t,g}$ and $\gamma_{t,g}\eta^z_{t,g}$. Depending on the functional form of choice, this assumption can imply restrictions on which variables $z_h$ influence the resource share. A related, but less general, assumption which is often made in the literature is that resource shares are invariant to expenditure (see DLP for a discussion of why this assumption is reasonably non-restrictive both theoretically and empirically).

**Assumption 3.1.4.** *$\gamma_{t,g}$ is a, potentially composite, preference parameter, and does not depend on unknown resource share parameters. A suitable restriction on $\gamma_{t,g}$ heterogeneity applies.*

We require that, while $a_{t,g}$ and $b^{\varphi}_{t,g}$ may be composite constant parameters which depend on both preference and resource share parameters, $\gamma_{t,g}$ can only be a composite preference parameter. This is because identifying $\eta^0_{t,g}$ and $\eta^z_{t,g}$ from $\gamma_{t,g}\eta^0_{t,g}$ and $\gamma_{t,g}\eta^z_{t,g}$ requires a restriction on heterogeneity in $\gamma_{t,g}$. If this is a composite preference parameter, then the identifying assumption restricts preference heterogeneity. $\gamma_{t,g}$ cannot depend on resource share parameters as that would require excessively strong restrictions on the sharing rule (such as equal sharing!). The candidate preference heterogeneity restrictions are discussed in section 3.5, and are in line with the identifying restrictions made in the literature.

## 3.5 Identifying assumption: SAP, SAT or SRAT

Identification of the sharing rule from individual-level demand data for a single assignable good requires a strong identifying assumption. It is important to note that the chosen assumption has an important impact on estimates, and should be chosen carefully. Assumptions could be made on the bargaining process e.g. Lise and Seitz (2011) assume that women and men with the same potential earnings divide full income equally. However, such assumptions are hard to justify and most of the literature has instead focused on restricting preference heterogeneity. Such assumptions are more transparent, realistic and capable of being tested to some extent. The identification result in this paper holds with any one of the following options for the identifying assumption. The first two have been used previously in the literature, while the third is suggested here as a somewhat milder alternative.

**Assignable good SAT (similarity across household types)**: preferences for the assignable good (relative to other goods) are similar between household compositions, in the sense that $\gamma_{t,g} = \gamma_t, \forall t, g$ (so men's preferences can be different from women's, but the preferences of single men are similar to those of men in couples). This allows us to identify the unknown (potentially composite) preference parameter $\gamma_{t,g}$ for each person type $t$ from singles data, allowing us to recover the resource share parameters. This assumption is weaker than requiring preferences to be identical within person type across household compositions. However, it is still a strong assumption, particularly since the set of goods available to singles differs from those available to larger households (joint leisure is not available for singles). Another difficulty with this assumption is that it cannot be employed for households where children are modelled as decision-makers, since children aren't observed living alone. SAT has been employed in previous literature e.g. Lewbel and Pendakur (2008) and Bargain and Donni (2012). However, we may be concerned by findings that, at least in some contexts, preference stability across household composition is rejected empirically (see Hubner (2020)).

**Assignable good SAP (similarity across people):** the unknown (potentially composite) preference parameter $\gamma_{t,g}$ is the same for people of different types $t$ within household category $g$: $\gamma_{t,g} = \gamma_g, \forall t \in h$ (women and men in couples have similar preferences, while single women may have different preferences from women in couples). This assumption still allows heterogeneity in all other utility parameters, as well as full heterogeneity between household categories. Depending on the granularity of the data, types and categories can be defined more granularly to allow for a greater degree of heterogeneity in preferences. This assumption has been employed in previous literature e.g. DLP and LPW. An indirect empirical test of this assumption can be conducted by testing whether SAP holds for a specific good in singles' data. If it does, this does not guarantee that it would hold for other household compositions, but it is at least reassuring. If it does not, then that makes it less realistic for other household compositions too. For instance, in my application to UK data I find that private leisure SAP holds for singles. The validity of this test effectively amounts to the SRAT assumption below.

**Assignable good SRAT (similarity of ratios across types)**: the ratio of the preference parameter $\gamma_{t,g}$ between different types of people $t$ remains stable across household compositions $g$: $\frac{\gamma_{t,g}}{\gamma_{st,g}} = \frac{\gamma_{t,single}}{\gamma_{st,single}}, \forall t, ts \in h$. This assumption implies that preferences on the assignable good change in a similar way for different types of people in the transition from singlehood to other household categories. This means that the ratio of men's to women's preference parameters is identified from singles data. The SRAT assumption allows preference levels to vary between different people and different household categories, while requiring that the ratio of the parameters is constant across household compositions. For instance, single women's clothing preference parameter may be $\alpha^c_{w,single} = 0.2$ and men's $\alpha^c_{m,single} = 0.1$, while in a working couple it may be that $\alpha^c_{w,couple} = 0.16$ and $\alpha^c_{m,couple} = 0.08$. SRAT is a weaker assumption than assignable good SAT or SAP, and has not been used in the literature to date. It is in practice identical to employing assignable good SAP where singles' preference parameters have successfully been tested for equality. Like assignable good SAT, it can only be applied to households without children (modelled as decision-makers), since children are not observed living alone.

SRAT (like testing SAP with singles data) is quite compelling for working couples without cohabiting children because there is no particular reason to suspect women's and men's preferences change in a different manner when transitioning from living alone to living in a couple. The assumption is stronger for households with children, since the idea that single men and women's preferences change similarly when they have children is less realistic (at least in Europe, the majority of gender differences in time-use and labour market outcomes emerge after the having of children).

## 3.6 Identification proof

I illustrate the semi-parametric identification argument assuming that the chosen assignable good is private leisure and the chosen function of consumption is expenditure: $f\left(c^a_{i,h}\right) = w_{i,h}l_{i,h}$. The same logic applies to different choices for (i) the assignable good and (ii) the function f of consumption. Substituting $\beta^0_{t,g} = \gamma_{t,g}\eta^0_{t,g}$ and $\beta^z_{t,g} = \gamma_{t,g}\eta^z_{t,g}$ into equation (3), we obtain the estimating equations:

$$\begin{aligned} w_{i,h}l_{i,h} = a_{t,g} &+ \sum_{\varphi} b^{\varphi}_{t,g} g^{\varphi}\left(p, w_{i,h}, Q_h, \left(y_h - R_hQ_h\right), \left(y_h - R_hQ_h\right)\left(z_h - \bar{z}\right)\right) \\ &+ \beta^0_{t,g} g^0\left(p, w_{i,h}, Q_h, \left(y_h - R_hQ_h\right)\right) + \sum_{z} \beta^z_{t,g} g^0\left(p, w_{i,h}, Q_h, \left(y_h - R_hQ_h\right)\right)\left(z_h - \bar{z}\right) \end{aligned} \tag{4}$$

*Proof.* By running seemingly unrelated linear regressions, one for each type in the household category, we identify the constants $a_{t,g}$ and $\beta^0_{t,g}$, and the vectors $b^{\varphi}_{t,g}$ and $\beta^z_{t,g}$. The $\beta$ coefficients are of interest to identify the resource share parameters $\eta$. However, because the resource share parameters $\eta$ enter into the $\beta$ coefficients multiplicatively with preference parameters, we require an identifying assumption to separately identify the resource share parameters. Under SAT, preference parameters are identified from singles data, so that $\eta^0_{t,g} = \frac{\beta^0_{t,g}}{\gamma_t}$ and $\eta^z_{t,g} = \frac{\beta^z_{t,g}}{\gamma_t}$ where $\gamma_t$ is known, so the resource share parameters are identified. Under SAP, the average resource share of each type is identified by $\eta^0_{t,g} = \frac{\beta^0_{t,g}}{\sum_{s\in h}\beta^0_{st,g}}$. This is because under SAP $\frac{\beta^0_{t,g}}{\sum_{s\in h}\beta^0_{st,g}} = \frac{\gamma_g\eta^0_{t,g}}{\sum_{s\in h}\gamma_g\eta^0_{st,g}} = \frac{\eta^0_{t,g}}{\sum_{s\in h}\eta^0_{st,g}} = \eta^0_{t,g}$ since, by definition, resource shares sum to one within household. The marginal impact of characteristic z is identified by $\eta^z_{t,g} = \frac{\beta^z_{t,g}}{\beta^0_{t,g}}\eta^0_{t,g}$. This is because under SAP $\frac{\beta^z_{t,g}}{\beta^0_{t,g}}\eta^0_{t,g} = \frac{\gamma_g\eta^z_{t,g}}{\gamma_g\eta^0_{t,g}}\eta^0_{t,g} = \eta^z_{t,g}$. Under SRAT, the proof is similar to SAP with the difference that we identify $\frac{\gamma_{t,g}}{\gamma_{st,g}} = \frac{\gamma_{t,single}}{\gamma_{st,single}} = \Lambda_{t,st}$ from singles data and then identify the average resource share of each type as $\eta^0_{t,g} = \frac{\beta^0_{t,g}}{\sum_{s\in h}\Lambda_{st,g}\beta^0_{st,g}}$.

□

### 3.7 Discussion of the identification result

See section 4 for a simple parametric (Cobb-Douglas) example of this identification result, which illustrates the intuition behind the more general result. I also illustrate the identification result with the Almost Ideal Demand System in appendix A. This identification result builds on LPW and extends it to a more general underlying structural model.[33] The extension to public goods creates some additional complication for identification, particularly in contexts where we wish to estimate the sharing rule from data that does not also include expenditure on public goods. This is illustrated in section 4, where I discuss an implementation with time-use data only.

I also note that the extension to contexts with price-variation (including individual-level wages) can complicate estimation. Firstly, it can greatly increase the number of regressors in the estimating equations, depending on how many prices are heterogenous within the sample and on the functional form of preferences. Moreover, additional data (on non-constant prices and wages) is required for estimation. Furthermore, where there is poor data on non-labour income, the wages of household members become colinear with the household budget, which can complicate or impede estimation under some functional form assumptions. However, allowing for price variation is necessary to enable using private leisure as the assignable good. Estimation with price variation is also likely to yield more accurate estimates in general. Finally, as this identification result does not hinge on price variation, the result in this paper includes as a special case a model where all prices are constant.

## 4 Implementation: a worked example

### 4.1 Data requirements

In general, regardless of the choices of assignable good and functional form of preferences, the minimum requirements for estimation of the sharing rule are:

[33]While all goods are non-public in the LPW setting, economies of scale from household size are modelled by letting the actual consumption vector of each member be equal to the vector of market purchases for that member multiplied by a matrix $A$. While I do not explicitly incorporate the sharing framework, the identification result in this section readily applies to a model with both public and shareable goods. The most notable effect of this inclusion, if there are cross-good economies of scale, is that estimating equations are likely to take on more complex functional forms and be less tractable.

- A cross-section household dataset with both household-level information and individual-level data for all members.
- Data on key characteristics which are likely to affect bargaining e.g. age, educational attainment and wages of all members. Wages may need to be estimated from earnings and hours worked.
- Individual-level data on demand / expenditure on an assignable good for all members. Sometimes assignable good demand or expenditure data will be directly available in the dataset. In other cases, it may require constructing. For instance, when using private leisure as the assignable good, it may be necessary to classify time-use data based on activity and co-presence information.
- Data on full household income. Full income will generally have to be constructed from information on wages, together with any non-labour income of members (recall that full income is not the same as earnings).
- If prices of some material good vary substantially in the sample, data on prices is generally required.[34]

## 4.2 A worked example

I illustrate an example of the identification result with (i) Cobb-Douglas functional form, (ii) leisure as the assignable good, (iii) SAP as the identifying assumption, and (iv) an additional restriction on preference heterogeneity allowing estimation of the sharing rule from time-use data alone. In section 5 I employ this very simple version of the method to estimate resource shares from UK time-use data. I focus only on heterosexual working couples without cohabiting children because, using the identification approach in this paper, private leisure can be used as the assignable good only for households where all members participate in labour markets.[35] This is a substantial limitation of this approach, and future work may develop ways of estimating resource shares from time-use data for broader family compositions. Even so, estimating the sharing rule for working couples provides important information on gender inequality in a country, as well as how different individual characteristics affect sharing in the household. The model seems to fit well and the restrictive assumptions made appear not to be strong-arming the data in this context.

[34] It may not be if assignable good preferences are separable from preferences of goods with price heterogeneity

[35] For further explanation see appendix C, which also argues why this sample selection is unlikely to introduce any bias in the estimates, at least in the UK context.

Utilities and domestic production functions are Cobb-Douglas, so that we can write:[36]

$$u_{t,g} = \sum_{j \epsilon \Omega^c} \left( \alpha_{t,g}^{cj} \ln(c_{i,h}^j) \right) + \sum_{j \epsilon \Omega^X} \left( \alpha_{t,g}^{Xj} \ln(X_h^j) \right) + \alpha_{t,g}^l \ln(l_{i,h}) + \sum_{s \epsilon h} \left( \alpha_{t,g}^{Tst,g} \ln jt_{s,h} \right) + \sum_{s \epsilon h} \left( \alpha_{t,g}^{Dst,g} \ln d_{s,h} \right)$$

Cobb-Douglas preferences are widely used in applied microeconomics. In the context of resource share estimation, they have several advantages. Importantly, they lead to parsimonious estimating equations, even with a rich characteristics vector $z_h$ affecting bargaining power. This enables estimation from realistic, widely available data, which generally involves small sample sizes. Small sample sizes are common in this literature, partly due to limitations of existing data, and partly by construction, because estimation must proceed separately for households of different categories, so that even with large data the sample is divided into smaller sub-samples for estimation.

Additionally, Cobb-Douglas preferences, unlike e.g. the Almost Ideal Demand System, have a direct utility representation, which presents several advantages. Firstly, it allows for very clear interpretation of identifying assumptions in terms of restrictions on preferences. Secondly, it allows broader applications than would otherwise be possible. For instance, the methodology of Chiappori, Okuyama et al. (2023) to estimate individual-level money-metric welfare indices can straightforwardly be implemented after estimating the sharing rule using the methodology in this paper under the assumption of Cobb-Douglas preferences. Moreover, the tractable direct utility representation of Cobb-Douglas enables estimation of the sharing rule from time-use data alone. Furthermore, under Cobb-Douglas preferences, resource shares are equal to Pareto weights scaled by a composite preference parameter. Therefore, finding that resource shares vary with market variables is equivalent to finding that Pareto weights do, allowing a test of the collective model over the unitary model of the household.

While it is possible to identify the sharing rule using more flexible functional forms than Cobb-Douglas, this often requires some additional linear approximations. For instance, Almost Ideal Demand System Engel curves need to be linearly approximated. Therefore, the degree to which alternative preferences would better capture any non-linearities is limited. Moreover, more flexible parametric forms often lead to additional practical issues for estimation because, in order to allow resource shares to depend on several characteristics $z_h$, estimation requires a large number of regressors, many of which are highly correlated with each other (see appendix A). For these reasons, implementation will often benefit from using very parsimonious functional form assumptions, such as Cobb-Douglas.

The price of the tractability and ease of application achieved with Cobb-Douglas is a strong

[36] See appendix D for a more detailed discussion, including normalising assumptions.

parametric assumption about preferences. In particular, Cobb-Douglas preferences impose homotheticity. We may be concerned that this is not a realistic assumption. To this end, I perform several tests of model fit in my application to UK data and find this assumption seems realistic in this context, and is not strong-arming results (see section 5.3.3). This echoes similar findings in Chiappori, Okuyama et al. (2023) with Japanese data. It is a strength of this approach that even using such a simple parametric form for preferences, the model and identification approach perform well as measured by a range of tests. Parametric assumptions for implementation should be judged depending on the context of application, and future work may explore the degree to which different functional form assumptions lead to differing resource share estimates, and compare their empirical performance.

## 4.3 Assumptions restricting preference heterogeneity

**Private leisure C-D SAP**. As the identifying assumption, I illustrate assignable good SAP. I note that the interpretation of the identifying assumptions differs depending on the functional form of preferences and the assignable good of choice. In this case, since it applies to private leisure as the assignable good, and to Cobb-Douglas preferences, I refer to the specific interpretation of the identifying assumption in this context as private leisure C-D SAP. Everyone, regardless of their type (in a given household category) is assumed to have the same Cobb-Douglas preference parameter for private leisure: $\alpha^l_{t,g} = \alpha^l_g$. Preference heterogeneity across types is maintained for all other preference parameters, and across household compositions. This means that a man and woman in the same couple will spend the same share of their individual second-stage budget on leisure, but will split the remaining budget differently between other private goods. As explained in section 5.3.1, this identifying assumption is consistent with UK data.

**Public good C-D SAP.** A novel implementation that is opened up by this paper is the possibility of estimating resource shares from time-use data alone (if using private leisure as the assignable good). However, if we only have time-use and no expenditure data, this generates a difficulty: the second-stage demand for assignable goods is conditional on public good expenditure, but we cannot control for the latter. In some cases, we may have access to some separate expenditure dataset which we can use to estimate public good expenditure for households with different characteristics, and apply those estimates to the time-use data. If we don't, additional parametric restrictions are required to identify resource shares in this case. In particular, we require public good expenditure not to vary with the distribution of bargaining power in the household. With Cobb-Douglas preferences, we can achieve this with public good C-D SAP, as explained in more detail in appendix D.3. Note that this assumption is in addition to those set out in the identification result in section 3.4, and is not required when public good expenditure data is available.

Public good C-D SAP imposes that the sum of all public good preference parameters is the same for all individuals of the same household category $g$:
$\left(\sum_{j\in\Omega^X} \alpha_{t,g}^{j} + \sum_{s\epsilon h} \alpha_{t,g}^{Tst,g} + \sum_{s\epsilon h} \alpha_{t,g}^{Dst,g}\right) = a_g^Q$[37]
This assumption allows heterogeneity in how different types would choose to divide the budget between specific private goods, and also between specific public goods, while requiring that they would choose the same overall split between private and public goods. As explained in section 5.3.2, this assumption is consistent with UK data.

## 4.4 Identification and estimation

Under the assumptions set out above, the system of estimating equations is:
$w_{i,h}l_{i,h} = \alpha_{t,g}^{l} y_h \left(\eta_{t,g}^{0} + \sum_z \eta_{t,g}^{z} \left(z_h - \bar{z}\right)\right)$
Re-writing this in terms of observables and regression coefficients:
$w_{i,h}l_{i,h} = \beta_{t,g}^{0} y_h + \sum_z \beta_{t,g}^{z} y_h \left(z_h - \bar{z}\right)$
where $\beta_t^0 = \alpha_{t,g}^{l}\eta_t^0$ and $\beta_t^z = \alpha_{t,g}^{l}\eta_t^z$
The linear and parsimonious expenditure functions resulting from the chosen parameterisation are advantageous for the purpose of estimation on small datasets, which are common in practical applications. It can be easily seen that the private leisure expenditure functions satisfy the assumptions of the identification result in section 3.4. Here, the assignable good is private leisure: $c_{i,h}^{a} = l_{i,h}$. The estimating equations are leisure expenditure functions $f\left(c_{i,h}^{a}\right) = w_{i,h}l_{i,h}$. Sub-function $f_{t,g}^{0}$ is a linear function of observables $f_{t,g}^{0} = \alpha_{t,g}^{l} y_h$, where the only unknown component $\alpha_{t,g}^{l}$ is a preference parameter, as required. There are no other sub-functions $f_{t,g}^{\varphi}$, so the requirement of no colinearity between $f_{t,g}^{\varphi}$ and $f_{t,g}^{0}\eta_{i,h}$ is met.

Empirically, identification of the levels of resource shares, and the marginal effects of different characteristics on them, is driven by different sources of variation in the data. For example, ceteris paribus, a higher wage for the man will reduce his leisure demand through a price effect, but increase it through a twofold income effect: (i) increased overall household budget, and (ii) increased own-resource share.[38] Therefore, the leisure patterns of two similar couples that differ in the man's wage help identify both the effect of male wage on sharing, and, by

[37]Equivalently, the sum of all private good parameters is also homogeneous within category: $\left(\sum_{j\in\Omega^c} \alpha_{t,g}^{j} + \alpha_{t,g}^{l}\right) = a_g^{c,l}$.

[38]Browning and Gørtz (2012) find that the 'unitary effect', that leisure demand falls as its price increases, dominates the 'collective effect' of the individual's bargaining power increasing in own-wage.

increasing the overall household budget, the levels of resource shares. Similarly, couples which are identical in characteristics apart from the age gap, and have different leisure patterns, help identify the effect of intra-couple age gaps on sharing.

### 4.4.1 A structural approach to the error term

To proceed to estimation, we must consider the source of any error terms. In this case, there are three likely sources of error in our estimating equations. The first is approximation error from linearly approximating the resource share based on the characteristics vector $z_h$. The second is household optimisation error at the first stage of the household problem. The third is individual optimisation error at the second stage of the household problem. As explained in appendix D.4 we can write $w_{i,h}l_{i,h} = \alpha^l_{t,g} y_h \left(\eta^0_{t,g} + \sum_z \eta^z_{t,g} (z_h - \bar{z})\right) + \epsilon_{i,h}$ where $\epsilon_{i,h}$ is mean-zero. If $z_h$ includes all key characteristics that affect bargaining (or at least all those that are correlated with regressors) then estimates should be unbiased. The errors are negatively correlated within household (with a correlation weaker in magnitude than -1), so that a SURE estimation approach is recommended.

### 4.4.2 Estimation procedure

Here I focus on the procedure when the sample used for estimation contains only one household category $g$, as in the empirical application in this paper, which focuses only on heterosexual working couples without cohabiting children. I therefore drop the $g$ subscripts. Where there are multiple household categories, the approach set out below should be carried out separately for each household category.

1. Run linear SURE regressions of leisure expenditure for the different types (e.g. men and women) with the restriction that $\sum_{i\epsilon h} \beta^z_t = 0$. This is because $\beta^z_t = \alpha^l_t \eta^z_t$ and from leisure SAP and the definition of the linear approximation parameters, $\sum_{i\epsilon h} \beta^z_t = \alpha^l_t \sum_{i\epsilon h} \eta^z_t = 0$. Note that there are other restrictions imposed by the model which we do not impose during estimation, and that can be used to test the fit of the model as discussed in section 5.3.

2. Estimate each type's average resource share as $\hat{\eta^0_t} = \frac{\hat{\beta^0_t}}{\sum_{s\epsilon h} \hat{\beta^0_{st}}}$. To see why, first note that $\frac{\beta^0_t}{\sum_{s\epsilon h} \beta^0_{st}} = \frac{\alpha^l_t \eta^0_t}{\sum_{s\epsilon h} \alpha^l_{st} \eta^0_{st}}$ and by the leisure SAP assumption the $\alpha$ parameters cancel out so that $\frac{\beta^0_t}{\sum_{s\epsilon h} \beta^0_{st}} = \frac{\eta^0_t}{\sum_{s\epsilon h} \eta^0_{st}} = \eta^0_t$ since resource shares sum to one by definition.

3. Estimate the marginal impact of different characteristics as follows. First, estimate $\hat{\alpha^l} = \frac{\hat{\beta^0_t}}{\hat{\eta^0_t}}$ since $\beta^0_t = \alpha^l \eta^0_t$. Then, we estimate $\hat{\eta^z_t} = \frac{\hat{\beta^z_t}}{\hat{\alpha^l}}$ since $\beta^z_t = \alpha^l \eta^z_t$.

4. The estimated parameters yield the sharing rule for households of category g: $\hat{\eta_g}$

Armed with the sharing rule, we can estimate individual-specific resource shares in the dataset used for estimation (the time-use dataset in this case): $\hat{\eta_{i,h}} = \hat{\eta_t^0} + \sum_z \hat{\eta_t^z} (z_h - \bar{z})$. Moreover, we can take the sharing rule across to other comparable datasets (e.g. expenditure data) and estimate household-specific resource shares there by applying the sharing rule.[39] This can be helpful to proceed to further applications of sharing rule estimation, as exemplified in section 6.

To fix the intuition behind identification, let us consider heterosexual working couples without cohabiting children. The identification problem we face is that the man's expenditure on private leisure may be more or less responsive to changes in the household budget for one of two reasons, or a combination of them. The first possibility is that the man receives a larger proportion of the household budget (net of public good expenditure), i.e. he has a higher resource share than the woman. The second possibility is that the man's preferences for private leisure are stronger relative to the woman's, so that he spends a larger proportion of his individual budget on private leisure. In order to disentangle these two channels, we shut down the preference channel by assuming a specific difference in preferences through the identifying assumption (none with SAP, a ratio estimated from singles with SAT and SRAT), allowing us to identify the sharing rule. Under CD-SAP, private leisure expenditure for individuals of any type (within a given household category) $l_i w_i = \alpha_g^l y_h \eta_{i,h}$ responds in the same way to the same increase in individual budget $y_h \eta_{i,h}$, so differences in responsiveness to changes in $y_h$ identify differences in sharing.

# 5 Estimating resource shares from UK time-use data

I illustrate how my methodology can be applied to estimate the sharing rule from time-use data alone. Using UK data, I find that women command lower resources than men, with substantial variation between households. The directions of marginal effects and their magnitudes are consistent with bargaining theory. To implement my identification result, I use the simple parametric form and identifying assumptions discussed in section 4. The model fits the data well, and key assumptions are found to be realistic in this context. This application illustrates how the method proposed in this paper can be implemented in a very simple way, with parsimonious estimating equations, using widely available time-use data only.

[39] To do so, we calculate how household characteristics in the expenditure dataset deviate from the averages in the time-use data, and substitute these deviations $(z_h - \bar{z})$ in the estimated sharing rule $\hat{\eta_g}$.

## 5.1 UKTUS

UKTUS 2000, 2014 (UK Time-Use Survey),[40] is a high-quality time-use survey that has been used in the economics literature (e.g. Kalenkoski et al. (2005)) but never before with the goal of estimating intra-household inequality. It is a national household-based study composed of: (i) a household questionnaire, (ii) an individual questionnaire, and (iii) individual time-diaries. A single household representative answered the household questionnaire, including questions on household characteristics such as composition, dwelling type, and location. The other components were answered by the individual in question. This is likely to substantially increase the quality of the data relative to datasets where a single members answers on behalf of all individuals. The individual questionnaire asks about individual characteristics including age, educational attainment and earnings. Each member completed a weekday and weekend time diary identifying primary and secondary activities for each 10-minute interval over the two days. The time-use data is extremely detailed, including very detailed activities, location, and co-presence of others (distinguishing between household and non-household members). This enables me to define private leisure very precisely, as time spent doing leisure activities without other household members co-present. The time-diaries are constructed carefully to minimise measurement error, for instance with the possibility of writing a simple sign to signify the same activity for multiple time intervals. The quality of the data is very high, with approximately 95% of observations having more than 5 distinct activities recorded in a day, and less than 90 minutes of unrecorded time. The data is nationally representative.

Using the identification approach from this paper, private leisure can be used as the assignable good only for households where all members participate in labour markets. The most common household type of this variety is heterosexual working couples without cohabiting children (they may have children, but if so they are no longer part of the household). I therefore focus my analysis on this group.[41] After cleaning the data, the final pooled (2000 and 2014) sample comprises 711 households (1422 individuals). While the sample size is not particularly large, it is of a good size relative to this literature.[42]

[40]Office for National Statistics. (2019b). United Kingdom Time Use Survey [data series] 2nd Release. https://doi.org/http://doi.org/10.5255/UKDA-Series-2000054.

[41]As explained in appendix C, this type of sample selection is unlikely to lead to any internal validity concerns. The analysis could be extended to working couples with small children, which can be modelled as public goods instead of decision-makers, but these would have to be analysed separately from working couples without cohabiting children, and would be a small sample. Moreover, the identifying assumptions required are harder to justify for households with children.

[42]e.g. Cherchye, De Rock and Vermeulen (2012) estimate a closely related model on a sample of 212 Dutch households. As discussed earlier, small sample sizes are to some extent inevitable in applications where different household compositions are analysed separately.

Summary statistics for some of the key variables are reported in table 1. Hourly pay is obtained by dividing labour income by actual hours worked, rather than contractual hours.[43] Women on average command lower hourly wages than men. Couples generally form with an older, higher-earning man and a younger, lower-earning woman. Household budget is full income: the sum of the labour endowment of members (hourly wage multiplied by 24 hours).[44] Women on average are more qualified.

| | | count | mean | sd | p25 | p50 | p75 |
|---|---|---|---|---|---|---|---|
| Hourly pay (2020 GBP) | female | 711 | 9.87 | 5.58 | 6.6 | 8.61 | 11.6 |
| | male | 711 | 11.6 | 8.74 | 7.51 | 9.78 | 13.1 |
| Age (years) | female | 711 | 41.9 | 13 | 30 | 43 | 53 |
| | male | 711 | 44 | 13.1 | 31 | 46 | 55 |
| Qualification (levels 0,1,2)[45] | female | 711 | .864 | .824 | 0 | 1 | 2 |
| | male | 711 | .816 | .813 | 0 | 1 | 2 |

Table 1: Summary statistics: UKTUS

## 5.2 Sharing rule estimates

Using the UKTUS pooled sample, I estimate the resource sharing rule for UK working couples. I find that households do not share resources equally: the mean resource share for women is 0.45 and 0.55 for men (the median is 0.44 and 0.56). Household-specific resource shares vary substantially (as can be seen in figure 1). This heterogeneity is driven by market variables and distribution factors. These findings confirm the importance of accounting for intra-household unequal sharing, and for household-specific variation in sharing as a function of characteristics. Moreover, these findings reinforce the prevalence of systematic inequality in sharing by gender, with the distribution of female resource shares to the left of that for men, as illustrated in figure 1.

### 5.2.1 Interpreting regression results and estimating marginal effects

I estimate leisure expenditure functions for men and women by seemingly unrelated regressions. The results are reported in table 2. I consider the following characteristics as likely

[43] This, together with self-employed labour, explains the lower end of hourly wages (which are sometimes lower than the official minimum hourly wage).

[44] Accurate non-labour income data is not available in UKTUS.

[45] Highest qualification obtained, simplified into three categories. Category 2 is equivalent to an undergraduate degree or higher. Category 1 is equivalent to end-of-school diplomas e.g. A levels, IBDP, or equivalent technical qualifications. Category 0 is anything less than that, including lower technical qualifications and school diplomas obtained before the end of school e.g. GCSEs.

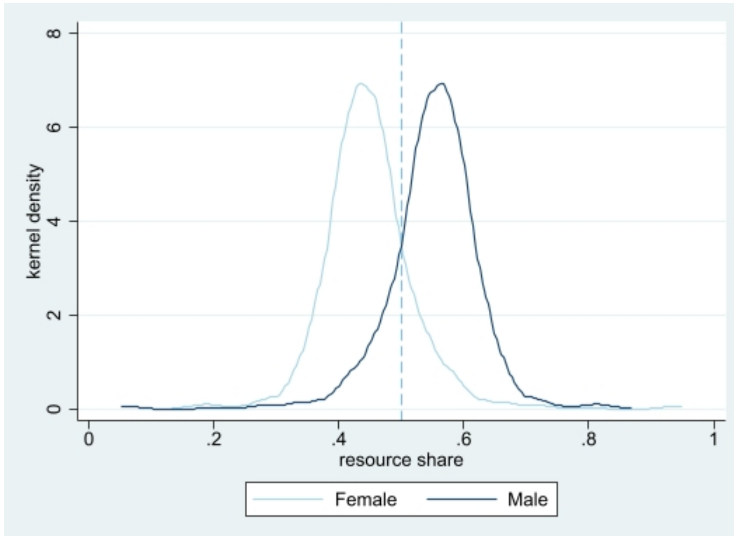


Figure 1: Baseline results, UKTUS. The distribution of male resource shares is a reflection of the distribution of female resource shares in the equal sharing (dotted) line. This is because each household's shares must sum to one.

to affect bargaining power: the hourly pay and educational attainment of each member, the age gap and average age of the couple[46] and regional wealth in the household's region.

The mean female resource share is calculated by dividing the coefficient on 'Budget' in the female regression by the sum of the 'Budget' coefficients across the two equations: $\hat{\eta_f^0} = \frac{0.198}{(0.198+0.243)}$, yielding 0.45. The preference coefficient is estimated by dividing the coefficient on 'Budget' in the female regression by the estimated average female resource share: $\hat{\alpha^l} = 0.198\frac{(0.198+0.243)}{0.198}$, coming to 0.44. This number is consistent with individuals choosing to take private leisure for a substantial proportion of their daily time endowment, where private leisure includes sleep, i.e. spending a large proportion of their budget on private leisure.

To interpret the regression coefficients, consider an example household $h$. The starting point for their resource shares are the averages $\hat{\eta_f^0} = 0.45, \hat{\eta_m^0} = 0.55$. We then adjust for any deviations from the sample mean for determinants of bargaining power. For instance, consider a deviation on female hourly pay. The effect of a unit deviation from the mean of the female wage $(wf)$ is calculated by dividing the coefficient on the 'Budget * dev. fem. hourly pay' term by the preference coefficient: $\hat{\eta_f^{wf}} = \frac{0.00559}{\hat{\alpha^l}}$. We estimate the resource share for a couple where the woman earns £15 an hour, instead of the mean of £9.87 as: $\hat{\eta_{f,h}} = \hat{\eta_f^0} + \frac{0.00559}{\hat{\alpha^l}} * (15 - 9.87)$, which is $\hat{\eta_{f,h}} = 0.51$ and hence $\hat{\eta_{m,h}} = 1 - 0.51 = 0.49$. Note that the impact of deviations from the mean female wage are very substantial. table 3 summarises the change in predicted female resource share for changes in different determinants of bargaining

[46]The age gap is the difference in years between the man and the woman

| Dependent variable | leisure expenditure | |
|---|---|---|
| Equation | male | female |
| Budget | 0.243*** | 0.198*** |
| | (0.00262) | (0.00236) |
| Budget * dev. fem. hourly pay | -0.00559*** | 0.00559*** |
| | (0.000198) | (0.000198) |
| Budget * dev. mal. hourly pay | 0.00215*** | -0.00215*** |
| | (6.87e-05) | (6.87e-05) |
| Budget * dev. fem. qualification | -0.00295 | 0.00295 |
| | (0.00253) | (0.00253) |
| Budget * dev. mal. qualification | 0.0154*** | -0.0154*** |
| | (0.00237) | (0.00237) |
| Budget * dev. average age | 0.000905*** | -0.000905*** |
| | (0.000151) | (0.000151) |
| Budget * dev. age gap | 0.000481 | -0.000481 |
| | (0.000338) | (0.000338) |
| Budget * dev. regional wealth | -5.44e-07** | 5.44e-07** |
| | (2.19e-07) | (2.19e-07) |
| Observations (households) | 711 | 711 |
| R-squared | 0.937 | 0.931 |

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 2: Regression results, baseline specification, UKTUS. The symmetry of coefficients on the interaction terms is imposed as a constraint during SUR estimation. Budget * dev. variable = budget * (household-specific value of variable - sample average of variable)

power.

The direction of estimated marginal effects is consistent both with bargaining theory and with previous findings in the literature. As shown in table 4, women's resource shares are higher in households with high female hourly pay and low male hourly pay. The impact of the former outweighs the latter, so that women in households with higher household budgets[47] have higher resource shares. Older couples (either because they are older or belong to more traditional generations) and couples with a larger age gap (older man relative to the woman) are characterised by a higher fraction of resources going to the man, although the latter effect is not statistically significant. Women have higher resource shares in wealthier regions, potentially because of better outside options for women in wealthier regions, or due to wealthier regions proxying more gender progressive regions. More educated men have

[47]The budget here is the full daily budget, i.e. the labour endowment of the couple (sum of male and female hourly pay, multiplied by 24 hours)

| variable | | mean | sd | impact of.. | on fem. share |
|---|---|---|---|---|---|
| Hourly pay (2020 GBP) | female | 9.87 | 5.58 | ↑ 1 s.d. | 0.0706 |
| | | | | ↑ £10 | 0.1266 |
| | male | 11.63 | 8.74 | ↑ 1 s.d. | -0.0426 |
| | | | | ↑ £10 | -0.0487 |
| Age (years) | average | 42.93 | 12.82 | ↑ 1 s.d. | -0.0263 |
| | | | | ↑ 10 years | -0.0205 |
| | gap | 2.06 | 4.72 | ↑ 1 s.d. | -0.0051 |
| | | | | ↑ 1 year | -0.0011 |
| Regional wealth (2020 GBP) | | 30,446 | 6,780 | ↑ 1 s.d. | 0.0084 |
| | | | | ↑ £10k | 0.0123 |
| Qualification (levels 0,1,2) | female | 0.86 | 0.82 | ↑ 1 s.d. | 0.0055 |
| | | | | ↑ 1 level | 0.0067 |
| | male | 0.82 | 0.81 | ↑ 1 s.d. | -0.0284 |
| | | | | ↑ 1 level | -0.0349 |

Table 3: Interpretation of regression coefficients

higher resources shares, and the same goes for females, although for the latter the effect is not statistically significant. A possible explanation is that, in the UK, female qualifications have trended upwards more strongly than male qualifications over time and generations, substantially reducing the matching market returns of female education.

Finally, I note that the significant effect of wages on the sharing rule (under the parametric assumptions discussed in section 4) is inconsistent with the unitary model and can be interpreted as evidence for the collective model. This finding is in keeping with several other findings in the household economics literature (see Cherchye, De Rock and Vermeulen (2012)).

| | | male hourly pay | |
|---|---|---|---|
| | | below median | above median |
| female hourly pay | below median | 0.43 | 0.39 |
| | above median | 0.50 | 0.47 |

Table 4: How the female resource share varies with male and female hourly pay

## 5.3 Empirical performance of the model

The model appears to fit the data well. Results are stable, and change in the theoretically consistent direction, when performing various robustness checks, including:

- Different definitions of private leisure, e.g. defining leisure in a much more narrow way, excluding time spent sleeping, eating, and work breaks. Consistently with the model, the estimated preference parameter on this narrower definition of leisure is much smaller in magnitude than the estimate for the baseline definition of private leisure.

- Estimation on different samples, e.g. excluding outliers with particularly low or high wages, and estimating the sharing rule separately on 2000 and 2014 data. Consistently with previous findings by Bargain, Donni and Hentati (2022) and Lise and Seitz (2011), I find that female resource shares in the UK are higher in more recent years.

- Running alternative regression specifications, e.g. substituting the gap between male and female qualification for the levels of male and female qualification, and excluding the age gap and regional wealth, as determinants of bargaining power.

Moreover, the model imposes several testable restrictions on the sign and magnitude both of the regression coefficients and the structural parameters recovered from them. These restrictions are not imposed by the estimation method, and testing them suggests the model fits the data well. Importantly, all resource shares fall within the unit interval.[48] Additionally, consistently with leisure being a good rather than a bad, the preference parameter on leisure is found to be positive. Moreover, in conformity to the way noise was incorporated in the model (see section 4.4.1), residuals are negatively correlated within the household, with a correlation coefficient of -0.09.[49]

The only restrictions that were imposed during estimation required that all coefficients apart from that on household budget sum to zero across equations. I run unconstrained SURE regressions and find the magnitudes of coefficients are similar across the two equations, and the signs are opposites of each other. This suggests that imposing the restriction that these coefficients sum to zero across the equations is not far off correct, and is not strong-arming the results (although some of the estimated coefficients are statistically significantly different from each other).

### 5.3.1 Testing the private leisure C-D SAP assumption

To test the validity of the private leisure C-D SAP assumption, I use data on working singles using UKTUS pooled data. I focus on working singles to ensure comparability in the way the budget is calculated, and also since non-participation leads to different interpretations of time-use choices, including involuntary leisure for the unemployed.

[48] I note that this is very much not mechanical: estimates using clothing and non-market-work fall outside of the unit interval at the extremes.

[49] A statistical test of cross-equation independence rejects at the 5% significance level

For singles, $w_i l_i = \alpha_t^l y_i$. I run this regression for men and women, finding the coefficients are approximately equal: $\hat{\alpha_m^l} \simeq \hat{\alpha_f^l} \simeq 0.6$.[50] I test the null hypothesis that $\alpha_m^l = \alpha_f^l$ and cannot reject the private leisure CD-SAP assumption at any of the usual significance levels. These findings suggest single working men and women satisfy private leisure C-D SAP, and hence that private leisure C-D SAP is plausible for working couples without cohabiting children. It is unclear why the preferences of men and women would change differently in this specific respect between being single (and working) and being in a couple (and still working). We may not be as sanguine about the validity of the test in other contexts, e.g. for couples with cohabiting children, since there are substantial gender differences in norms around childcare.

### 5.3.2 Testing the public good C-D SAP assumption

Recall that (in order to estimate sharing from time-use data alone) we also assumed that preference parameters over all public goods sum to the same quantity for men and women. Equivalently, we can test whether the preference parameters over all private goods sum to the same quantity for men and women.[51]

We have already tested that private leisure preferences are similar across types. We therefore only need to test that preferences for aggregate material private good consumption are the same for men and women. I test this using a comparable UK expenditure dataset (Living Costs and Food Survey, LCF, see section 6) for working singles.[52] For singles, $\sum_{j \in \Omega^c} p^j c_{i,h} = \left(\sum_{j \in \Omega^c} \alpha_t^j\right) y_{i,h} = \alpha_t^c y_{i,h}$. The test therefore requires regressing aggregate expenditure on material private goods on the budget, and checking whether the coefficients are the same for men and women. The coefficients are very similar in magnitude ($\hat{\alpha_m^c} \simeq \alpha_m^c \simeq 0.07$) and a test of equality cannot reject at any usual significance level. This test is consistent with the public good SAP assumption.

### 5.3.3 Homotheticity tests

Cobb-Douglas is clearly a strong simplification of underlying preferences, and we may be particularly concerned about imposing homotheticity. I test this assumption by estimating

[50] Not only are these preference parameters similar to each other but, comfortingly, they are higher than the estimated preference parameter for private leisure for working couples. We would expect that to be the case because joint leisure is not an option for singles, so we would expect singles to have a higher relative preference (higher preference coefficient, under the normalising assumption that preference coefficients sum to 1) for private leisure.

[51] The latter implication is preferable for testing because data on private good consumption is less lumpy, and collected more accurately, in most expenditure surveys, including the one used here.

[52] For singles there isn't a distinction between private and public goods in practice, but we can still distinguish theoretically between goods which are non-rivalrous and those that aren't, categorising goods in the same way for singles and couples.

singles' leisure expenditure separately for higher and lower income singles. I cannot reject equality of coefficients at any of the usual significance levels. As a further check, I estimate resource shares separately for higher and lower budget couples. Parameter estimates are broadly similar to the baseline, and the resulting estimated resource shares are highly correlated (correlation of 0.85)[53] with the baseline estimates, suggesting that the homotheticity assumption is not strong-arming results. Finally, I relax the homotheticity assumption by estimating the sharing rule under the assumption of Stone-Geary preferences.[54] Comparing the results to the baseline estimates, the direction of marginal effects is the same, and the estimated resource shares are very highly correlated (correlation of 0.97) with baseline estimates. Overall, it appears that, in this context, assuming homotheticity is not strong-arming results.

## 5.4 Comparison to other approaches

As well as estimating my baseline results with private leisure as the assignable good, I also estimate resource shares using two alternative assignable goods: (i) non-market-labour time, and (ii) clothing, and compare the findings. Finally, I consider how my findings relate to existing estimates in the literature on UK data.

### 5.4.1 Non-market-work hours as the assignable good

Survey data on usual working hours is available in many datasets, including expenditure datasets. Where detailed time-use data is not available, time not spent on market work can be considered an imperfect substitute for private leisure as an assignable good, and was used in this manner by Lise and Seitz (2011). Because this choice of assignable good also includes joint leisure and time spent on domestic work (and childcare, where there are cohabiting children) the assumption that it is a private good is unrealistic. I assess how much of a problem this is for UK working couples using two different measures of non-market-work. First, I construct non-market-work time from detailed UKTUS diary data. Then, I construct it from a question on usual hours worked from the LCF expenditure data (see section 6).

Using the first, results are very similar to those obtained by using private leisure (they are very highly correlated, with a correlation coefficient of 0.97, and regression coefficients have similar signs and magnitudes), although they slightly overestimate female resource shares

[53] This is for the sample excluding outliers. The correlation coefficient including outliers is still high but falls to 0.75 as each of the two sub-samples only contains outliers in one direction.

[54] In order to avoid colinearity issues due to the lack of accurate non-labour income data, this version of the estimates assumes all goods are private and that subsistence levels for private leisure are equal to zero (while they can be non-zero for other goods).

(by less than a percentage point on average). Using the second, results are still highly correlated with my baseline results, but to a much lesser extent (the correlation coefficient is 0.76),[55] and the signs and significance of regression coefficients differ in a couple of cases from the baseline regression. The mean resource share is estimated at 0.49 for women, substantially higher than baseline. This suggests that, for working couples, the main source of inaccuracy from estimation using labour supply data, instead of detailed time-use data, is the measurement error in the former rather than the inability to disentangle between private leisure, joint leisure, and domestic work. For other household compositions, in particular with cohabiting children, we may expect an additional strong source of bias: women in these households tend to spend substantially more time than men on childcare and domestic work, and less time on work. This would lead to further overestimation of female resource shares from labour supply data.

Taken together, this suggests (i) where possible, using detailed time-use data to construct private leisure, and (ii) where it is not, using less accurate data containing a signal of non-market-work time, but being particularly cautious when interpreting results for households categories which are likely to exhibit gender differences in time spent on activities other than leisure and work.

### 5.4.2 Clothing as the assignable good

I repeat my analysis using clothing as the assignable good, using the LCF expenditure data. Clothing is of interest because it has been used as the assignable good almost universally in this literature. I find a mean female resource share of 0.63, substantially higher than my baseline finding of 0.45. As can be seen in figure 2, the whole distribution of female resource shares is shifted upwards, and the extremes of the distribution violate the testable restriction that resource shares lie in the unit interval. Using singles data, I find that women's preference parameter for clothing is approximately twice as large as men's, and clothing CD-SAP is rejected at the 5% significance level. Therefore, I adopt an adjusted approach with C-D SRAT instead of C-D SAP as the identifying assumption. The resulting distribution of adjusted clothing resource shares is much closer to the baseline, as can be seen from figure 2, and the mean resource share for women falls to 0.46. While adjusting the identifying assumption is helpful in reducing the gap between clothing and baseline estimates, the results remain hard to reconcile. Firstly, the regression coefficients for clothing are in contrast with bargaining theory: female hourly pay decreases female resource shares, and male hourly

[55] My baseline resource share estimates for LCF are obtained by applying my baseline estimate of the sharing rule, from UKTUS, to LCF data. As explained in section 6, the datasets are comparable and the distribution of baseline resource shares is very similar.

pay decreases male resource shares. Secondly, the estimated resource shares are strongly negatively correlated with those estimated from private leisure.

Investigating the raw correlations between clothing expenditure and hourly wages, we observe that expenditure on female (male) clothing is increasing in both female (male) and male (female) hourly pay, but more strongly in the latter. It appears that clothing is not credibly private and that it may have important externalities, status effects or gift-giving effects. To confirm this, I run different candidate structural regressions of clothing expenditure on resource share estimates and household budget as explained in appendix E. I find that clothing expenditure is best explained by (i) using baseline resource share estimates instead of clothing resource share estimates, and (ii) treating clothing as a public, rather than private, good. Therefore it seems that for UK working couples without cohabiting children clothing is not a reliable assignable good.

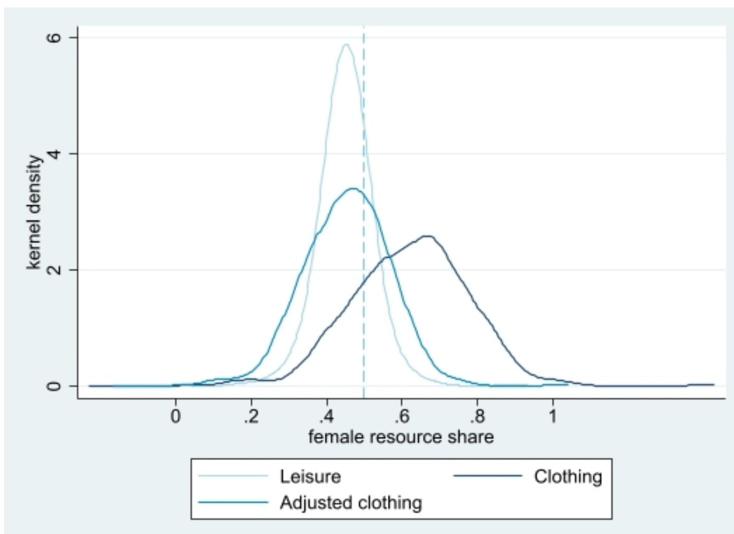


Figure 2: Using C-D SRAT instead of C-D SAP narrows the gap between leisure and clothing estimates

### 5.4.3 Estimates using different identification approaches

Reassuringly, my findings are similar to those in Lise and Seitz (2011), who use non-labour-hours as the assignable good. Their framework differs from this paper in several respects, including parametric specification and estimation approach. A key identifying assumption in Lise and Seitz (2011) is the symmetry assumption that women and men with the same potential earnings have the same resource shares. In a traditionally patriarchal society this assumption seems strong.[56]

Regardless of the different approach taken, their estimates are similar in magnitude to mine. Their estimates include couples with non-participation and cohabiting children, so are not directly comparable. Their estimated share for the last cohort in their data, born in the 1960s, has a female resource share of 44.2% on average. This is not quite comparable to my estimates as women are likely to have lower resource shares in households with non-participation and cohabiting children, but the magnitude is reassuringly similar. This suggests that findings are reasonably robust across the different approaches recently developed in the literature.

However, the results of Bargain, Donni and Hentati (2022), using clothing as the assignable good (as well as a different methodology) are substantially different. They find the average resource share for women in heterosexual couples without children (including non-participants) in 1978-2007 is 51.7%. This is an estimate for the pooled 1978-2007 sample, with an upward trend over time, implying a higher estimate for 2000-14. Therefore, it appears that the approach in Bargain, Donni and Hentati (2022) (which differs from this paper in several respects, including parametric specification and estimation approach) leads to much higher estimates than the adjusted clothing approach discussed above. Moreover, it differs substantially from my baseline estimates using private leisure as the assignable good. Qualitatively, my baseline result that, on average, women have a lower resource share than men is reversed. It seems unlikely that women would have higher resource shares than men on average in a society which, while comparatively gender progressive, still has a patriarchal tradition and norms. This further raises questions about the issues associated with clothing being used as the assignable good, and whether it may lead to inaccuracies in certain contexts. More systematic and thorough investigation of how different approaches compare is a priority for future work, to enable additional progress in this field.

# 6 Individual-level consumption for UK couples

Having estimated the sharing rule from time-use data, this can be of interest in itself, or can be applied to other comparable datasets to investigate additional questions. A natural goal is to estimate the total cost of all material goods consumed by an individual (individual-level consumption).[57] For each individual, this is defined as the sum of household material

[56] Empirically, this assumption is not supported by my findings. A woman with an hourly wage equal to the average male wage in the sample is estimated to have only a 47% resource share, substantially below the average male resource share of 55%.

[57] This terminology is somewhat imprecise since I am not referring to the consumption bundles of different individuals. However, it captures the essence of this measure, which compares the value of different consumption bundles at their market prices. I avoid the term individual-level expenditure because this could

public expenditure and the individual's share of the household's material private expenditure, similarly to Lise and Seitz (2011). This metric allows us to compare the objective (preference-independent) value of material consumption of different individuals. While this is not equivalent to welfare comparisons, precisely because it does not take differences of preferences into account, it is a clearly policy-relevant measure. It captures the monetary value of all private and public material goods consumed by the individual.

The following process can be implemented to estimate individual-level consumption. I illustrate this by estimating individual-level consumption for UK working heterosexual couples without cohabiting children. Further detail is provided in appendix B.

1. A cross-section household expenditure dataset is required.
   (a) I use the LCF (Living Costs and Food Survey)[58]; a high-quality, large-scale survey that is used to estimate official government statistics.
2. If the expenditure data is contained in a separate dataset from the estimating dataset (e.g. a time-use dataset) apply the estimated sharing rule to the expenditure dataset. It is important that the dataset used to estimate the sharing rule and the expenditure dataset be comparable, e.g. nationally representative data for the same country in the same year. The data should include household characteristics and member's characteristics which influence the sharing rule (e.g. members' wages, ages, etc.)
   (a) The LCF, like UKTUS, is nationally representative for the UK. I use data for the year 2014 to ensure time-period comparability with the UKTUS data used for resource share estimation.[59] The LCF contains information on individual and household characteristics, individual labour supply and detailed income data, which allow me to estimate individual-specific resource shares using my estimated sharing rule. The distribution of resource shares in the LCF is similar to that in the UKTUS, and the mean female resource share remains 45%.
3. In the expenditure dataset, divide household expenditure into public and private expenditures. Moreover, some outflows should be excluded if they are incompatible with the static nature of the model (e.g. saving, insurance and investments). To enable

be confused with a different approach, taking into account individual Lindahl prices for public goods.

[58]Office for National Statistics. (2019a). Living Costs and Food Survey [data series] 3rd Release. http://doi.org/10.5255/UKDA-Series-2000028.

[59]Since the sharing rule was estimated using data from 2000 and 2014, this choice of year ensure comparability of the expenditure and time-use data used to estimate the sharing rule. I do not additionally analyse LCF data from 2000 as there were substantial changes in the dataset in 2001 which undermine the feasibility of accurately pooling the 2000 and 2014 LCF datasets.

accurate classification of expenditures, the dataset should record household expenditure by sufficiently granular categories. Ideally, the data would allow inclusion both of durable and non-durable expenditure. Additionally, it is helpful if the data contains information on vehicles owned and the characteristics of the home the family lives in to enable imputing rent and lease prices consistently across the sample. This is important because expenditures on purchasing or renting a home, and purchasing vehicles, are extremely large in relative terms (especially in some contexts, including the UK).

(a) The LCF contains household expenditure divided into very granular categories (COICOP codes). In addition, the LCF records a two-week expenditure data, recall questions on infrequent expenditures, data on the number of vehicles owned, and the characteristics of the home the family lives in.

4. In the expenditure dataset, estimate individual-level consumption for each household member by summing (i) public good expenditure and (ii) private good expenditure weighted by the individual's resource share: $C\hat{C}_{i,h} = \hat{\eta_{i,h}} \left( \sum_{j \epsilon \Omega^c} p^j c^j_h \right) + \left( \sum_{j \epsilon \Omega^X} r^j X^j_h \right)$ This yields a distribution of individual-level consumption, which can then be used to estimate different measures of inequality.

## 6.1 Results

For heterosexual working couples without cohabiting children, the distribution of individual-level consumption for women is to the left of the distribution for men. On average, the gender gap in consumption between men and women is 7.86%. The reason this gap is smaller than the average gender gap in resource shares is that the sharing rule only applies to private expenditures, while consumption is also inclusive of public expenditure. I also estimate two common measures of inequality: the Gini coefficient is 0.21, and 6% of the group live in relative poverty (consuming less than 60% of the median individual-level consumption). My baseline estimates of inequality are low relative to standard estimates of inequality, partly because I focus on the sub-sample of working couples without cohabiting children. The degree of intra-household inequality estimated in this paper likely underestimates the degree of intra-household inequality in the UK as a whole, for instance Bargain, Donni and Hentati (2022), suggests that women's resource shares are lower in couples with children than without. Another likely driver of my inequality estimates being lower than some of the available estimates is that I attempt to fully account for public expenditure, and assign all public expenditure to each household member. Focusing only on private consumption, or treating voices of public expenditure as private, substantially overestimates inequality (especially absolute measures of poverty, as these mechanically increase if some expenditures are excluded). Excluding public expenditure, the Gini coefficient increases to 0.30 and the relative poverty rate to 16%. When treating public expenditure as private, the Gini coefficient increases to 0.24 and the relative poverty rate to 9%.

## 6.2 Other applications

### 6.2.1 Individual-level full consumption (including time-use)

We may be interested in the monetary value of individual-level consumption including private leisure, joint leisure, and the domestic good. Following Becker (1965) and Lise and Seitz (2011) I term this 'full consumption'.[60] This may be of particular interest in contexts where different types of people are thought to have different preferences for time-use, so that a comparison of material consumption only may be misleading (with private leisure SAP, we may worry less about this, although there could still be a difference in preferences over joint leisure and the domestic good).

Including expenditure on private leisure is straightforward as long as we have time-use data. We need only sum $w_{i,h}l_{i,h}$ to individual i's personalised expenditure. With joint leisure and domestic work, we can follow a similar procedure. While individuals consume domestically produced goods, for which we do not estimate the production functions, the expenditure required to generate those good is calculated simply by the amount of time spent on the relevant activities multiplied by the price of that time (wages). For instance, the expenditure on the domestic good is $\sum_{i\in h} w_{i,h}d_{i,h}$, regardless of what the production function for the domestic good is.

The time-inclusive measure of individual-level consumption is given by:

$$T\hat{CC}_{i,h} = w_{i,h}l_{i,h} + \hat{\eta_{i,h}}\left(\sum_{j\epsilon\Omega^c} p^j c_h^j\right) + \sum_{j\epsilon\Omega^X} r^j X_h^j + \sum_i w_{i,h}\left(d_{i,h} + jt_{i,h}\right)$$

In order to estimate time-inclusive measures of individual-level consumption, at least some time-use data is required. When time-use and expenditure datasets are separate, it may still be possible to estimate time-inclusive individual-level consumption, as discussed in appendix D.5 for the case of Cobb-Douglas preferences.

### 6.2.2 Money-metric welfare estimates

Instead of individual-level consumption, we may be interested in a money-metric measure of individual welfare. This captures the expenditure that would be necessary for an individual

[60] Lise and Seitz (2011) distinguish only between market labour and non-labour time, but the concept is similar.

to achieve, as a single, the same level of utility they currently enjoy (see Chiappori, Okuyama et al. (2023)). This measure takes preferences into account, and in particular the fact that different people may enjoy the same level of public good consumption to varying extents.

This application requires more restrictive preference assumptions than the identification result in this paper, but can be readily undertaken with the Cobb-Douglas implementation in this paper. Note that money-metric welfare measurement relies on intra-personal utility comparisons between the same person when single and not. We may worry about similar concerns as discussed with using the SAT assumption, but on a larger scale, as the whole utility function must be the stable across household compositions to make this comparison.

# 7 Conclusion

My findings add to the growing literature on the importance of estimating intra-household inequality and the importance of considering the dimensions, such as gender, as well as the levels, of inequality. The approach set out in this paper is grounded in the well-established collective model of the household, has a clear source of identification, and is straight-forward to implement on widely available data. Moreover, while the approach is simple, tests of model fit in my application to UK data suggest that it is empirically sound. This approach is also well-suited to straightforwardly estimating a variety of measures of individual-level well-being. These characteristics may contribute to the goal of facilitating adoption of individual-level inequality measures outside of academia.

To further aid this goal, further work is required to investigate the empirical performance of the different approaches that have been proposed in the literature. Bargain, Lacroix et al. (2021) test a specific approach against reported individual-level consumption. Future work might systematically compare the performance of different approaches using simulated and empirical data, as well as comparing their more general advantages in terms of applicability. This exercise would be very valuable in a literature which has been growing in several different directions, and may benefit from consolidation into something like a mainstream approach.

The methodology in this paper builds on, and bridges between, work in multiple strands of the literature. It retains the linear identification from a single assignable good of LPW, but extends it to more general contexts, which are modelled in other papers with more complex and hard-to-implement identification. There may be important advantages from further bridging between different strands of the literature to yield an approach which retains the strongest features of each. This paper argues that there are important advantages of using private leisure, instead of clothing, as the assignable good. However, using the identification approach in this paper, this restricts analysis to households where all members participate in market work. Future work might extend analysis to more household compositions, while endeavouring to maintain a simple estimation approach, by exploring new approaches using multiple assignable goods jointly for identification.

Such approaches would also have the advantage of potentially reducing reliance on the, rather strong, identifying assumptions required in this paper and similar approaches in the literature. These assumptions have a large impact on the magnitude of estimates and even on qualitative conclusions, as illustrated with clothing-based estimates of the sharing rule in the application to UK data. An important direction for future work is to retain the simplicity and ability to point-identify of this strand of the literature while weakening its reliance on these identifying assumptions, or testing them in as much as possible.

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# Appendix

## A Identification worked example: Almost Ideal Demand System

Consider preferences which are separable in public and private goods, and where the private good sub-utility corresponds to the Almost Ideal Demand System (Deaton and Muellbauer (1980)).[61] Then Engel curves for the assignable good take the form:
$\frac{p^a_{i,h}c^a_{i,h}}{\rho_{i,h}} = \alpha_{t,g} + \sum_j \gamma^j_{t,g} \ln p^j_{i,h} + \beta_t \ln \rho_{i,h} - \beta_{t,g}\left(\alpha^0_{t,g} + \sum_j \alpha^j_{t,g} \ln p^j_{i,h} + \frac{1}{2}\sum_j \sum_k \gamma_{kj} \ln p^k_{i,h} \ln p^j_{i,h}\right)$
Here the notation for material private goods is used to also include private leisure, so that $p^l_{i,h} = w_{i,h}$. Hence, the $i$ subscript (some other material good prices may vary at the household level). Similarly, the prices of public goods are indexed by $h$: $R_h$.

We re-write the Engel curves in terms of observables and the object of interest, remembering that $\rho_{i,h} = \eta_{i,h}(y_h - R_h Q_h)$ and multiplying through by the resource share:

$$\frac{p^a_{i,h}c^a_{i,h}}{y_h - R_h Q_h} = \eta_{i,h}\left(\alpha_{t,g} - \beta_{t,g}\alpha^0_{t,g}\right) + \eta_{i,h}\sum_j \left(\left(\gamma^j_{t,g} - \beta_{t,g}\alpha^j_{t,g}\right)\ln p^j_{i,h}\right)$$
$$- \eta_{i,h}\beta_{t,g}\frac{1}{2}\sum_j \sum_k \gamma_{kj} \ln p^k_{i,h} \ln p^j_{i,h} + \eta_{i,h}\beta_{t,g} \ln \eta_{i,h} + \eta_{i,h}\beta_{t,g} \ln (y_h - R_h Q_h)$$

This is all linear in unknowns with the exception of the non-linear $\eta_{i,h}\beta_{t,g} \ln \eta_{i,h}$ term. We can linearly approximate $\beta_{t,g} \ln \eta_{i,h}$ as $\kappa^0_{t,g} + \sum_\omega \kappa^z_{t,g}\omega_h$, where $\omega_h$ could coincide with the characteristics $z_h$ used in the resource share approximation, or be transformations of $z_h$. Substituting this in and rearranging we obtain:

[61] This is an extension of the set-up in LPW to public goods. If also wishing to incorporate the shareable goods framework, this can be done as in LPW, with their assumption that A is block-diagonal so that there are no cross-good complementarities relating to the assignable good $A_g = \begin{matrix} A^X & 0 \\ 0 & A^c \end{matrix}$. This assumption ensures that the assignable good Engel curves have the usual form, instead of the more complex form which would result from cross-good complementarities with other goods.

$$\frac{p^a_{i,h}c^a_{i,h}}{y_h - R_hQ_h} = \eta_{i,h}\left(\alpha_{t,g} - \beta_{t,g}\alpha^0_{t,g} + \kappa^0_{t,g}\right) + \eta_i \sum_j \left(\left(\gamma^j_{t,g} - \beta_{t,g}\alpha^j_{t,g}\right)\ln p^j_{i,h}\right)$$
$$- \eta_{i,h}\beta_{t,g}\frac{1}{2}\sum_j\sum_k \gamma_{kj}\ln p^k_{i,h}\ln p^j_{i,h} + \eta_{i,h}\left(\sum_\omega \kappa^z_{t,g}\omega_h\right) + \eta_{i,h}\beta_{t,g}\ln\left(y_h - R_hQ_h\right)$$

where $\eta_{i,h}$ is approximated by $\eta^0_{t,g} + \sum_z \eta^z_{t,g}\left(z_h - \bar{z}\right)$
We can see that this satisfies the assumptions for identification, as long as $(y_h - R_hQ_h)$ is not an element of $\omega_h$ (a related assumption in LPW is that resource shares don't depend on the household budget):

- $f\left(c^a_{i,h}\right)$ is the Engel curve $\frac{p^a_{i,h}c^a_{i,h}}{y_h - R_hQ_h}$
- unknowns enter linearly (after the approximation of the log resource share term, and of the resource share)
- $f^0_{t,g} = \beta_{t,g}\ln\left(y_h - R_hQ_h\right)$, which satisfies:
  - the requirement that the only unknown parameter $\beta_{t,g}$ is a preference parameter, and
  - that $\beta_{t,g}\ln\left(y_h - R_hQ_h\right)\left(\eta^0_{t,g} + \sum_z \eta^z_{t,g}\left(z_h - \bar{z}\right)\right)$ is not no-colinear with any other term in the Engel curve, as long as $(y_h - R_hQ_h)$ is not an element of $\omega_h$
- the identifying assumption restricting preference heterogeneity is made on $\beta_{t,g}$

The Almost Ideal Demand System Engel curves are not very tractable if all prices vary in the sample. This is both because it becomes highly multi-dimensional and because estimating it requires high-quality, detailed price data. As a result, DLP and LPW assume no price variation and drop the price index terms into the constant. This was possible because time-use was not modelled in those papers. In a model with time-use, there will be some price variation in the form of individual-level wages, even if we can assume all other prices to be constant in the sample.[62] If this is the case, we can simplify the estimating equations to:
$\frac{p^a_{i,h}c^a_{i,h}}{y_h - R_hQ_h} = a^0_{t,g}\eta_{i,h} + a^1_{t,g}\eta_{i,h}\ln w_i + \sum_\omega a^\omega_{t,g}\omega_h\eta_{i,h} + \eta_{i,h}\beta_{t,g}\ln\left(y_h - R_hQ_h\right)$
Even so, resource shares are likely to depend on the wage, age, and educational attainment of all household members at least. With an N person household, that is 3N terms for the

[62] To avoid dealing with this, if the assignable good is material, we could assume preferences that are separable in material goods and time-use.

resource share approximation, plus the constant term. Assuming the log resource share term is also approximated by 3N terms plus a constant, the estimating equations are composed of $(3N + 1)(3N + 3)$ terms.

For a couple, that is 63 terms. Many of these terms will also likely be highly correlated with each other. Without yet more approximation, this is not a tractable specification, especially for estimation with small samples. This is problematic since small samples are almost inevitable in this literature since estimation must be conducted separately for households of different categories. Yet, more approximation is also unpalatable, as it undermines the link between the structural model and the estimated regression. A more restrictive, but more parsimonious functional specification for preferences may be a preferable approach.

# B Expenditure data and individual-level consumption estimation

## B.1 The dataset

The LCF (Living Costs and Food Survey) (previously FES) is a UK survey containing information on individual and household characteristics, individual labour supply, detailed income data, and very detailed expenditure data. It is a repeated cross-section available yearly since 1978. I illustrate this application with 2014 data.

It is a high-quality, nationally representative, large-scale survey that is used to estimate official government statistics. The FES/LCF data has been widely used both for academic and policy applications due both to its scale and high quality. For instance, Bargain, Donni and Hentati (2022) and Lise and Seitz (2011) use this data.

The survey has multiple components: (i) a household survey recording household characteristics and retrospective questions on irregular expenses (rent, clothing, vehicles...); (ii) an individual questionnaire with individual characteristics, including demographic characteristics, hours worked and sources of income;[63] (iii) a detailed two-week expenditure diary for all members older than 7 (simplified diary for people aged 7-15, full diary for people aged 16 or above). The household questionnaire is answered by the reference person either alone, or together with other household members. Individual surveys, and expenditure diaries, are

[63] In rare instances, income is top-coded. I adjust top-coded values using data on after-tax income percentiles from HM Revenue & Customs. HM Revenue & Customs. (2023). Percentile points from 1 to 99 for total income before and after tax. https://www.gov.uk/government/statistics/percentile-points-from-1-to-99-for-total-income-before-and-after-tax

answered by the relevant person. The expenditure diaries are kept for two weeks by all household members.

The expenditure diary records the type of good in detail, and receipts are attached. Clothing and footwear is divided into male and female, children.[64] Household expenditure is obtained by summing expenditure over all members. Additional information on expenditure on large infrequent expenses, such as house repairs, and regular expenses, such as rent, is obtained during the household survey. These expenditures are transformed to an equivalent weekly value to make them comparable to other categories.

After restricting the LCF 2014 sample to heterosexual working couples and cleaning the data, the final sample comprises 583 households (i.e. 1,166 individuals).

## B.2 Categorisation of expenditures into private and public

The next step towards estimating individual-level inequality is to divide the LCF household-level expenditure data into (i) private expenditure, (ii) public expenditure, and (iii) expenditure to be excluded from consideration. In deciding how to do so, it is important to consider the goal of the exercise. In the case of this paper, the aim is to compare the material standard of living of different individuals in the UK.

### B.2.1 Excluded categories

I exclude expenditure categories which have almost no immediate consumption value and cannot easily be squared with the static model underlying the methodology in this paper. This includes: savings, insurance, investments, major house works including renovation (minor repairs are included), financial gifts, bets, gambling and expenditure on education (the latter is minor for working couples without cohabiting children). Future work considering dynamic aspects would enable incorporating these categories into overall resources in a theory-consistent manner. Again with in mind the goal of measuring material well-being, I focus on expenditures gross of any government refund or subsidy (sometimes expenditures are partly funded by the government and this is visible in the expenditure data for some goods). Where this is the case, I do not detract any subsidies or refunds from expenditure, since these still contribute to material well-being.[65]

[64] Note that any particular good may be purchased by a household member but consumed by any combination of them and/or other members, so personal expenditures do not measure personal consumption.

[65] For other applications, e.g. to estimate what proportion of the population has access to a minimal standard of living, we might not consider the consumption of 'bads' to contribute to that standard of living, and hence also exclude those.

### B.2.2 Private vs public consumption

Based on the detailed COICOP plus codes into which expenditure is divided in the LCF, I divide expenditure between private and public. While it would be possible to categorise expenditure based on less granular data, the granularity aids accuracy. For instance, most house-cleaning products are categorised as public (as they contribute to the public cleanliness of the house) but washing powders are categorised as private as more is needed to wash the clothes of more members.

Discretion is needed in categorising goods, as most goods have at least some public element, including externalities of consumption on other household members. While categorising partly public goods is likely to create some inaccuracies, it is important to recall these are much smaller in magnitude than simply assuming all goods are private. In some cases this categorisation should be made conditionally on household characteristics. For instance, given this application focuses on couples, I categorise holiday accommodation as a public good, since the price of a hotel room is typically similar for one or two people. This categorisation may not be as accurate for larger families.

In some cases, it might be possible to use other data (e.g. data on car occupancy) to estimate to what degree a good (e.g. cars) is public and to what degree it is private. A car and related expenses (insurance, fuel, etc.) could be purely public if everyone in the household only used it together (e.g. to drive to a holiday home). Alternatively it could be purely private if only one household member used it to drive to work. As discussed in more detail below, I approximate the likely economies of scale of car-related expenditures using UK car occupancy data (we can think of car-related expenditures as separate goods depending on the type of trip, where some of them are private and some of them are public).

### B.2.3 Treatment of durables, including homes and vehicles

While some papers in this literature focus on non-durable consumables, I suggest it is important to also consider durables. In particular, housing is a very durable and infrequent purchase, but it is important to take it into account since it is such a large expenditure for many households, and since it is a very important component of public consumption and household economies of scale.

For less expensive durables such as clothing and phones, I use the LCF expenditure data without adjustments. While for any specific category this is likely to lead either to over-estimates or under-estimates, the overall expenditure across categories is likely to be a reasonably accurate estimate of usual expenditure. For very large expenses (buying a home or a vehicle), a different approach is needed, since the magnitude of these expenditures dwarves regular weekly purchases. Moreover, these expenditures are often diluted over some periods of time, with a mortgage or loan, with repayments depending not only on the quality of the good being purchased but also on factors which are not directly relevant to material well-being, such as macroeconomic conditions at the time of purchase and individual credit score. Keeping in mind the goal of measuring material well-being, we wish to estimate the value of the good being consumed (e.g. a home in a certain area with a certain number of rooms) while abstracting from extraneous considerations (e.g. whether the home is owned or rented). Of course the household may financially benefit from owning outright instead of renting, through decreased monthly expenditures on housing. However, this will translate into increased expenditures on other goods, and hence will still be taken into account insofar as it affects material well-being.

**Imputed rents for homes** Housing is a particularly complex good from the viewpoint of measuring inequality in the UK. The prices of homes with similar attributes in different regions differs vastly, but households generally have limited choice as to their location, especially in the short-run (due to jobs, relationships, and coordination between members). While it is approximately true that goods like food are similarly priced across the UK, assuming price homogeneity for homes across the UK would be too unrealistic (e.g. homes in the London area command a very large premium). Another source of difficulty is that homes are sometimes owned outright, sometimes they are purchased with a mortgage (the interest on which varies vastly by year and credit score of the purchasing party), sometimes they are rented privately, and sometimes publicly (e.g. council housing provided as a benefit). To avoid both the issue of infrequency and the problem of comparability between renters, outright owners, and owners paying back a mortgage, I impute a weekly standardised rental price. This price increases in the quality of the home (as measured by the number of bedrooms, centrality of location, etc.) but is standardised across different purchasing conditions, as well as for regional house price differences (the latter being mostly something that affects the investment value of the property rather than reflecting the underlying quality of the home).

I suggest taking the following approach to balance between the two opposing goals of comparability between households and granularity of household-specific expenditure:

- Using data on private renters, I regress rent expenditure on

    1. Number of bedrooms. This is the best proxy for home size available in the data.
    2. Area type. I divide areas into four types based on granular OAC codes: rural, more desirable urban, less desirable urban, and suburban. The 2011 Area Classification

for Output Areas (OAC) categorises postcodes into types e.g. areas dominated by 'urban professional and families'.

3. Council tax band. Council tax bands are available for homes in England, Wales, and Scotland (I impute them for Northern Ireland). Council tax bands are based on legacy valuations of homes, and provide a good signal of the quality of the home.

4. Region (Wales, Scotland, Northern Ireland, and the nine regions of England). There are substantial regional disparities in the prices of homes in the UK.

- I impute the rent for all households in the data (including those who own, rather than rent) based on these characteristics.

- Next, I standardise imputed rents across regions by indexing them to the region with the lowest median imputed rent (the North East of England). I deflate the imputed rents for homes in other regions by the ratio of the median imputed rent in their region to the median imputed rent in the North East of England. This procedure can be thought of as uncovering the fundamental quality of the consumption obtained from the house, disentangling it from other considerations like investment value of property. We can also motivate this choice by noting that households often cannot move region (at least in the short-run), but can choose the specific location of their home (with different associated OAC code), its size (proxied by the number of bedrooms) and broader quality (proxied by council tax band).

- For each household I record the standardised imputed rent.

- A small number of households have a second home, but the data contains no information on the second home. Where this is the case, I double the standardised imputed rent of the main home, since the value of the main home is the best available signal in the data of the likely value of the second home.

- I add the standardised imputed rents to public household expenses.

**Imputed lease price for vehicles** Car purchases are the second largest expenditure items after homes for many UK households. For this reason, it seems important not to exclude this expenditure. Following a principle similar to the one outlined above for housing, I propose a rental approach to vehicles (cars, vans and motorcycles):

- I calculate the median weekly lease price paid by households that lease a vehicle.

- For each household, I observe how many vehicles they own, and estimate their vehicle lease expenditure by multiplying the average lease price by the number of vehicles owned.

- I add 54% of this imputed lease to private expenditures and the remaining 46% to public expenditures. This approximation is based on data from the National Travel Survey[66] on the proportion of car trips by purpose, and data on the occupancy rate of cars by trip purpose.[67] This method aims to capture the likely economies of scale of cars, although of course further granularity could be achieved if the data contained information on how the household uses cars. We can think of this as dividing car-related expenditures into multiple goods, some of which are private (e.g. car trips for solo work trips) and some of which are public (e.g. car trips for family holidays).[68]

# C Why the assignable good must be private, and the issue of non-participation

The identification approach in this paper relies on having a credibly private assignable good. If this is the case, then the second-stage demand of each household member depends on their own resource share, and does not directly depend on any bargaining fundamentals. However, when the chosen assignable good is not really private, each members' demand for the good is a function of the Pareto weights of all members. In this case, sharing cannot be identified only from data on these demands. If a good that is not really private is treated as such, and used as the assignable good, that results in mis-specification and unreliable estimates.

The chosen assignable good may not be private if there are substantial public elements to it, as appears to be the case for clothing in UK couples. This is also the substantial risk in using non-labour-time instead of private leisure (as in Lise and Seitz (2011)).[69]

Alternatively, the chosen assignable good may fail to be private in the case of corner solutions. This could potentially be an issue for a material assignable good if the individual was consuming zero quantities for all other private goods. This is unlikely if material private

[66]Department for Transport. (2021). National Travel Survey: 2020. https://www.gov.uk/government/statistics/national-travel-survey-2020/national-travel-survey-2020#trends-in-car-trips.

[67]Department for Transport. (2022). Car or van occupancy and lone driver rate by trip purpose, England: 2002 onwards. https://www.gov.uk/government/statistical-data-sets/nts09-vehicle-mileage-and-occupancy.

[68]I do not attempt to adjust this split based on the numbers of vehicles owned as I am not aware of data that would enable such an adjustment.

[69]Empirically, in my application to UK working couples, this seems to yield similar estimates as private leisure. However, for more general contexts of application, that is unlikely to be the case.

goods are defined in a reasonably granular way. It is also unlikely in a model with private time-use. However, the issue of non-privateness induced by a corner solution is likely to materialise for some households categories when private leisure is used as the assignable good. If in the second stage of the household the individual chooses how to allocate their remaining time endowment between market work and private leisure, but they are at a corner solution with no market work, then their private leisure demand is fully determined by the first-stage (public) decisions about their joint leisure and domestic work. Hence, for individuals who do not supply market work, private leisure cannot be reliably used as the assignable good.

The difference between the case of participation and non-participation is illustrated in 3. The left panel shows the case of an interior solution with market participation, while the right panel exemplifies non-participation. To simplify exposition, I group domestic work and joint leisure into a single public time category. This is to be understood as the optimal combination of domestic work and of joint leisure. In the interior solution case, there is an internal market for the individual's time, so that the first stage optimisation and the second stage optimisation can be considered separately, using the market wage as the price of time in both cases. This leads to neat second-stage leisure demand functions, which can be used directly for estimation of resource shares. In the case of market non-participation, the market wage is lower than the returns from optimal non-market-work time-use over the whole unit interval. This substantially complicates the problem. The price of public time-use is no longer the market wage, and leisure demand is no longer derived from a first order condition, but simply falls out from the household's first stage choices of domestic time and joint leisure (public time uses). This means that, in practice, leisure can no longer be treated as a private good.

Therefore, this paper focuses only on households where all members participate in market work. It is best not to extrapolate results to households with non-participants due to the likely substantial non-linearities between the interior and corner contexts, which would weaken the viability of the linear approximation of the resource share function. An important direction for future work may be to extend the methodology set out in this paper to the context of non-participation.

## C.1 Why excluding couples with non-participation is unlikely to bias estimates

In my application to UK data, I focus only on couples without cohabiting children. In this setting, I restrict my sample only to couples where both members supply some market work. It is important to note that this is unlikely to bias the estimates, for the following reason.

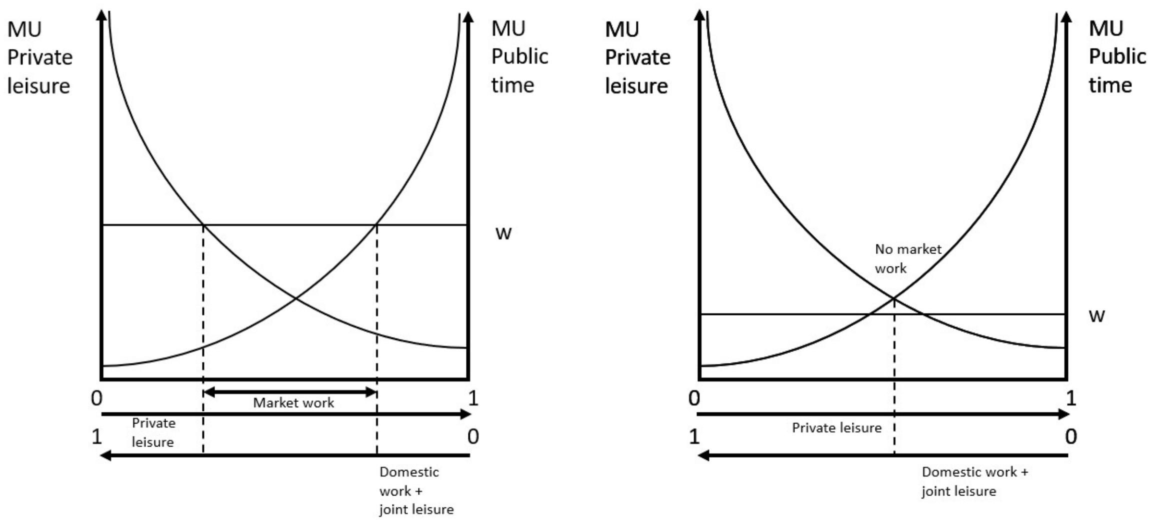


Figure 3: Interior solution vs. corner solution for time-use

In the UK, for couples without dependent children, men and women have very similar, and high, participation rates. Non-participation is due to reasonably exogenous drivers such as long-term illness, disability or temporary unemployment. Importantly, non-participation is not driven by a member having a particularly high, or low, resource share. Hence, excluding non-participating couples does not bias resource share estimates for participating couples.

For couples with dependent children, it would be much more problematic to exclude non-participating couples. This is because, in the UK, there is a substantial gap in participation between men and women with dependent (especially very young) children. It is realistic to think that part of this phenomenon is driven by women with very low potential wages, and low resource shares, taking on a full-time childcare and domestic work commitment, leaving them too little leisure time to additionally take on market work. In my application, I do not incur the latter difficulty because I focus only on couples without cohabiting children. While the having of children may itself be endogenous, it would require a very different, dynamic, model to take this into account appropriately, and difficulties would arise in modelling the preference changes associated with changes in household composition.

Finally, I note that, if using clothing as the assignable good, excluding households with zero expenditure on clothing may bias results because zero expenditure on clothing may be driven either by infrequency of purchase or, problematically, by actual zeros due to low resource shares.

# D Cobb-Douglas: additional material

## D.1 Cobb-Douglas Domestic production functions

Household economics, despite its emphasis on households, frequently does not involve modelling domestic production because it introduces identification issues. However, it is clear that domestic production does play an important role in understanding individual-level resources (see for instance Apps and Savage (1989)). One way to model domestic production within a collective model with relative ease is to assume that the domestic good is marketable - it can be bought and sold. This assumption has the advantage of leading to separability between the production and consumption functions of the household as the price of the domestic good is exogenously determined by the market, and not endogenously within the household (see e.g. Browning, Chiappori and Weiss (2014) section 4.6.2).

However, this assumption cannot easily be reconciled with empirical facts. If market work has a constant return of $w_{i,h}$ and domestic work has a constant return $w_{i,h}^{dm}$, then each individual would either supply market work or domestic work, and not both. This is in contrast with empirical evidence that a very substantial proportion of the population do both.

One way of trying to reconcile marketability and this empirical fact is if market work is constrained in terms of hours, a fact which is consistent with some of the literature on elasticity of labour supply. This assumption is sensible in some contexts, but is less likely realistic for low skilled work, where one is more likely to be able to ask to do overtime shifts, or have multiple jobs alongside each other to make ends meet. Moreover, this assumption would still not be reconcilable with the empirical fact that many people work part-time and also do domestic work. These people are clearly not constrained in terms of number of hours doing market work (especially as part-time hours are quite heterogeneous) and yet they do both types of work.

This is suggestive that we should not think of domestic work as having constant returns. If we wish to maintain the assumption of constant returns to market work, the most coherent way forward is to avoid modelling domestic work as marketable, as it would then become hard to justify decreasing returns to one type of marketable work and constant returns to another. Moreover, it is simply not realistic to model domestic work as being marketable. We can substitute between it and market purchased goods to some degree, but they are not the same good. Cleaning is perhaps the example where the boundary is most blurred, but making a home-cooked meal while looking after one's own child are clearly not the same as buying take-out food and sending the child to a nursery. Empirically, even people with high wages spend some time on domestic work - the reason is that it cannot be purchased, and it is desirable.

Therefore, we are left with the problem of modelling domestic production. Modelling domestic production is a complex task, and one that has historically not received nearly as much attention as aspects relating to market work. The literature which does exist has used a variety of production functions, depending on the aims of the analysis and the key features from the data of interest. For instance, Griffith et al. (2022) employ a Leontieff production function for home cooked food where the inputs are market purchased ingredients and domestic time. They further assume constant returns to domestic time, and perfect substitution between different household members' time (time spent cooking is just the sum of individual time spent cooking, and household leisure is just the sum of individual time spent on leisure).

For the purpose of this paper, it is instead important to model decreasing returns to domestic work to avoid the model being irreconcilable with the empirical facts described above. It is also important to model the domestic time of different members as not being perfect substitutes, again to explain patterns of behaviour with market and domestic work. I propose using a simple Cobb-Douglas production function. The productivity of each type of household member (and of the same types in different household categories) is heterogeneous. The concavity of the production function ensures that everyone in the household will do some domestic work.

An advantage of this approach is that the same functional form also seems appropriate to model non-private leisure. We want everyone in the household to have at least some time together (e.g. many households insist on a dinner with everyone, even those who are very busy), and there are higher returns to public leisure if others are able to join in. The Cobb-Douglas production function captures this concavity, and also allows us to model possible heterogeneity in the returns to joint leisure (e.g. the quality of joint leisure might be higher if children are involved, as it avoids the negative externalities of children being away from their parents). The model lends itself to greater granularity where relevant, for instance separating out leisure that is joint between strict subsets of household members.

However, for the purpose of this paper, consider the following production functions:

$D_h = \delta_g \prod_{i \epsilon h} (d_{i,h})^{\delta t,g}$

$T_h = \phi_g \prod_{i \epsilon h} (jt_{i,h})^{\phi t,g}$

The Cobb-Douglas production functions, substituted into the Cobb-Douglas utility functions, yield a Cobb-Douglas functional form, preserving the advantages of the Cobb-Douglas preferences:

$u_{t,g} = \sum_{j \epsilon \Omega^c} \left(\alpha_{t,g}^{cj} \ln(c_{i,h}^{j})\right) + \sum_{j \epsilon \Omega^X} \left(\alpha_{t,g}^{Xj} \ln(X_h^j)\right) + \alpha_{t,g}^{l} \ln(l_{i,h}) + \alpha_{t,g}^{T} \ln(T_h) + \alpha_{t,g}^{D} \ln(D_h)$

$$u_{t,g} = \sum_{j\epsilon\Omega^c} \left(\alpha_{t,g}^{cj} \ln(c_{i,h}^j)\right) + \sum_{j\epsilon\Omega^X} \left(\alpha_{t,g}^{Xj} \ln(X_h^j)\right) + \alpha_{t,g}^l \ln(l_{i,h})$$
$$+ \alpha_{t,g}^T \ln(\phi_g \prod_{s\epsilon h} (jt_{s,g})^{\phi st,g}) + \alpha_{t,g}^D \ln(\delta_g \prod_{s\epsilon h} (d_{s,g})^{\delta st,g})$$

The above can be re-written as a standard Cobb-Douglas utility function as a direct function of time-use:

$$u_{t,g} = \sum_{j\epsilon\Omega^c} \left(\alpha_{t,g}^{cj} \ln(c_{i,h}^j)\right) + \sum_{j\epsilon\Omega^X} \left(\alpha_{t,g}^{Xj} \ln(X_h^j)\right) + \alpha_{t,g}^l \ln(l_{i,h})$$
$$+ \left(\alpha_{t,g}^T \ln(\phi_g) + \alpha_{t,g}^D \ln(\delta_g)\right) + \alpha_{t,g}^T \sum_{s\epsilon h} \phi_{st,g} \ln jt_{s,g} + \alpha_{t,g}^D \sum_{s\epsilon h} \delta_{st,g} \ln d_{s,g}$$

We can work with this utility function and drop the production function constraints. We may wish to re-normalise the utility functions by setting the constant terms
$\left(\alpha_{t,g}^T \ln(\phi_g) + \alpha_{t,g}^D \ln(\delta_g)\right)$
to 0, and define some new notation to simplify the coefficients on domestic time and joint leisure:
$u_{t,g} = \sum_{j\epsilon\Omega^c} \left(\alpha_{t,g}^{cj} \ln(c_i^j)\right) + \sum_{j\epsilon\Omega^X} \left(\alpha_{t,g}^{Xj} \ln(X_h^j)\right) + \alpha_{t,g}^l \ln(l_i) + \sum_{s\epsilon h} \left(\alpha_{t,g}^{Tst,g} \ln jt_s\right) + \sum_{s\epsilon h} \left(\alpha_{t,g}^{Dst,g} \ln d_s\right)$

## D.2 Normalising assumptions

Observationally, household behaviour is equivalent up to (i) positive affine transformations of individual utility functions, and (ii) any positive monotonic function of the sum of the individual utilities weighted by their respective Pareto weights.[70] I normalise the model as follows:

- the constant term in the utility functions is set to zero (omitted in the equations above)
- the sum of each person's preference parameters over all goods (material and time-use, public and private) is set to one $\sum_j \alpha_{t,g}^j = 1$
- the Pareto weights are set to sum to one: $\sum_{i\in h} \mu_{i,h} = 1$

[70]Note that there is no uncertainty in this model so that the overall optimisation problem is unchanged by a positive monotonic transformation. The same cannot be said for the individual utilities because the household's optimisation problem is not to maximise a single utility but the weighted sum of all of the utilities, so that each individual utility can only be transformed up to positive affine transformations.

## D.3 Public good C-D SAP

### D.3.1 Under public good C-D SAP, the resource share and Pareto weight coincide

Note that $\rho_{i,h} = y_h \mu_{i,h} \left( \sum_{j \in \Omega^c} \alpha^j_{t,g} + \alpha^l_{t,g} \right)$. Also recall that:

$\eta_{i,h} = \frac{\rho_{i,h}}{\sum_{s \in h} \rho_{s,h}} = \frac{y_h \mu_{i,h} \left( \sum_{j \in \Omega^c} \alpha^j_{t,g} + \alpha^l_{t,g} \right)}{y_h \sum_{s \in h} \mu_{s,h} \left( \sum_{j \in \Omega^c} \alpha^j_{st,g} + \alpha^l_{st,g} \right)} = \frac{\mu_{i,h} \left( \sum_{j \in \Omega^c} \alpha^j_{t,g} + \alpha^l_{t,g} \right)}{\sum_{s \in h} \mu_{s,h} \left( \sum_{j \in \Omega^c} \alpha^j_{st,g} + \alpha^l_{st,g} \right)}$.

Now assume public good SAP so that $\left( \sum_{j \in \Omega^X} \alpha^j_{t,g} + \sum_{s \epsilon h} \alpha^{Tst,g}_{t,g} + \sum_{s \epsilon h} \alpha^{Dst,g}_{t,g} \right) = a^Q_g$

Equivalently, $\left( \sum_{j \in \Omega^c} \alpha^j_{t,g} + \alpha^l_{t,g} \right) = a^{c,l}_g$

Having made this assumption, we can write $\eta_{i,h} = \frac{\mu_{i,h} a^{c,l}_g}{a^{c,l}_g \sum_{s \in h} \mu_{s,h}}$. By definition, $\sum_{s \in h} \mu_{s,h} = 1$ and the $a^{c,l}_g$ terms cancel out, so that $\eta_{i,h} = \mu_{i,h}$.

In this case, finding that resource shares vary with market variables implies that Pareto weights do too, and hence is evidence in favour of the collective model over the unitary model.

### D.3.2 Under public good C-D SAP, second-stage demands do not depend on public expenditure

Note that $l_{i,h} w_{i,h} = \frac{\mu_{i,h} \alpha^l_{t,g}}{\sum_{s \in h} \mu_{s,h} \sum_j \alpha^j_{st,g}} y_h$, where $\sum_j \alpha^j_{st,g}$is the sum of preference coefficients over all different goods, and was normalised to 1, and $\sum_{s \in h} \mu_{s,h} = 1$ . Therefore, $l_{i,h} w_{i,h} = \alpha^l_{t,g} \mu_{i,h} y_{i,h}$. Without assuming public good C-D SAP, re-writing this equation in terms of resource shares, rather than Pareto weights, (this is necessary to be able to estimate the resource shares, which are our object of interest) requires including public expenditure $R_h Q_h$ as a term (or writing the latter in terms of bargaining fundamentals).

With public good C-D SAP, as discussed above, $\mu_{i,h} = \eta_{i,h}$ and hence we can write $l_{i,h} w_{i,h} = \alpha^l_{t,g} \eta_{i,h} y_h$. In this case, we are able to estimate resource shares from data that does not contain public good expenditure. Intuitively, this is driven by the assumption that changing bargaining power may affect how the household divides expenditure on specific public goods, but not on the aggregate public budget, so that we can ignore public good expenditure for the purpose of estimating resource shares.

## D.4 Structural approach to the error term

It is good practice to include sources of noise in the structural model (see for instance Reiss and Wolak (2007) for an excellent discussion of this point). I suggest doing this in two ways here.

Firstly, it is unlikely that we can control for all characteristics that affect bargaining within the household. Even if we have many characteristics in our data, there are likely unobserved factors. Moreover, recall that we are using a linear approximation of the Pareto weight, and hence there is likely an approximation error. We can write:

$w_{i,h}l_{i,h} = \alpha^l_{t,g} y_h \left(\eta^0_{t,g} + \sum_z \eta^z_{t,g} (z_h - \bar{z}) + e_{i,h}\right)$

Where by definition $e_{i,h}$ is mean-zero. As long as and we assume $e_{i,h}$ is independent of $y_h$ and $z_h$ (or, more weakly, that it is uncorrelated with $y_h, y_h^2, z_h, y_h z_h$) then $\alpha^l_{t,g} y_h e_{i,h}$ is uncorrelated with the other terms. Note that, by definition, the shocks sum to zero within each household: $\sum_{i\in h} e_{i,h} = 0$. Hence in a two-person household this implies a correlation of -1 between the errors of members of a couple.

Secondly, there may be some optimisation error $u_i, u^l_i$ so that:

$w_{i,h}l_{i,h} = \alpha^l_{t,g} \left(y_h \left(\eta^0_{t,g} + \sum_z \eta^z_{t,g} (z_h - \bar{z}) + e_{i,h}\right) + u_{i,h}\right) + u^l_{i,h}$

We can think of it in these terms:

1. In the first stage of the household problem, the household makes optimisation errors in the division of resources between public goods and individual budgets. This means that a member's individual budget will deviate by $u_{i,h}$ relative to the optimal budget. These errors needn't sum to zero across household members' individual budgets since there is also scope for error in public good expenditure. Note that $\sum u_{i,h} + u_{X,h} = 0$ so that the optimisation errors in the first stage must sum to zero. The $u_{i,h}$ errors will be negatively correlated across household members, with a correlation weakly smaller in magnitude than -1.

2. In the second stage of the household problem, each individual member makes optimisation errors in the division of their budget between private goods for their personal consumption. This means that each private expenditure may deviate from optimal by $u^j_{i,h}$ where $\sum_{j\in\Omega_c} u^j_{i,h} = 0$. Specifically, leisure expenditure may deviate from optimum by $u^l_{i,h}$. These errors could potentially be correlated across members, but there is no clear reason why they would be.

For both sources of optimisation error, as is standard, assume they are mean-zero and uncorrelated with each other and all other variables.

Taking these three sources of error, the final model can be written as:

$w_{i,h}l_{i,h} = \alpha^l_{t,g} y_h \left(\eta^0_{t,g} + \sum_z \eta^z_{t,g} (z_h - \bar{z})\right) + \left(\alpha^l_{t,g} y_h e_{i,h} + a^l_{t,g} u_{i,h} + u^l_{i,h}\right)$

Define $\epsilon_i = \alpha^l_{t,g} y_h e_{i,h} + a^l_{t,g} u_{i,h} + u^l_{i,h}$ so that $w_{i,h}l_{i,h} = \alpha^l_{t,g} y_h \left(\eta^0_{t,g} + \sum_z \eta^z_{t,g} (z_h - \bar{z})\right) + \epsilon_{i,h}$ where $\epsilon_{i,h}$ is mean-zero and uncorrelated with all other regressors. The errors are negatively correlated within household (with a correlation that is negative but smaller in magnitude than -1), so that a SURE estimation approach is recommended.

### D.5 Individual-level full consumption when expenditure data is separate from time-use data

Under Cobb-Douglas preferences, we can estimate time-inclusive individual-level consumption even when expenditure and time-use data are separate, as long as we observe hours worked $m_{i,h}$ and hourly wage $w_{i,h}$ in the expenditure data.[71] This is the case in many expenditure datasets which do not contain any other information relating to time-use. The procedure is simple:

- Recall that leisure expenditure is $w_{i,h}l_{i,h} = a^l y_h \eta_{i,h}$ and that we have estimated $\hat{\eta_{i,h}}$ and $\hat{\alpha^l}$ from the time-use data
- Hence we can estimate $l_{i,h}$ in the expenditure data as $\hat{l_{i,h}} = \frac{\hat{a^l} y_h \hat{\eta_{i,h}}}{w_{i,h}}$
- Then from the time feasibility constraint $l_{i,h} + m_{i,h} = 1 - (d_{i,h} + jt_{i,h})$ we can estimate public time-use as $(d_{i,h} + jt_{i,h}) = 1 - \left(\hat{l_{i,h}} + m_{i,h}\right)$
- We therefore obtain expenditure on public time-use $w_{i,h}\left(d_{i,h} + jt_{i,h}\right)$

## E Clothing as a public good

Under Cobb-Douglas with C-D public SAP, if clothing is private in the sense that the woman in the couple enjoys expenditure on female clothing and not on male clothing (and vice versa) then we can write:

$c^c_{f,h}p^c = \alpha^c_f \mu_f y_h$, or equivalently $c^c_{f,h}p^c = \alpha^c_f y_h - \alpha^c_f \mu_m y_h$

$c^c_{m,h}p^c = \alpha^c_m \mu_m y_h$, or equivalently $c^c_{m,h}p^c = \alpha^c_m y_h - \alpha^c_m \mu_f y_h$

We can regress clothing expenditure on estimated pareto weights (resource shares)[72] and observe the estimated preference parameters. If using estimates obtained by using clothing as the assignable good then the estimated coefficients have the correct sign (positive for the

[71] If the data has income from work $m_{i,h}w_{i,h}$ and hours worked $m_{i,h}$ then we can estimate $w_{i,h} = \frac{m_i w_{i,h}}{m_{i,h}}$. Where expenditure data only has higher level information, such as part-time or full-time work, employment status and sector, we can estimate hours worked based on those variables.

[72] Recall that under this specific model resource shares and Pareto weights are equivalent: $\mu_{i,h} = \eta_{i,h}$. Here I write everything in terms of Pareto weights as it leads to a more natural interpretation of the public expenditure equations.

first formulation, one positive and one negative for the second formulation). However, if using baseline estimates obtained by using private leisure as the assignable good, we find positive coefficients on all terms. This suggests that the assumption that clothing is private may be incorrect.

If clothing is public, then we can write:

$c^c_{f,h}p^c = \alpha^{c,f}_f \mu_f y_h + \alpha^{c,f}_m \mu_m y_h = \alpha^{c,f}_m y_h + (\alpha^{c,f}_f - \alpha^{c,f}_m)\mu_f y_h$

$c^c_{m,h}p^c = \alpha^{c,m}_f \mu_f y_h + \alpha^{c,m}_m \mu_m y_h = \alpha^{c,m}_m y_h + (\alpha^{c,m}_f - \alpha^{c,m}_m)\mu_f y_h$

The explanatory power of these regressions is substantially better than treating clothing as private. The signs of the estimated coefficients imply that people have stronger preferences for clothing expenditure on their partner than on themselves. This finding is consistent with the raw correlations between expenditure on clothing and female and male hourly wages; which are all positive, but stronger for opposite-sex clothing expenditure. This suggests that, at least for UK working couples without cohabiting children, clothing is not an appropriate assignable good as externalities of consumption appear not only strong, but even to outweigh the enjoyment from own-consumption.